Influence Values for Vertical Stresses in a Semi-infinite Mass due to an Embankment Loading

Lignes d'Influence des Contraintes Verticales Produites dans une Massif Semi-infini Charge par Remblai

J. O. OSTERBERG, Northwestern University, Evanston, Illinois, U.S.A.

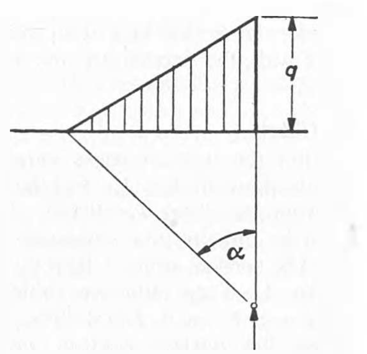
Summary

Influence values in the form of a chart are given for rapid calculation of vertical normal stresses in foundations due to embankment loadings. Examples are given to illustrate the use of the chart.

Sommaire

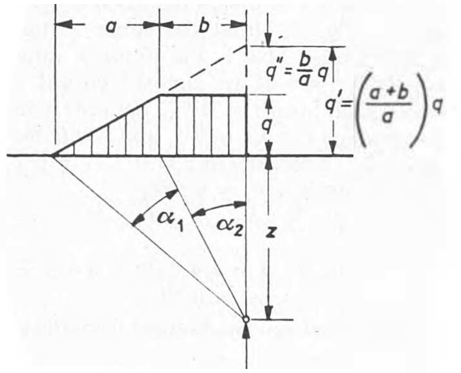
Des lignes d'influence sont données sous forme graphique pour le calcul rapide des contraintes verticales normales engendrées dans les fondations par la charge d'un remblai. L'emploi du graphique est illustré par des exemples.

Computation of vertical normal stresses in soil masses constitutes an important task in estimating settlements due to surface loadings. Formulae giving stresses for various shapes and load distributions are obtained from integrating the Boussinesq solution for stresses due to a single concentrated vertical load on a semi-infinite homogeneous isotropic mass.



$$\sigma_z = \frac{q}{\pi}\alpha$$

Fig. 1 Equations for the construction of the influence chart
Equations pour l'établissement du graphique des lignes d'influence



$$\sigma_z = I \cdot q$$

$$\sigma_z = \frac{q}{\pi}\left[\left(\frac{a+b}{a}\right)(\alpha_1 + \alpha_2) - \frac{b}{a}\alpha_2\right]$$

$$\therefore I = \frac{1}{\pi}\left[\left(\frac{a+b}{a}\right)(\alpha_1 + \alpha_2) - \frac{b}{a}\alpha_2\right]$$

$$I = f\left(\frac{a}{z}, \frac{b}{z}\right)$$

Fig. 2 Equations for the construction of the influence chart
Equations pour l'établissement du graphique des lignes d'influence

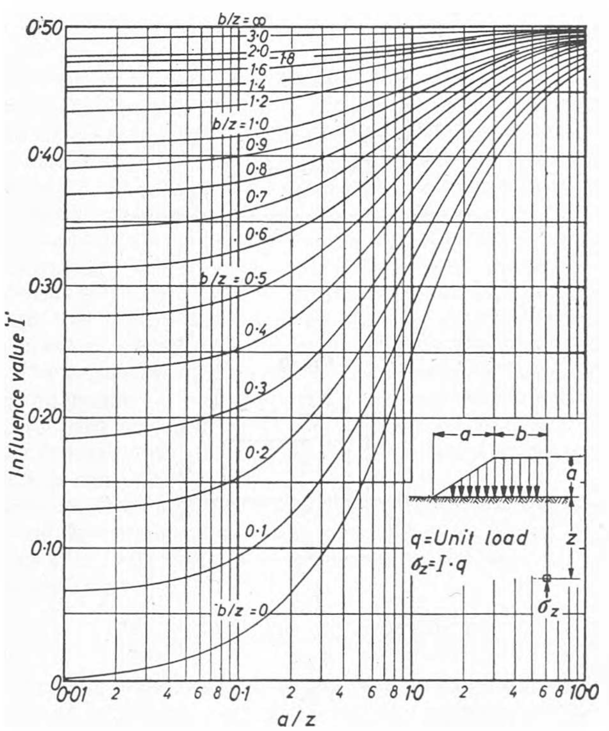


Fig. 3 Influence chart for vertical stress embankment loading—infinite extent. Boussinesq case
Graphique des lignes d'influence des contraintes verticales engendrées par la charge d'un remblai indéfini. Cas Boussinesq.

These have been summarized by JURGENSEN (1934), GRAY (1936) and NEWMARK (1940). NEWMARK (1942) also constructed an influence chart based on a uniformly distributed circular loading, from which the vertical normal stresses due to a uniformly loaded area of any shape can be found by counting the elements of the area covered by the loading plan drawn to the proper scale. FADUM (1948) reviewed the contributions from the theory of elasticity and presented charts giving influence values for vertical stresses under the corner of a rectangular load, and under the end of a line load of finite extent. A chart giving influence values for vertical stresses for an embankment type loading is presented here with examples of application by simple superposition.

