The Bearing Capacity of Sands under Deep Foundations

Capacité Portante des Fondations Profondes dans le Sable

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Summary

Investigations on the stability of sands under deep and shallow foundations have been carried out in the Transport Construction Scientific Research Institute and Leningrad Institute of Railway Engineers.

In the first section the results of the experimental tests on the character of deformation and stability of dense and loose sands are discussed. One may observe the pushing out of the dense sand caused by surface and shallow foundations. In loose sands the foundation of any depth ratio causes interaction of the sliding zone and compacting zone above or below the base of the footing. The same process occurs in dense sands under deep foundations.

The bearing capacity of footings increases considerably with the increasing depth ratio and density of sand.

In the second section of the report, on the basis of experimental investigation, a new theoretical solution for determining the ultimate bearing capacity of deep foundations has been found. The methods for continuous and circular footings were found on the basis of the limit equilibrium theory.

Theoretical values of bearing capacity coincide satisfactorily with the data of the experimental tests.

Sommaire

Les recherchés de la stabilité des sables sous les fondations profondes et directes furent exécutées au Centre de Recherche du Ministère de Construction des chemins de fer à Moscou et à l'Institut des Ingénieurs des chemins de fer à Léningrade.

La première partie du mémoire expose les résultats des essais, consacrés à l'étude des déformations et de la stabilité des sables denses et légers.

Dans le cas de fondation directe dans les sables denses on observe un soulèvement du sol. Dans les sables friables, les fondations à n'importe quelle profondeur provoquent l'action réciproque des zones de glissement et des zones de tassement du sol au-dessus ou au-dessous de la base de fondation. Le même effet a lieu dans les sables denses sous les fondations profondes.

Avec l'augmentation de profondeur et de densité des sables, la stabilité du sol s'accroît considérablement.

Les résultats des recherches expérimentales sont utilisés pour la création des méthodes théoriques de calcul de la stabilité du sol sous les fondations profondes. Ces résultats sont exposés dans la seconde partie du mémoire.

Les solutions théoriques sont données pour les fondations continues ou circulaires, en partant de la théorie sur l'équilibre limite du sol.

Les evaluations théoriques de la capacité portante coïncident assez bien avec les résultats des essais.

Experimental Investigation of the Bearing Capacity of Sands

Experimental procedure of the tests—The deformation of sands was investigated in boxes with glass walls by means of still and ciné photography, which enabled observation of the trajectories of particles, and of the successive forming of a compacted core and sliding surfaces to be made.

The ultimate bearing capacity of sands was investigated under centrally loaded strip and circular foundations of different sizes. The foundations were tested *in situ* (cand. techn. I. F. Razorenov) by means of the centrifuge (cand. techn. A. G. Prokopovitch) and in the laboratory.

During the tests measurements were taken of the upheaval of the ground surface as well as displacements to one side and settlements of foundations.

Assistant professor N. N. Sidorov established the correlation of the angle of internal friction with the density by means of shearing and triaxial tests.

Results of experimental investigations—At the commencement of the vertical settlement of a foundation (at any values of soil density and depth ratio D/B), the trajectories of displacement of the sand particles are continuous (except for small areas near the very edge of the foundation), and are directed downwards with small deviation from the vertical axis. The zone of displacement of particles (zone of compaction) spreads to a great depth. For equal settlements of a foundation, the zone of compaction grows with the increasing sand density. The foundation settles smoothly without displacement to one side, or upheaval of the ground surface (see a–b on the curve $s = f(\sigma)$ and a'–b' on the curve $s = f'(\sigma)$, Fig. 2).

The first phase of deformation is thus chiefly a phase of compaction: there is no sudden increase of the settlement or lateral movement that might be dangerous to the structure. Further settlement of a foundation on dense sand, as well as on loose sand, is attended by the formation of a compacted core that is

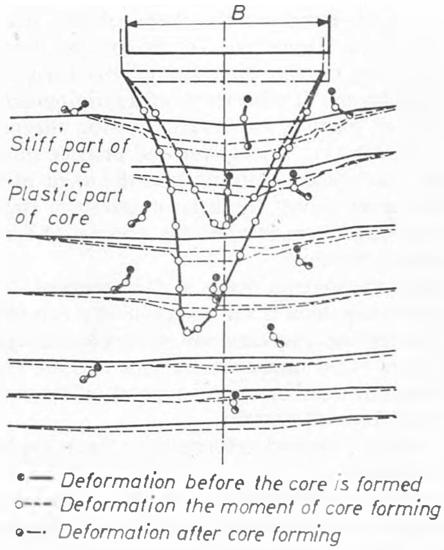


Fig. 1

separated by the rupture surfaces. The compacted core causes the displacement of soil along the slip surfaces which either reach the ground surface or terminate in the thickness of the soil, depending on the depth ratio and soil density (Figs. 2 and 3). The settlement of the foundation then increases suddenly and when the failure surface is observed at ground level the foundation slides to one side (see b–c on the curve $s = f(\sigma)$ and b'–c' on the curve $s = f'(\sigma)$, Fig. 2). Therefore the phase of deformation of sands after the compacted core has been formed is the one of sliding, which is accompanied by a different degree of compaction. There are significant and rapidly increasing settlements that might be dangerous for the structure.

