Calculation of the Distribution of Soil Reactions Underneath Eccentrically Loaded Footings

Le Calcul de la Répartition des Réactions du Sol sous des Fondations Chargées Excentriquement

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Summary

The aim of the present contribution is to complete the study of the distribution of the reactions under a beam resting on soil characterized by a constant modulus of elasticity.

Two methods are proposed for the use of electrical or electronic calculating machines to solve systems of a great number of equations with many unknown quantities—the method of steps for beams of infinite rigidity and the analytical method for beams with any rigidity—which impose the concordance between beam and soil in a great number of points.

Sommaire

La présente contribution a pour but de compléter l'étude de la distribution des réactions sous une poutre reposant sur un sol caractérisé par un module d'élasticité constant.

L'emploi de machines électriques ou électroniques permettant de résoudre des systèmes d'un grand nombre d'équations à multiples inconnues, nous proposons deux méthodes — la méthode des gradins pour des poutres de raideur infinie, et une méthode analytique pour des poutres de raideur quelconque — imposant la concordance sol-poutre en un grand nombre de points.

Introduction

The problem of the distribution of reactions underneath beams resting on soil has not yet been rigorously treated.

The case of beams, eccentrically loaded, has been treated with great approximation by DE BEER and KRSMANOVIC (1951, 1952). It is proposed to complete this work by the study of an 'approached' analytical solution, i.e. realizing the concordance between soil and raft on a limited number of points, in the case of a beam (i.e. the width short with regard to the length and the transverse distribution of the reactions supposed uniform)

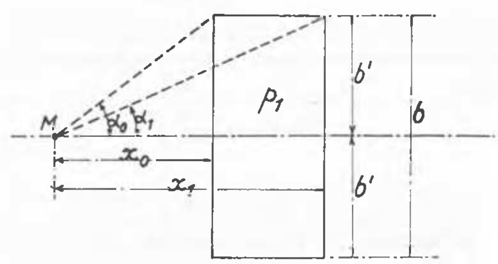

Fig. 1

resting on a soil characterized by a modulus of elasticity constant with depth (E_s = constant) (DE BEER, LOUSBERG and VAN BEVEREN, 1956).

The expression of the settlement of a point M (Fig. 1) located on the axis of a rectangle, uniformly loaded, has been obtained by Sleicher after integration of the law of Boussinesq:

$$s_M = \frac{2p_1}{\pi E_s}\left\{\left[x_1 \log_e \operatorname{cotan}\left(\frac{\pi}{4} - \frac{\alpha_1}{2}\right) + b' \log_e \operatorname{cotan}\frac{\alpha_1}{2}\right] - \left[x_0 \log_e \operatorname{cotan}\left(\frac{\pi}{4} - \frac{\alpha_0}{2}\right) + b' \log_e \operatorname{cotan}\frac{\alpha_0}{2}\right]\right\} \quad \ldots\ldots (1)$$

with

$$\tan\alpha = \frac{b'}{x} \quad \ldots\ldots (2)$$

Case of Beams with any Rigidity—Analytical Method

In this case the curve of the distribution of the reactions may be assimilated with a parabolical law of even degree n (Fig. 2):

$$p = f(x) = \rho\,.\,p_m = p_m \sum_{i=0}^{i=n} a_i\left(\frac{x}{l'}\right)^i = p_m \sum_{i=0}^{i=n} a_i k^i \quad \ldots\ldots (3)$$