Construction of Influence Chart

The influence chart is based on the equation for a triangular load of infinite extent (Newmark, 1942) as shown in Fig. 1. By superposition, the loading in Fig. 2 is obtained. The vertical

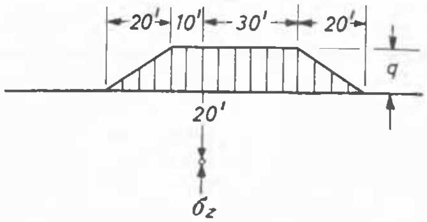


Fig. 4

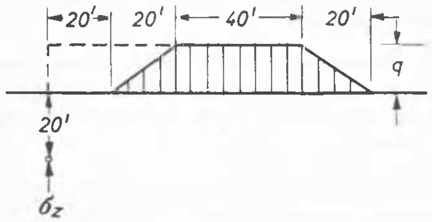


Fig. 5

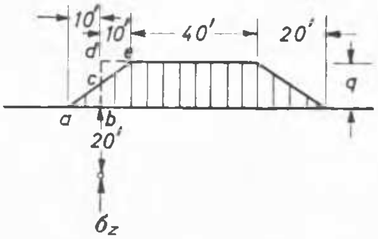


Fig. 6

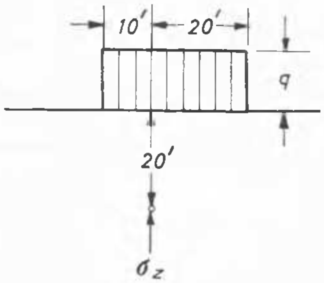


Fig. 7

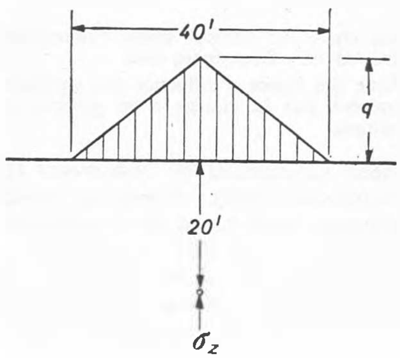


Fig. 8

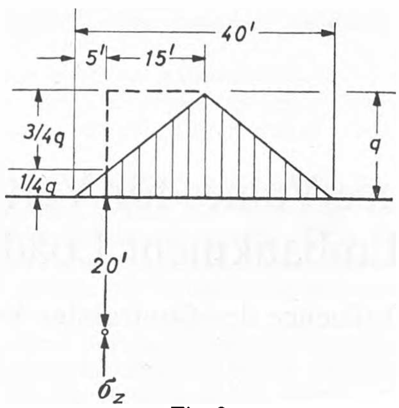


Fig. 9

Figs. 4–9 Examples showing uses of the influence chart
Exemples de l'emploi du graphique

stress at the location shown is a function of a/z, b/z and the unit load 'q'. From this relationship an influence chart (Fig. 3) has been constructed which simplifies the computation of vertical stresses beneath embankment loadings. The stress given by the chart is the vertical stress directly under the vertical face of a portion of an embankment of infinite extent. Vertical stresses for any point in the foundation can be found by superposition as illustrated below. For stresses under a corner such as under the vertical face of an embankment ending abruptly against a wall, the stresses are one half of those given in the chart.

Use of Influence Chart

Example 1. Find the vertical stress beneath an embankment at the location shown in Fig. 4. For the left part $a/z = 1$, $b/z = 0{\cdot}5$, and from the chart $I = 0{\cdot}397$. Similarly for the right side $I = 0{\cdot}478$, and the total influence value is $0{\cdot}397 + 0{\cdot}478 = 0{\cdot}875$. The vertical stress is then $\sigma_z = 0{\cdot}875q$.

Example 2. Fig. 5. Find influence value for dashed and solid portion ($a/z = 1$, $b/z = 4$, $I = 0{\cdot}499$). Subtract the influence value for the dashed portion ($a/z = 1$, $b/z = 1$, $I = 0{\cdot}455$). The stress is then $\sigma_z = 0{\cdot}044q$.

Example 3. Fig. 6. Stress due to *abc* is the same as due to *cde*. Since one is plus and the other is minus the stress is the same as if the embankment were vertical at *B*. Therefore ($a/z = 1$, $b/z = 2{\cdot}5$, $I = 0{\cdot}492$) the stress is $\sigma_z = 0{\cdot}492q$.

Example 4. Fig. 7. Find the stress at the point shown under a strip load. This is the limiting value for $a/z = 0$. The values for $a/z = 0{\cdot}01$ are almost identical, the error being 4 per cent too high for $b/z = 0{\cdot}1$, 2 per cent high for $b/z = 0{\cdot}3$ and 1 per cent high for $b/z = 0{\cdot}5$, and negligible for all values greater than 0·5. Therefore ($b/z = 0{\cdot}5$, $I_1 = 0{\cdot}278$; $b/z = 1{\cdot}0$, $I_2 = 0{\cdot}410$) the stress is $\sigma_z = 0{\cdot}688q$.

Example 5. Fig. 8. b/z is zero, $a/z = 1$, $I = 0{\cdot}25$, $\sigma_z = 0{\cdot}50q$.

Example 6. Fig. 9. $I = 1/4\ 0{\cdot}08 + 0{\cdot}434 - 3/4\ 0{\cdot}203$

$$\sigma_z = 0{\cdot}302q.$$

A copy of the chart can be obtained from the author.

References

Fadum, R. E. (1948). Influence values for estimating stresses in elastic foundations. *Proc. 2nd International Conference on Soil Mechanics and Foundation Engineering*, Vol. 3, pp. 77–84

Gray, H. (1936). Stress distribution in elastic solids. *Proc. 1st International Conference on Soil Mechanics and Foundation Engineering*, Vol. 2, pp. 157–168

Jurgenson, L. (1934). The application of elasticity and plasticity to foundation problems. *J. Boston Soc. Civ. Eng.*, **21**, 206

Newmark, N. M. (1940). Stress distribution in soils. *Proc. Purdue Conference on Soil Mechanics and its Applications*, pp. 295–303

— (1942). Influence charts for computation of stresses in elastic foundations. *Bull. Univ. Illinois Eng. Exp. Station*, No. 338