The compacted core formed in the process of transition from the phase of compaction to the phase of sliding consists of two parts; 'stiff' and 'plastic' (Fig. 1). The stiff part of the core moves as a part of the foundation. Its contour is symmetrical and has almost a triangular form with the top angle of 85 degrees for a surface foundation on dense sand to 60 degrees for a deep foundation on loose sand. The magnitude and direction of the displacements of the sand particles in the 'plastic' part of the core differ from the magnitude and direction of the displacement of the foundation. The distinguishing feature of this part of the core is that it is not symmetrical about the axis of foundation; the sliding surfaces are immediate continuations of the lower part of the core (Fig. 1).

angle $\alpha > 45$ degrees $- \phi/2$ (Fig. 2). It is observed with shallow foundations on dense sand ($0{\cdot}5 \leqslant D_f/B \leqslant 1$, at $B > 1$ m) and surface foundations on medium sand.

Case 2a Deep displacements of large volumes of soils occur adjacent to the compacted core, accompanied by the compaction of the overburden (Fig. 3).

The settlement of the foundation at the moment of failure increases rapidly, sometimes without increase of the bearing capacity. Considerable sideways displacement of the foundation is not observed. On further loading of the foundation the diagram $s = f(\sigma)$ becomes almost linear (Fig. 3). Case 2a is observed with shallow foundations on dense and medium sands ($1 \leqslant D_f/B \leqslant 3$ by $B > 1$ m).

In Case 2b deep displacements occur of small volumes of soil directly adjacent to the compacted core, accompanied by the compaction of sand mainly below the base. At the moment when the process of forming of the compacted core is finished (at a 'failure') the settlement of the foundation and the bearing capacity are increasing. On further loading of the foundation the diagram $s = f(\sigma)$ becomes parabolic. There are no lateral displacements of the foundations. Case 2b is observed with

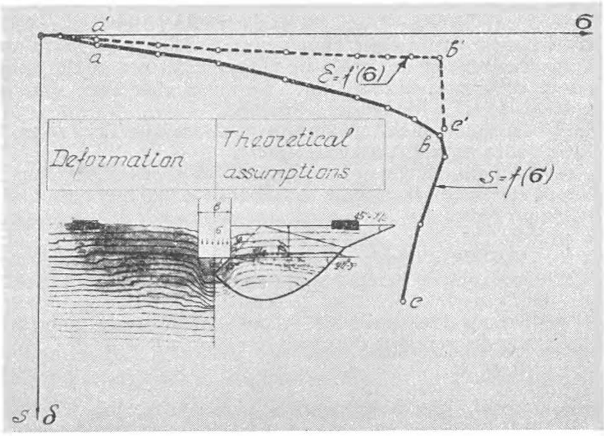


Fig. 2

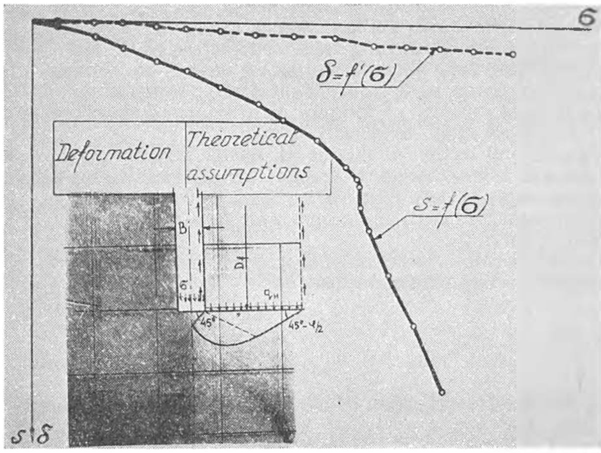


Fig. 3

In the process of settlement the shape of the 'plastic' part of the core and the disposition of its top are permanently changed, depending on the direction of the least resistance. The core causes lateral displacement of adjoining sand zones; the trajectories of particle movement outside the core get a distinct turn (see Fig. 1). Increasing the density of sand and decreasing the overburden both increase the angle of turning.

Experimental data permit the determination of the moment of sand failure, which begins when the process of forming the compacted core is finished.