if one puts

$$k = x/l' \quad \ldots\ldots (4)$$

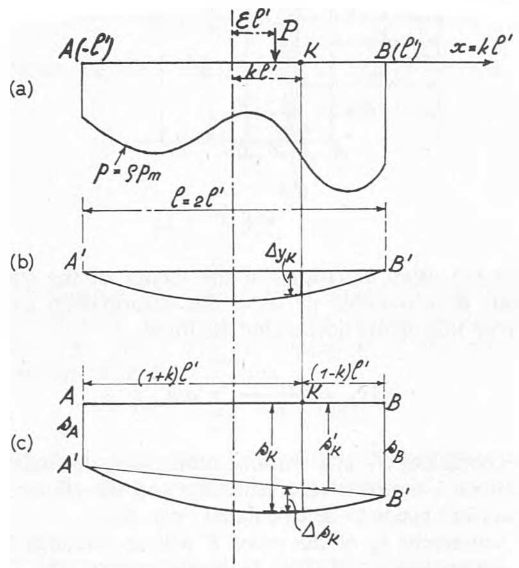

Fig. 2

Under the eccentric load P and the reactions p, the beam AB will be deformed; let Δy_K be the displacement of the point K (abscissa kl') along $A'B'$ (Fig. 2b), the point K, supposedly belonging to the soil, will settle a quantity s_K with regard to the starting line AB. One may put (Fig. 2c)

$$\Delta s_K = s_K - s'_K \quad \ldots\ldots (5)$$

with s'_K given as a function of s_A and s_B by

$$s'_K = \tfrac{1}{2}[s_B(1 + k) + s_A(1 - k)] \quad \ldots\ldots (6)$$

In equation 3, the $n + 1$ coefficients a_i are unknown quantities. The $n + 1$ equations which will permit solution of the system are:

The expression of the vertical equilibrium

$$\frac{P}{2bl'} = p_m \sum_{j'=0}^{j'=n/2} \frac{a_{2j'}}{2j' + 1} \qquad \ldots. \ (7)$$

or, with

$$p_m = \frac{P}{2bl'} \qquad \ldots. \ (8)$$

$$1 = \sum_{j'=0}^{j'=n/2} \frac{a_{2j'}}{2j' + 1} \qquad \ldots. \ (9)$$

the expression of the equilibrium of rotation

$$\epsilon = \sum_{j'=0}^{j'=n/2-1} \frac{a_{2j'+1}}{2j' + 3} \qquad \ldots. \ (10)$$

with expression 8 and

$$\epsilon = e/l' \qquad \ldots. \ (11)$$

the concordance of the deformations of the soil-beam at $n - 1$ points other than A and B. We write at $n - 1$ arbitrary points K

$$\Delta y_K = \Delta s_K = s_K - s'_K \qquad \ldots. \ (12)$$

The expressions Δy_K and s_K must be investigated.

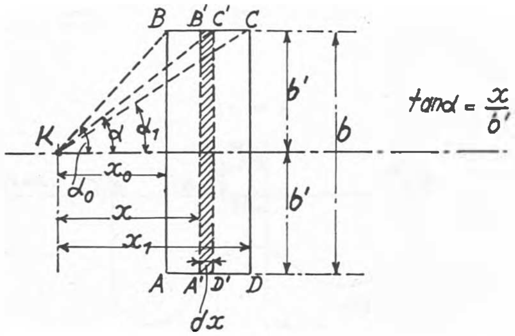

Fig. 3

From the usual equations of the theory of the strength of materials it is possible to write the deformation Δy_K. One may write this expression under the form

$$\Delta y_K = \frac{p_m b l'^4}{EI} \sum_{i=0}^{i=n} a_i Z_{i,\,k,\,\epsilon} \qquad \ldots. \ (13)$$

The coefficient $Z_{i,\,k,\,\epsilon}$, without dimension, depends only on the number i, the relative eccentricity and the chosen point K and therefore needs to be calculated only once.

The settlement s_K of the point K will be obtained from the law in expression 1. If (Fig. 3) instead of applying a uniform load p to the rectangle $ABCD$, we load the elementary rectangle $A'B'C'D'$, the elementary settlement is written as follows:

$$ds_K = \frac{p}{\pi E_s} F(x) dx \qquad \ldots. \ (14)$$

with

$$F(x) = 2 \log_e \cot\mathrm{an}\left(\frac{\pi}{4} - \frac{\alpha}{2}\right) \qquad \ldots. \ (15)$$

Now, if the law $p = f(x)$ is valid in the interval (x_0, x_1), the settlement becomes:

$$s_K = \frac{1}{\pi E_s} \int_{x_0}^{x_1} f(x) . F(x) dx \qquad \ldots. \ (16)$$

