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Danger of Frost Heaving of Skating-rink Foundations

Le Danger de Gonflement des Fondations des Pistes de Patinage sous l'Action du Gel

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Summary

A new method of computation of the freezing progress of 'sliced' (formed by successive different layers) foundations for skating-rinks is presented. The method, based in general on Fourier's law, enables the determination of the frost heaving danger and is illustrated by an actual example of foundation design.

Sommaire

Une nouvelle méthode est présentée pour calculer le progrès de pénétration du gel dans une fondation formée de plusieurs tranches horizontales pour piste de patinage. La méthode, basée sur la loi de Fourier, permet de déterminer le danger de gonflement du sol gelé. La méthode est illustrée par un exemple réel d'une fondation de piste de patinage.

The problem was raised as to whether the foundation of a skating-rink in known soil conditions would be menaced by soil heaving. General methods for computing the progress of thermal penetration in the ground are known. In uniform soil conditions some deduced formulae (Ruckli) for solving these problems are available.

The author in his consulting practice has had to deal with some questions which have required a special study; the following presents one of them.

The foundation slab was designed as a 'sliced' construction shown in Fig. 1. The temperature on the level α was assumed − 8° C. acting steadily each year during 210 days.

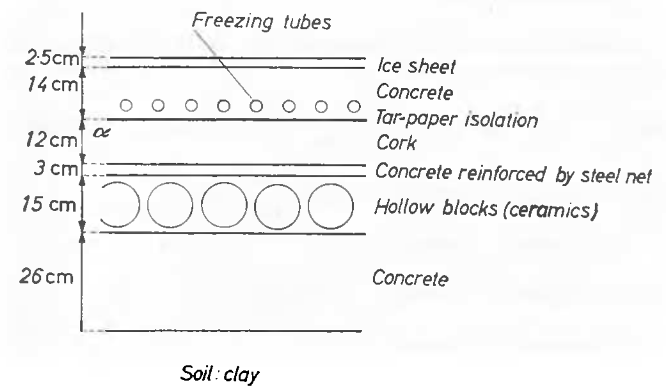

Fig. 1 Design of 'sliced' foundation slab
Fondation en 'tranches'

The characteristic figures relating to the thermic computations were assumed, as shown in Table 1.

Table 1

	λ	γ	c	$c_v = c\gamma$
Cork (ground, asphalt glued)	0·5	0·2	0·45	0·09
Concrete	11	2·2	0·21	0·46
Hollow ceramic blocks	6·6	1·3	0·21	0·27
Soil: clay	19	2·0	0·36	0·72
$W = 24{\cdot}7$				
$n = 0{\cdot}4$				
$k_w = 1{\cdot}10^{-7}$ cm/min				

The symbols used here and in Table 1 signify:

λ, heat conductivity, in cal. $cm^{-1}.h^{-1}$ °C.$^{-1}$;
c, specific heat, in cal. g^{-1} °C.$^{-1}$;
γ, bulk density, in g. cm^{-3};
h, hours;
°C., degree centigrade;
c_v, volumetric heat capacity, in cal. cm^{-3} °C.$^{-1}$.

The upper ice sheet is formed by means of cold liquid flowing in freezing tubes placed in a concrete layer. Beneath this layer, on the top of cork, the temperature was assumed $\vartheta_I = -8°$ C. All computations will take into account that only that quantity of heat which will pass through the isolating cork layer can flow underneath. The intensity of heat flow will decrease in conformity with deeper penetration of frost.

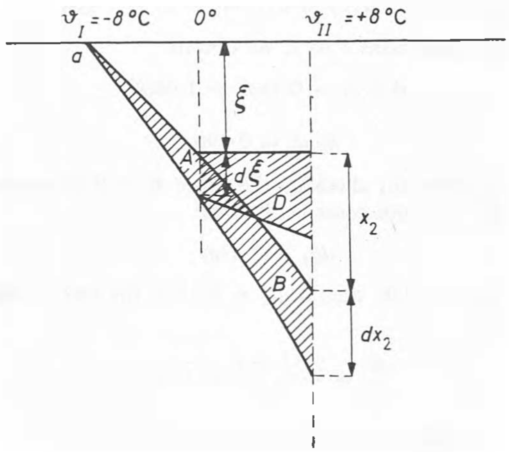

Fig. 2 Calculation of the freezing progress for a 'sliced' foundation
Calcul de la pénétration du gel dans une fondation en 'tranches'

The process of freezing is presented generally in Fig. 2.

If we assume that at the moment t temperature equal to zero is reached in some uniform material to depth ξ, then in the next infinitely small space of time dt this temperature will penetrate further by $d\xi$. The quantity of heat (here of cold) dq flowing according to Fourier's law will be:

$$dq = -\lambda\frac{\vartheta_I}{\xi}dt$$

if calculated for 1 cm² of horizontal section. Some more detailed investigations show that dq can be graphically equalized to area of contours A and B. So we can write:

$$dq = c\gamma - \frac{\vartheta_I}{2}d\xi + c\gamma\vartheta_{II}d\xi$$

where $\vartheta_{II} = +8°$ C. is the temperature of ground and taking approximately contour B equal to D.