The following phenomena occur at this moment:

(1) In a case where there is soil upheaval one can observe the beginning of general soil displacement along the sliding surfaces;

(2) When there is no upheaval one can observe the intense increase in foundation settlement on account of the sliding zone and compaction zone interaction.

According to the observed deformations there are four main cases of sand failure.

Case 1a is characterized by displacement of the soil adjacent to the compacted core along the well-defined sliding surfaces that reach ground level at an angle $\alpha = 45$ degrees $- \phi/2$. The foundation fails by sliding to one side, with a sudden increase of the settlement and decrease of the bearing capacity. This case is observed only with surface foundations on dense sand.

Case 1b generally is similar to Case 1a with the only difference that the rupture surface and the ground level intersect at an

deep foundations on dense and medium sands ($D_fB > 3$) and even with surface foundations on loose sand.

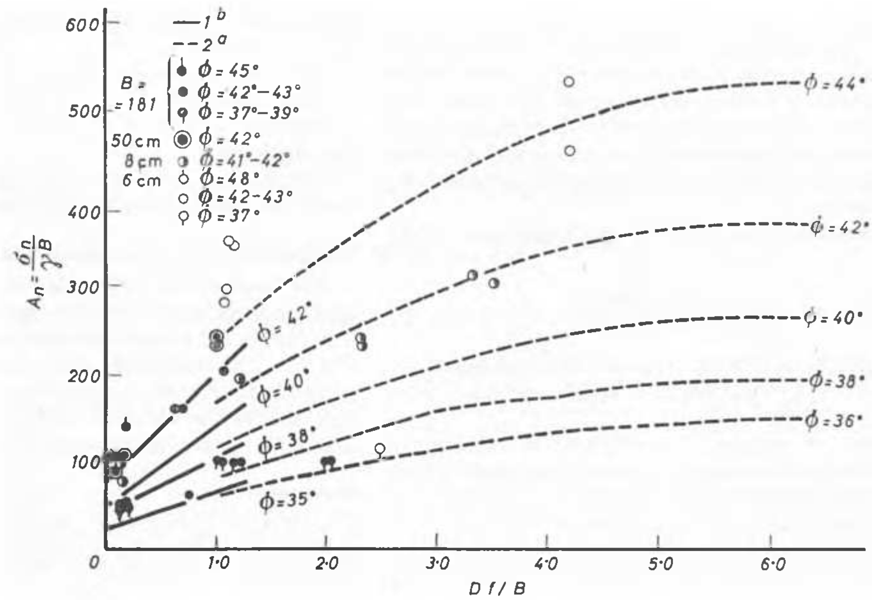
The experimental results confirm that the compressibility of sand increases with increasing size of the foundation, for the same soil density and depth ratio D_f/B, and a failure with a well-defined rupture surface and an upheaval of the ground is observed only with surface and shallow foundations on dense sand. On account of the production of an extended shear zone and a large compaction zone the period of deformation becomes longer. The curve $s = f(\sigma)$ becomes more smooth without a distinct indication of having reached the ultimate bearing capacity (the failure). The same phenomenon is observed with circular and square foundations.

Theoretical Investigations of Bearing Capacity of Sands

Different theoretical diagrams correspond to different cases of reaching the ultimate bearing capacity of sands as shown in the tests.

Case 1a—This shows the possibility of using the limit equilibrium theory for the strip and circular foundations [1] and [2]. Changes of boundary conditions that take place as a result of the formation of a compacted core are to be considered when dealing with central loading. Theoretical determination of the core shape leads to an investigation of a complex state of stress. It is therefore necessary to take an approximate shape of core, based on experimental data, and to consider the core surface

Fig. 4



as the surface of a rough retaining wall. In solving the equation for Case 1a the compacted core was taken as a three-sided prism for a strip foundation, and as a cone for a circular foundation. The core has as its vertical section a right angled isosceles triangle.

The approximate shape of slip lines spreading from the tip of the core were outlined by means of the analysis of slip lines obtained by numerical step-by-step computation (V. V. Sokolovsky); slip lines were composed of a logarithmic spiral and a straight line, meeting the ground level at an angle 45 degrees $- \phi/2$.

The ultimate bearing capacity can be expressed by equations:

For a strip foundation

$$\sigma_n = A_{n_0} . \gamma . B \qquad \ldots . \ (1)$$

For a circular foundation

$$\sigma_K = A_{K_0} . \gamma . a \qquad \ldots . \ (2)$$

where γ = unit weight of the ground; B = width of strip foundation; a = radius of circular foundation; A_{n_0} and A_{K_0} = surface bearing capacity factors dependent on ϕ (see Fig. 4, where $D_f/B = 0$).