With $p = f(x)$ given by law 3, the integration, 16, is laborious and may be written:

$$s_K = \frac{l' p_m}{\pi E_s} \sum_{i=0}^{i=n} a_i t_k^{(i)} \qquad \ldots. \ (17)$$

with

$$t_k^{(i)} = \sum_{j=0}^{j=i} C_i^j k^{i-j} M_j \qquad \ldots. \ (18)$$

$$C_i^j = \frac{i(i-1) \ldots (i-j+1)}{j!} \qquad \ldots. \ (19)$$

$$M_{j=2m} = \frac{2}{2m+1}[(1-k)^{2m+1} D_{1-k} + (1+k)^{2m+1} D_{1+k} - \delta^{2m+1} \frac{(-1)^m}{2m} \sum_{p=0}^{p=m} (-1)^p C_m^p (T_{1-k}^{(2p)} + T_{1+k}^{(2p)})] \ \ldots \ (20)$$

$$M_{j=2m+1} = \frac{2}{2m+2}[(1-k)^{2m+2} D_{1-k} - (1+k)^{2m+2} D_{1+k} + \delta^{2m+2} (-1)^m \sum_{p=0}^{p=m} (-1)^p C_m^p (S_{1-k}^{(2p+1)} - S_{1+k}^{(2p+1)})] \ .. \ (21)$$

In the expressions for M, the following abbreviations are used:

$$\delta = \frac{b}{l} = \frac{b'}{l'} \qquad \ldots. \ (22)$$

$$D = \log_e \cot\mathrm{an}\left(\frac{\pi}{4} - \frac{\alpha}{2}\right) \qquad \ldots. \ (23)$$

$$T^{(2p)} = \begin{cases} \log_e \tan\frac{\alpha}{2} \ (\text{when } 2p = 0) & \ldots. \ (24) \\ \frac{1}{2p}\left(\tan^{2p}\frac{\alpha}{2} - \cot\mathrm{an}^{2p}\frac{\alpha}{2}\right) \ (\text{when } 2p \neq 0) & \ldots. \ (25) \end{cases}$$

$$S^{(2p+1)} = \frac{1}{(2p+1)\sin^{2p+1}\alpha} \qquad \ldots. \ (26)$$

The suffixes $(1 - k)$ and $(1 + k)$ of the letters D, T and S indicate that the angle α must be written respectively α_1 and α_2 defined by

$$\tan\alpha_1 = \frac{\delta}{1-k} \qquad \ldots. \ (27)$$

$$\tan\alpha_2 = \frac{\delta}{1+k} \qquad \ldots. \ (28)$$

The coefficients $t_k^{(i)}$, 17, are constant for a given value of $\delta = \frac{b}{l}$, a chosen point K on the axis of the beam and the number i of the considered term of law 3.

Example—We consider a beam with $l = 6{\cdot}00$ m, $b = 1{\cdot}50$ m charged by a load of 100 tons with an eccentricity $e = 0{\cdot}50$ m resting on a soil $E_s = 1000$ kg/cm². We suppose three rigidities ($I = 2{\cdot}5 . 10^6$ cm⁴, $I = 5{\cdot}10^6$ cm⁴ and $I = \infty$):

We have from expressions 8 and 11:

$$p_m = \frac{P}{2bl'} = 1{\cdot}11 \text{ kg/cm}^2 \qquad \ldots. \ (29)$$

$$\text{and} \quad \epsilon = \frac{e}{l'} = \frac{1}{6} \qquad \ldots. \ (30)$$

The curve of reactions is assimilated with a parabolic law, 3, of the fourth degree:

$$p = p_m(a_0 + a_1 k + a_2 k^2 + a_3 k^3 + a_4 k^4) \quad \ldots. \ (31)$$

The five unknown quantities $a_0, \ldots a_4$ are determined from equations 9, 10 and 12:

The vertical equilibrium

$$1 = a_0 + \frac{a_2}{3} + \frac{a_4}{5} \qquad \ldots. \ (32)$$

The equilibrium of rotation

$$\epsilon = \frac{1}{6} = \frac{a_1}{3} + \frac{a_3}{5} \qquad \ldots . (33)$$

The concordance of soil and beam in three points

$C(k = 0)$, $\quad K_1(k = 0{\cdot}7)$ and $\quad K_2(k = -0{\cdot}7)$ (Fig. 4)

$$y_C = \frac{bl'^4 p_m}{EI}(0{\cdot}13735a_0 - 0{\cdot}05093a_1 + 0{\cdot}07634a_2 - 0{\cdot}03056a_3 + 0{\cdot}05306a_4)$$

$$= \Delta s_C = s_C - s'_C$$

$$= \frac{l'p_m}{\pi E_s}(1{\cdot}19758a_0 - 0{\cdot}51330a_2 - 0{\cdot}49340a_4) \qquad \ldots . (34)$$

$$y_{K_1} = \frac{bl'^4 p_m}{EI}(0{\cdot}05429a_0 - 0{\cdot}00837a_1 + 0{\cdot}03124a_2 - 0{\cdot}00397a_3 + 0{\cdot}02209a_4)$$

$$= \Delta_{sK_1} = s_{K_1} - s'_{K_1}$$

$$= \frac{l'p_m}{\pi E_s}(0{\cdot}88202a_0 + 0{\cdot}58834a_1 + 0{\cdot}12945a_2 + 0{\cdot}16948a_3 - 0{\cdot}09968a_4) \qquad \ldots . (35)$$

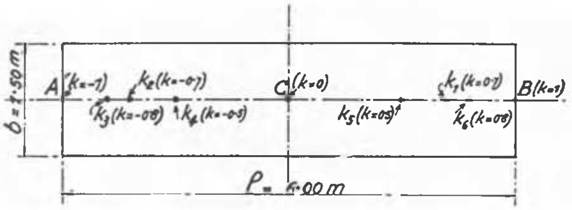

Fig. 4

and

$$y_{K_2} = \frac{bl'^4 p_m}{EI}(0{\cdot}05337a_0 - 0{\cdot}02496a_1 + 0{\cdot}03094a_2 - 0{\cdot}01603a_3 + 0{\cdot}02190a_4)$$

$$= \Delta s_{K_2} = s_{K_2} - s'_{K_2}$$

$$= \frac{l'p_m}{\pi E_s}(0{\cdot}88202a_0 - 0{\cdot}58834a_1 + 0{\cdot}12945a_2 - 0{\cdot}16948a_3 - 0{\cdot}09968a_4) \qquad \ldots . (36)$$

The solution of equations 32 to 36 gives the values of $a_0 \ldots a_4$. In Table 1 we have written for several rigidities the values of the reactions p_A, p_C and p_B, the settlements s_A, s_C and s_B, the angle β and the error of concordance defined by:

$$e_K = \frac{\Delta s_K - y_K}{y_K} \qquad \ldots . (37)$$

at the points $K_3(k = -0{\cdot}8)$; $K_4(k = -0{\cdot}5)$; $K_5(k = 0{\cdot}5)$; and $K_6(k = 0{\cdot}8)$ (Fig. 4).

In Fig. 5 we have drawn these results for the chosen rigidities. Curve 1 represents the reactions with the law of the fourth degree, curve 2 the reactions obtained from the method of trial (De Beer, 1951, 1952), curves 3 and 4 respectively the settlements of the soil and the deformation of the beam.

Case of Beams with Infinite Rigidity ($I = \infty$)—Method of Steps

For this case another method is proposed. We divide the beam into $2n$ equal sections (Fig. 6).

The law of distribution of the reaction p is replaced by $2n$ steps with $p_{-n}, \ldots, p_{-1}, p_1, \ldots, p_n$ (Fig. 6a). Under the load P, the soil will settle and the beam AB will take the position $A'B'$ characterized by a medium settlement S_m and by an angle β.

There are $2n + 2$ unknown quantities ($p_1 \ldots p_n, p_{-1} \ldots p_n$, s_m and β). The $2n + 2$ equations which will permit solution of the system are:

The vertical equilibrium

$$P = \frac{2bl}{2n}\sum_{j=1}^{j=n}(p_j + p_{-j}) \qquad \ldots . (38)$$

the equilibrium of rotation

$$P.e = \frac{bl'^2}{n^2}\sum_{j=1}^{j=n}(j - \tfrac{1}{2})(p_j - p_{-j}) \qquad \ldots . (39)$$