By equalizing the two expressions the general solution of heat balance is obtained:

$$dq = -\frac{\lambda\vartheta_I}{\xi}dt = c\gamma\left(\vartheta_{II} - \frac{\vartheta_I}{2}\right)d\xi$$

For uniform material this equation is solved easily, but for 'sliced' construction it must be adequately adapted.

In this latter case the computation proceeds by small cuts aiming to find at the point of cut in each medium the relation of freezing progress ($d\xi/dt$). When this figure is confirmed it is possible to obtain by reversal what interval of time Δt corresponds to the freezing depth increase $\Delta\xi$. Summing finally particular values of Δt it is easy to obtain freezing depth on each time.

This general method requires to be explained by partial computations relating to a given task.

(1) Time needed for freezing to depth $\xi = 7$ cm. Layer: cork. Of the cold coming from point a at time interval dt some quantity, namely $\vartheta_I d\xi(c\gamma)/2$, shall be absorbed by this layer and the rest dq_1 allowed to pass down.

$$dq_1 = \frac{\lambda\vartheta_I}{\xi}dt - \vartheta_I d\xi\frac{c\gamma}{2} = \frac{0{\cdot}5.8}{7}dt - 8d\xi.\frac{0{\cdot}09}{2}$$
$$= 0{\cdot}57dt - 0{\cdot}36d\xi \quad \ldots . \text{ (1)}$$

According to Fig. 2 the quantity dq_1 will be expended for cooling an amount of cork $(d\xi + dx)/2_1$ from temperature $+8°$ to $0°$. There are sufficient reasons (not discussed here) to assume $dx = 2d\xi$

$$dq_1 = \vartheta_{II}.1{\cdot}5d\xi c\gamma = 8.1{\cdot}5d\xi.0{\cdot}09 = 1{\cdot}08d\xi \quad \ldots . \text{ (2)}$$

Equalizing expressions 1 to 2, we obtain

$$0{\cdot}57dt - 0{\cdot}36d\xi = 1{\cdot}08d\xi$$

and

$$d\xi/dt = 0{\cdot}396$$

By substituting for checking the value $d\xi = 0{\cdot}396dt$ in equations 1 and 2 in both cases is obtained

$$dq_1 = 0{\cdot}43dt$$

When $d\xi/dt = 0{\cdot}396$, then $dt/d\xi = 2{\cdot}53$ at the end of interval. Mean value

$$\frac{dt}{d\xi} = \frac{2{\cdot}53 + 0}{2} = 1{\cdot}26$$

Freezing of layer $\xi = 7$ cm will need $1{\cdot}26.7 = 9$ hours.

(2) In a similar way it will be found that freezing to 11 cm will require $7 + 29 = 36$ hours.

(3) Taking next $\xi = 14$ cm we shall cool 12 cm of cork and 2 cm of concrete. By preliminary trial computations it can be proved that total loss of cold from $\vartheta_I = -8°$ C. to 0 degrees will be divided: in cork 7·95 degrees and in concrete 0·05 degrees.

As we have explained, we compute quantities of cold dq_1 and dq_2 reaching in time interval dt: (a) to bottom of cork layer and (b) to depth $\xi = 14$ cm.

$$dq_1 = \frac{0{\cdot}5.7{\cdot}95}{12}dt - 7{\cdot}95d\xi\frac{12^*}{14}.\frac{0{\cdot}09}{2} = 0{\cdot}331dt - 0{\cdot}307d\xi$$

$$dq_2 = 0{\cdot}331dt - 0{\cdot}307d\xi - 0{\cdot}05.d\xi\frac{13^*}{14}.0{\cdot}46 = 0{\cdot}331dt - 0{\cdot}328d\xi$$

* Multipliers $\frac{12}{14}$ and $\frac{13}{14}$ are introduced on geometric reasons (Fig. 2).

This quantity will be absorbed by hollow ceramic blocks by lowering their temperature from + 8 degrees to 0 degrees

$$dq_2 = 0{\cdot}27.8.d\xi.1{\cdot}5 = 3{\cdot}24d\xi$$
$$0{\cdot}331dt - 0{\cdot}328d\xi = 3{\cdot}24d\xi;$$
$$\frac{d\xi}{dt} = 0{\cdot}093$$

Checking the assumed distribution of temperatures

$$dq_2 = 3{\cdot}24.0{\cdot}093dt = 0{\cdot}30dt$$
$$\frac{11.0{\cdot}05}{2}dt = 0{\cdot}275dt = \sim 0{\cdot}30dt$$

Time computation

$$\frac{dt}{d\xi} = 10{\cdot}8;\ \left(\text{preceding } \frac{dt}{d\xi} = 12{\cdot}0\right)$$

mean

$$\frac{dt}{d\xi} = \frac{12{\cdot}0 + 10{\cdot}8}{2} = 11{\cdot}4$$
$$11{\cdot}4.3 = 34 \text{ hours}$$