Equation 1 is to be used for a rectangular foundation of length/width ratio greater than 3·0; while equation 2 may be used approximately for a square foundation with side 2a, [2].

Case 1b—This case required a special theoretical analysis, for strip and circular foundations, because the overburden changes the shape of the shear pattern.

On account of the principal stress acting in the ground at a base level not being vertical the assumption that the overburden at the base level of a footing at a depth D_f is equivalent to a uniform surcharge $\gamma . D_f$ is not valid (see Fig. 2).

The angle of stress declination from the vertical was defined by the requirement that there should be uniform conjunction of the slip lines at a base level. This angle is equal to ϕ.

Thus finding the bearing capacity of deep foundation in the case when ground upheaval occurred resulted in solving two new problems of limit equilibrium theory, e.g. (1) the problem of equilibrium overburden D_f under a load acting upward at an angle ϕ to the vertical, and (2) the problem of equilibrium of the footing when the uniform surcharge is inclined.

Solving the first problem proved that in the overburden a limited tension is connected with the formation of an inclined rupture line below which straight slip lines are inclined to the vertical at an angle ϕ. Above this line the straight slip lines were inclined to the vertical at an angle 45 degrees $+ \phi/2$.

The second problem may be solved exactly by numerical step-by-step computation (Sokolovsky's method). An approximate solution is obtained by the substitution of logarithmic spirals for the exact slip lines. The parameters of these spirals are specially chosen. In this case the bearing capacity is found to be:

For a strip foundation

$$\sigma'_n = A_n . \gamma . B \qquad \ldots . \ (3)$$

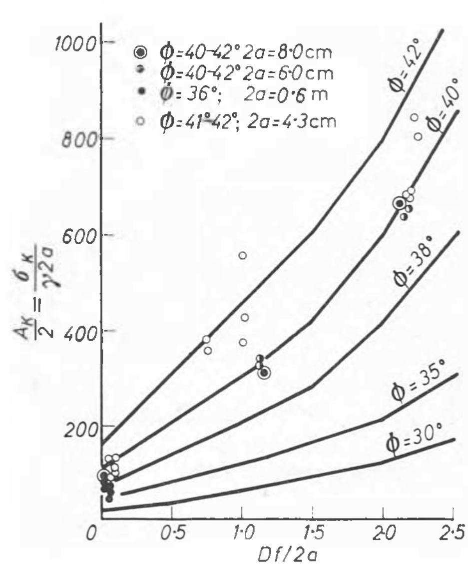


Fig. 5

For a circular foundation

$$\sigma'_K = A_K . \gamma . A \qquad \ldots . \ (4)$$

In equations 3 and 4 A_n and A_K are the resultant bearing capacity factors depending on the foundation depth/width (D_f/B and $D_f/2a$) as well as ϕ (see Fig. 4).

Case 2a—Here expanding shear zones gradually reach the base level of the footing and compact the overburden of depth D_f. In the process of compaction the boundaries of the shear zones move upward in the overburden. Thus the deformation of the overburden is the failure of a medium which has undergone compaction.

On account of mathematical difficulties an analysis of this compaction process is not yet available and the problem can be solved at present only by determining the limits of pressure that make the overburden compact. The minimum pressure on the base that causes intensive settlement of foundation is determined as bearing capacity for Case 1a by assuming the surcharge q_H acts at the base level.

For a strip foundation the uniform surcharge can be expressed by:

$$q_H = \gamma D_f\left(1 - \frac{D_f}{B}.\frac{\xi \tan \phi}{K}\right) \quad \ldots. (5)$$

where ξ = the factor of lateral pressure (average value for sand $-$ 0·4); $K = l/B$; l = the length of failure surface at base level.

The second term of equation 5 represents the influence of friction between the overburden and the remaining part of soil, as well as on the lateral surface of foundation.

Thus, the equation for a strip foundation is:

$$\sigma''_n = A_{n_0}.\gamma.B + B_n.q_H = A'_n.\gamma B \quad \ldots. (6)$$

The values of resultant bearing capacity factor A'_n, which depends on foundation depth/width as well as ϕ, are shown in Fig. 4 (dotted line).

The ultimate bearing capacity in Case 2a is equivalent to the bearing capacity in Case 1b.

The bearing capacity in Case 2b is determined only by foundation settlement.

In this case it is necessary to solve the mixed problem of the theory of limit equilibrium and theory of compaction.

Comparison of the Theoretical and Experimental Results

The theoretical and experimental data for surface and shallow strip foundation are shown in Fig. 4 (1a, 1b and 2a). The results for circular foundations (1a and 1b) are shown in Fig. 5. The theoretical data were found to be in reasonable agreement with the experiments. According to theory and experiments the ultimate bearing capacity increases considerably with increasing depth ratio and density of sand.

References

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