If one puts

$$\rho_j = \frac{p_j}{p_m} \qquad \ldots . (40)$$

$$\rho_{-j} = \frac{p_{-j}}{p_m} \qquad \ldots . (41)$$

$$u_j = (\rho_j + \rho_{-j}) \qquad \ldots . (42)$$

$$v_j = (\rho_j - \rho_{-j}) \qquad \ldots . (43)$$

with expression 8 and 11, the relations 38 and 39 become

$$1 = \frac{1}{n}\sum_{j=1}^{j=n}\mu_j \qquad \ldots . (44)$$

$$\epsilon = \frac{1}{n^2}\sum_{j=1}^{j=n}\nu_j(j - \tfrac{1}{2}) \qquad \ldots . (45)$$

the concordance of the deformation of soil and beam on $2n$ points of the axis, by occurrence the centres M_j of the $2n$ rectangles (Fig. 6b). The settlement s_j of the point M_j will be written (Fig. 6c)

$$s_j = s_m + (j - \tfrac{1}{2})\frac{l'}{n}\tan\beta \qquad \ldots . (46)$$

This settlement is the sum of the settlements resulting by the loads $p_{-n} \ldots p_{-1}\, p_1 \ldots p_n$ on all the rectangles of Fig. 6b.

Table 1

$b = 1{\cdot}50$ m $\quad l = 6{\cdot}00$ m $\quad \delta = b/l = 1/4 \quad e = 0{\cdot}50$ m $\quad \epsilon = 1/6 \quad p = 100$ t $\quad p_m = 1{\cdot}11$ kg/cm² $\quad E_s = 1000$ kg/cm²

I	p_A	p_C	p_B	s_A	s_C	s_B	β	e_{K_3} $(k = -0{\cdot}8)$	e_{K_4} $(k = -0{\cdot}5)$	e_{K_5} $(k = 0{\cdot}5)$	e_{K_6} $(k = 0{\cdot}8)$
cm⁴	kg/cm²	kg/cm²	kg/cm²	cm	cm	cm	degree	%	%	%	%
∞	2·18	0·97	4·71	0·2117	0·2847	0·3577	0° 0′ 50″	∞	∞	∞	∞
5×10^6	0·59	1·37	2·76	0·1430	0·3565	0·2774	0° 0′ 46″	− 2·6	13·0	− 29·1	53·2
$2{\cdot}5 \times 10^6$	(− 0·15)	1·59	1·71	0·1096	0·3954	0·2388	0° 0′ 44″	− 16·0	14·1	− 13·2	21·0