(4) Computation for the freezing point at depth $\xi = 100$ cm. Results obtained from six increasing depths of freezing and from renewed trial computations for this case allow us to assume a sequence of temperature decreases as follows:

cork 12 cm thick . . .	6·38 degree
concrete 3 cm thick . . .	0·05 degree
hollow blocks 15 cm thick . .	0·37 degree
concrete 26 cm thick . . .	0·63 degree
clay 44 cm thick . . .	0·57 degree

Quantities dq of cold passing to the bottom of each of the five layers are computed.

$$dq_1 = \frac{0{\cdot}5.6{\cdot}38}{12}dt - 6{\cdot}38.d\xi.\frac{12}{100}.\frac{0{\cdot}09}{2} = 0{\cdot}266dt - 0{\cdot}035d\xi$$

$$dq_2 = 0{\cdot}266dt - 0{\cdot}035d\xi - 0{\cdot}05d\xi\frac{13{\cdot}5}{100}.0{\cdot}46$$
$$= 0{\cdot}266dt - 0{\cdot}038d\xi$$

$$dq_3 = 0{\cdot}266dt - 0{\cdot}038d\xi - 0{\cdot}37d\xi\frac{22{\cdot}5}{100}.0{\cdot}27$$
$$= 0{\cdot}266dt - 0{\cdot}061d\xi$$

$$dq_4 = 0{\cdot}266dt - 0{\cdot}061d\xi - 0{\cdot}62d\xi\frac{43}{100}.0{\cdot}46$$
$$= 0{\cdot}266dt - 0{\cdot}183d\xi$$

$$dq_5 = 0{\cdot}266dt - 0{\cdot}183d\xi - 0{\cdot}58d\xi\frac{78}{100}.0{\cdot}72$$
$$= 0{\cdot}266dt - 0{\cdot}508d\xi$$

The quantity dq_5 is absorbed for decreasing the soil temperature from + 8 degrees to 0 degrees beneath $\xi = 100$ cm and can be shown as formerly:

$$dq_5 = 0{\cdot}72.8.d\xi.1{\cdot}5 = 8{\cdot}64d\xi$$
$$0{\cdot}266dt - 0{\cdot}508d\xi = 8{\cdot}64d\xi$$
$$\frac{d\xi}{dt} = 0{\cdot}029$$

Checking

$$dq_5 = 8{\cdot}64.0{\cdot}029 = 0{\cdot}250dt$$
$$\frac{19.0{\cdot}58}{44}dt = 0{\cdot}250dt$$

Time computation

$$\frac{dt}{d\xi} = 34{\cdot}5 \left(\text{preceding } \frac{dt}{d\xi} = 25{\cdot}6\right)$$

mean

$$\frac{dt}{d\xi} = \frac{25{\cdot}6 + 34{\cdot}5}{2} = 30{\cdot}0$$

$$30{\cdot}0 . 44 = 1320 \text{ hours.}$$

The method used here gives results shown in Table 2.

Table 2

ξ cm	$\Delta\xi$ cm	$\frac{d\xi \text{ cm}}{dt \text{ hour}}$	$\frac{dt \text{ hour}}{d\xi \text{ cm}}$	$\frac{dt}{d\xi}$ *mean*	$\frac{dt\,\Delta\xi}{d\xi}$ hours	*Hours summed*
0			0			
	7			1·25	9	
7		0·40	2·50			9
	4			7·25	29	
11		0·083	12·0			38
	3			11·4	34	
14		0·093	10·8			72
	8			12·65	101	
22		0·069	14·5			173
	13			14·7	191	
35		0·067	14·9			364
	21			20·2	424	
56		0·039	25·6			788
	44			30·0	1320	
100		0·029	34·5			2108
	70			36·9	2580	
170		0·0255	39·3			4688
	63			41·6	2620	
233		0·0228	43·9			7308

In conclusion it can be expected that during 7 months = 5040 hours of continuous freezing the temperature equal to zero will reach to the depth

$$\xi = 170 + \frac{(5040 - 4688).63}{2620} = 178 \text{ cm}$$

and a layer of soil 178 − 56 = 122 cm thick will be frozen.

Taking into consideration that capillary suction of water will not appear in clay in any remarkable degree owing to its very small permeability, we shall obtain a frost heaving of the foundation caused only by freezing of water contained in the soil. This heaving can be evaluated as follows:

$$s = 122.0{\cdot}40.0{\cdot}09 = 4{\cdot}4 \text{ cm}$$

The phenomenon of frost heaving limited only to this figure would not present a menace to the skating-rink foundation, but the soil there was not uniform. If any water suction could take place, the freezing process would be slower but heaving would increase. Provision was therefore made to allow the passage of warm air through the hollow blocks in order to prevent the freezing of the soil beneath the foundation.