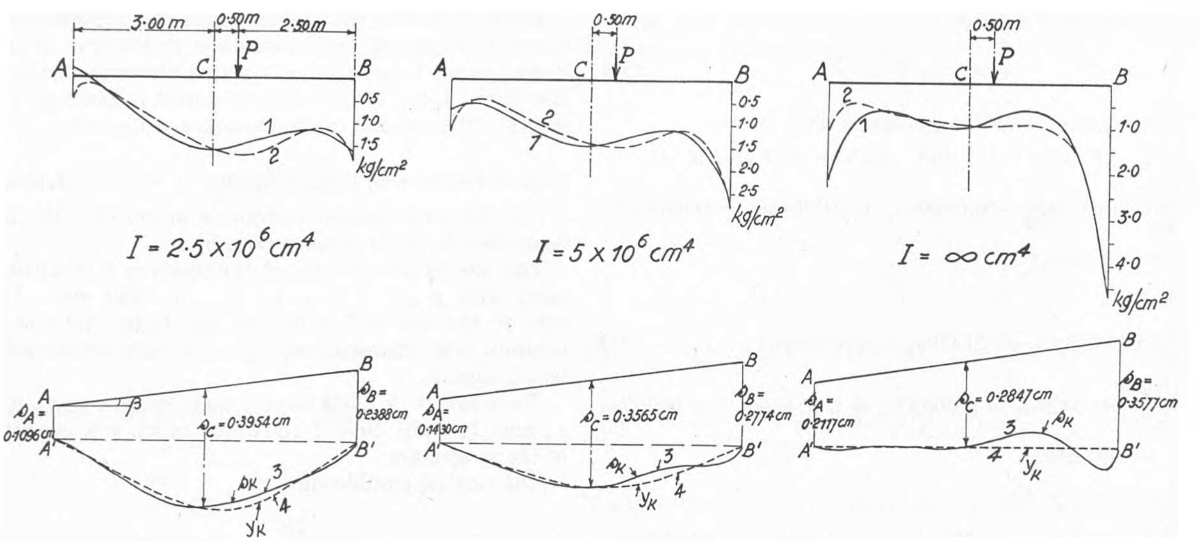

Fig. 5 Curve 1, distribution of the reactions with the analytical method; curve 2, distribution of the reactions with the method of trial; curve 3, settlement of the soil; curve 4, deformation of the beam

Courbe 1, Répartition des réactions par la méthode analytique; courbe 2, répartitions des réactions par la méthode par tâtonnements, courbe 3, tassement du sol; courbe 4, déformée de la poutre

By means of expression 8, 22, 40 and 41 one has

$$s_j = \frac{l p_m}{\pi E_s}\left[\sum_{i=1}^{i=n-j+1} \rho_{j+i-1} T_i + \sum_{k=1}^{k=j} \rho_{j-k+1} T_k + \sum_{k=j+1}^{k=n+j} \rho_{-(k-j)} T\right] \quad \ldots. \ (47)$$

with

$$T_1 = \frac{1}{2n}\left[\log_e \operatorname{cotan}\left(\frac{\pi}{4} - \frac{\alpha_1}{2}\right) + \delta \log_e \operatorname{cotan} \frac{\alpha_1}{2}\right] \quad \ldots. \ (48)$$

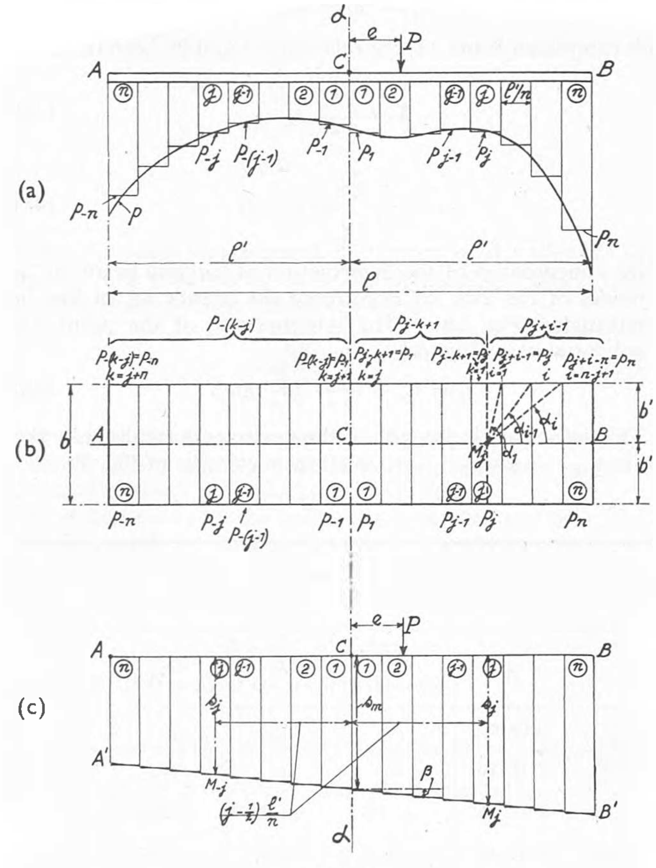

Fig. 6

$$T_i \text{ (for } i > 1) = \frac{1}{n}\Big[\left(i - \frac{1}{2}\right) \log_e \operatorname{cotan}\left(\frac{\pi}{4} - \frac{\alpha_i}{2}\right) - \left(i - \frac{3}{2}\right) \log_e \operatorname{cotan}\left(\frac{\pi}{4} - \frac{\alpha_{i-1}}{2}\right) + \delta\left(\log_e \operatorname{cotan} \frac{\alpha_i}{2} - \log_e \operatorname{cotan} \frac{\alpha_{i-1}}{2}\right)\Big] \quad \ldots. \ (49)$$

Expressions 46 and 47 may also be written for the symmetric point M_{-j} centre of the left rectangle j.

$$s_{-j} = s_m - \left(j - \frac{1}{2}\right)\frac{l'}{n} \tan \beta \quad \ldots. \ (50)$$

$$s_{-j} = \frac{l p_m}{\pi E_s}\left[\sum_{i=1}^{i=n-j+1} \rho_{-(j+i-1)} T_i + \sum_{k=1}^{k=j} \rho_{-(j-k+1)} T_k + \sum_{k=j+1}^{k=n+j} \rho_{k-j} T_k\right] \quad \ldots. \ (51)$$

The n groups of equations 46 and 47 and the n groups of equations 50 and 51 with the expressions 44 and 45 form the system to solve.

This system may be separated into two systems of $n + 1$ equations with $n + 1$ unknown quantities.

If we add and substract respectively 46 and 50, 47 and 51, and by means of 42, 43, 44, 45 and putting $\nu_j = \epsilon . \nu'_j$ we find the two systems

$n + 1$ equations

$$1 = \frac{1}{n}\sum_{j=1}^{j=n} \mu_j \quad \ldots. \ (44)$$

$n + 1$ unknown quantities $\mu_1 \ldots \mu_n, s_m$

$$n\left\{ s_m = \frac{l p_m}{\pi E_s} \sum_{j,i} \mu_j T_i \right. \quad \ldots. \ (52)$$

$n + 1$ equations

$$1 = \frac{1}{n^2}\sum_{j-1}^{j=n} \nu'_j\left(j - \frac{1}{2}\right) \quad \ldots. \ (53)$$

$$n\left\{ \left(j - \frac{1}{2}\right)\frac{\tan \beta}{n} = \frac{2 p_m \epsilon}{n E_s} \sum_{j,i} \nu'_j T_i \right. \quad \ldots. \ (54)$$

$n + 1$ unknown quantities $\nu'_1 \ldots \nu'_n \tan \beta$

The expressions T_l, 48 and 49, depend only on the number of steps $2n$, the position of the chosen rectangle and $\delta = \frac{b}{l}$ and need be calculated only once. Thus, when we give $2n$ and δ the terms $\mu_1 \ldots \mu_n$ and $\nu'_1 \ldots \nu'_n$ may be calculated.

Example—We choose the following example: $b = 1{\cdot}50$ m, $l = 6{\cdot}00$ m, $\delta = 1/4$, $p_m = 1$ kg/cm², $e = 0{\cdot}5$ m. We have divided the beam into 20 sections $n = 10$. The values of μ_j and ν'_j are given in Table 2. With $\epsilon = e/l = 1/6$ we may calculate

$$\nu_j = \epsilon\nu'_j, \quad \rho_j = \frac{p_j}{p_m} \quad \text{and} \quad \rho_{-j} = \frac{p_{-j}}{p_m}$$

also reproduced in Table 2.

Table 2

$2n = 20$	$\delta = 1/4$	$\epsilon = 1/6$		
		$\nu_j = \epsilon\nu'_j$	$\rho_j = \mu_j + \nu_j$	$\rho_{-j} = \mu_j - \nu_j$
$\mu_1 = 0{\cdot}831$	$\nu'_1 = 0{\cdot}103$	$\nu_1 = 0{\cdot}017$	$\rho_1 = 0{\cdot}848$	$\rho_{-1} = 0{\cdot}814$
$\mu_2 = 0{\cdot}820$	$\nu'_2 = 0{\cdot}292$	$\nu_2 = 0{\cdot}049$	$\rho_2 = 0{\cdot}869$	$\rho_{-2} = 0{\cdot}731$
$\mu_3 = 0{\cdot}841$	$\nu'_3 = 0{\cdot}492$	$\nu_3 = 0{\cdot}082$	$\rho_3 = 0{\cdot}923$	$\rho_{-3} = 0{\cdot}759$
$\mu_4 = 0{\cdot}832$	$\nu'_4 = 0{\cdot}703$	$\nu_4 = 0{\cdot}117$	$\rho_4 = 0{\cdot}949$	$\rho_{-4} = 0{\cdot}715$
$\mu_5 = 0{\cdot}888$	$\nu'_5 = 0{\cdot}914$	$\nu_5 = 0{\cdot}152$	$\rho_5 = 1{\cdot}040$	$\rho_{-5} = 0{\cdot}736$
$\mu_6 = 0{\cdot}870$	$\nu'_6 = 1{\cdot}135$	$\nu_6 = 0{\cdot}189$	$\rho_6 = 1{\cdot}059$	$\rho_{-6} = 0{\cdot}681$
$\mu_7 = 0{\cdot}911$	$\nu'_7 = 1{\cdot}421$	$\nu_7 = 0{\cdot}237$	$\rho_7 = 1{\cdot}148$	$\rho_{-7} = 0{\cdot}674$
$\mu_8 = 0{\cdot}911$	$\nu'_8 = 7{\cdot}741$	$\nu_8 = 0{\cdot}290$	$\rho_8 = 1{\cdot}201$	$\rho_{-8} = 0{\cdot}621$
$\mu_9 = 1{\cdot}205$	$\nu'_9 = 2{\cdot}173$	$\nu_9 = 0{\cdot}362$	$\rho_9 = 1{\cdot}567$	$\rho_{-9} = 0{\cdot}843$
$\mu_{10} = 1{\cdot}891$	$\nu'_{10} = 4{\cdot}703$	$\nu_{10} = 0{\cdot}784$	$\rho_{10} = 2{\cdot}675$	$\rho_{-10} = 1{\cdot}107$

In Fig. 7 curve 1 represents the distribution of reactions obtained by the method of steps, curves 2 and 3 the same obtained with the method of trial (De Beer, 1951, 1952) and the analytical method respectively (see Fig. 5).

Conclusions

The difference between curves 1 and 2 of Fig. 5, obtained by the analytical method and the methods of trial respectively, arises from the fact that a parabolical law of the fourth degree differs from the real distribution. It is necessary to chose a parabolic law of a higher degree if we are to get a nearer approach to reality.

In Fig. 7 there is also a difference between curves 1 and 2 obtained by the method of the steps and the method of trial respectively. This difference results from the great difference between the values of contiguous steps at the ends of the beams. In order to get a better approach, it is necessary to take a greater number of steps at these ends; but this makes the calculation rather laborious.

The two methods described require the solution of many linear equations with many unknown quantities. This solution is possible when electrical or electronic machines are available.

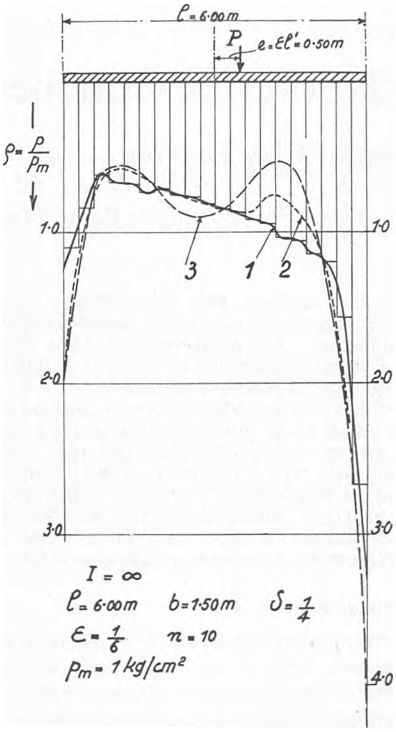

Fig. 7 Distribution of the reactions: curve 1, method of steps; curve 2, method of trial; curve 3, analytical method

Répartition des réactions: courbe 1, méthode des gradins; courbe 2, Méthode par tâtonnements; courbe 3, méthode analytique

This contribution is an excerpt from a more general study undertaken at the Belgian Governmental Institutute of Soil Mechanics under the direction of Professor E. De Beer and is published with his permission.

References

De Beer, E. (1951, 1952). Calcul de poutres reposant sur le sol. Cas des charges uniformément réparties, des charges équidistantes et des charges excentrées. *Annales des Travaux Publics de Belgique*, **52**, 981; **53**, 9

— Lousberg, E. and van Beveren, P. (1956). Le calcul de poutrés et de plaques appuyées sur le sol. *Annales des Travaux Publics de Belgique* (press)