The Axial and Lateral Load Bearing Capacity, and Failure by Buckling of Piles in Soft Clay

La Force Portante Axiale et Latérale et la Rupture par Flambage de Pieux Battus en Argile Tendre

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Summary

An analysis is made of some hundreds of vertical loading tests on piles in soft clay and for these the formula $P = \beta \int \tau . U . dl$ is valid, where U is the perimeter of the pile section.

For two vertical piles and a pile trestle subjected to horizontal forces, pressure measurements were made at ten points along the piles. It seems possible to consider the piles as beams on elastic foundations. For the trestle it was observed that there is a risk that the lateral forces may break the piles. The spring modulus kB was 50 to 10 kg/cm² for a clay with $\tau = 1{\cdot}3$ ton/m², the lower value for increasing or repeated loading.

Buckling of steel piles with a solid cross-section is possible in soft clay. Tests have shown that the buckling load is given by $P = 8$ to $10(\tau . EI)^{\frac{1}{2}}$.

Sommaire

Une assimilation est faite entre quelques centaines de charges d'essai sur pieux en argilemolle, et la formule $P = \beta \int \tau . U . dl$, où U est la circonférence du pieu, montre que celle-ci est valide pour ces pieux.

Deux pieux verticaux et un chevalet sont soumis à des forces horizontales et des mesures sont faites en dix points le long des pieux. Il est possible de calculer les pieux comme des poutres sur fondations élastiques, dans des constructions de ce genre ou risque la rupture sous des charges latérales. Le module de ressort kB était de 50 à 10 kg/cm² pour une argile de $\tau = 1{\cdot}3$ t/m², la valeur inférieure correspondant à une augmentation ou à une répetition de la charge.

Le flambage des pieux en acier à section pleine est possible en argile molle. Les essais ont indiqué que la charge de flambage est $P =$ 8 à $10(\tau . EI)^{\frac{1}{2}}$.

Bearing Capacity for Axial Loads on Floating Piles in Soft Clay

When calculating the axial bearing capacity of a pile one generally uses some formula starting either from the driving resistance or from the properties of the soil and the pile dimensions, that is

$$P = Q . h . f\left(\frac{1}{\delta} \ldots\right) \qquad \ldots . \text{(1a)}$$

or

$$P = N_s A + \beta \int_0^l (c + \sigma . \tan \phi) . U . dl. \qquad \ldots . \text{(1b)}$$

Formula 1a for the driving resistance contains not only the external work of the hammer of weight Q falling height h, and a function depending on the sinking, δ, of the pile, but also the relation of hammer weight to pile weight, impact losses and blow intervals, the dimensions and elastic qualities of the pile, and so on.

Formula 1b containing the soil properties is generally expressed in two terms: the first for the point resistance given as cross-sectional area, A, times some expression N_s for soil resistance; and the second term for skin friction modified by a correction factor, β.

For piles driven in soft clay without reaching any firm layer, the formula 1a is, generally, not valid, whilst 1b gives better results. In the Port of Gothenburg, where such piling is very common, this has long been accepted (see papers by WENDEL, 1901 and HULTIN, 1928, 1937). As the first term of equation 1b was found to be very small, only 0·5 to 1·0 ton, the pile formula was simplified to:

$$P = \int \tau U . dl \qquad \ldots . \text{(2)}$$

The writer has collected together the results of all piling tests made by the Port of Gothenburg since Wendel's tests in the 1890s and has compared them with the clay properties at the various sites. Some clay tests were made on samples taken after the piling was completed, but during the past 20 years the tests have been made on samples taken during the test piling. A small number of the pile records from within the harbour district are shown in Fig. 1, where the theoretical failure load is compared with applied test load at failure. It is seen that the agreement is good, i.e. the points of the diagram are situated near the 45 degree line. Most of the test loads were applied by anchor piles nearby but kentledge was used to load others.

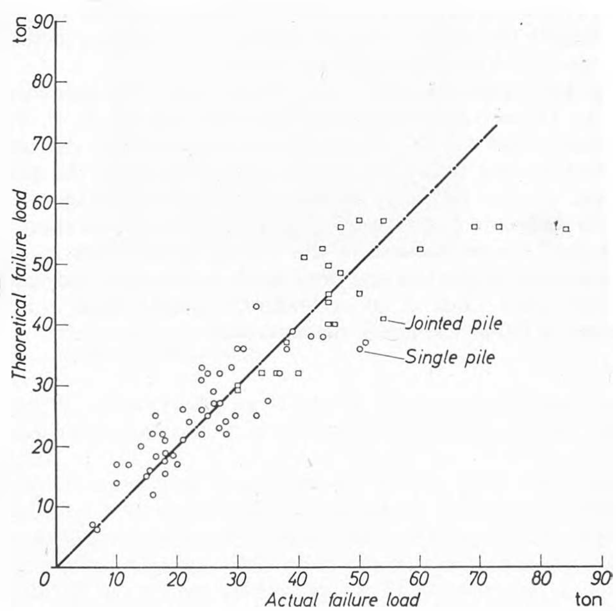

Fig. 1 Axial failure loads of piles, calculated and actual
Charge de rupture pour des pieux, calculée et mesurée

No marked difference in the results from the two test procedures have been observed.

The correction factor β in equation 1b is presumed to be 1·0 in Fig. 1. That part of the factor dependent on time is of little significance here as the piles were generally tested one month after driving. Any increase in bearing capacity after that period would be relatively small in the fine-grained clays of Gothenburg. The author's investigations showed β to be dependent also on the sensitivity and the liquid limit of the clay, however, the clay in the Port is generally such that the factor is normally close to 1·0. Investigations also showed that the factor was influenced by the pile material, whether wood, concrete or steel, probably as a result of drainage of pore water from the clay, and of disturbance and compression of the clay.

The bearing capacity of individual piles in Gothenburg clay (at least for short-term tests) is thus well defined and so to some extent is the settlement of pile foundations. The settlement varies from small values to over 100 cm in 50 years. It seems as if the extent to which the bearing capacity is utilized has only a small effect on the settlement, at least so long as the factor of safety is greater than 2. The settlement then depends mostly on the compressibility of the ground below the piles. The writer intends to publish a paper on this settlement problem at a later date.

Resistance and Deformation of Piles Loaded Transversely

In the paper by WENDEL (1901), the results of tests of horizontal loads on vertical timber piles of different lengths in a very soft clay and in a firm one were recorded. The displacement and the angle of the inclination of the piles at the soil surface were measured. It was observed that an increase in pile length over 5 m had no influence and that the deformation occurred almost only a few m below the surface. Under increasing loads the piles were broken at 1·5 to 2·1 m below the soil surface. The results of several similar tests have been published, for example those by FEAGIN (1937) and JAMPEL (1947).

GRANHOLM (1929) showed that the deformation could be calculated with good approximation from the equation for a beam on an elastic foundation. Others assumed the pile to be fixed at some depth below the ground surface (CUMMINGS, 1938). Even in the latter case, it is only possible of course to get good agreement if the lateral soil resistance is taken into consideration. For other methods of calculation see JAMPEL (1949) and HRENNIKOFF (1949).

In connection with the building of some warehouses in the Port, the author has had the opportunity to make investigations of the lateral resistance of piles. The areas of the sheds have varied from 30 × 105 to 35 × 40 m, and the sheds have been built without special foundation slabs or beams. The timber piles were driven about 14 m down into the clay, with 4 m protruding above soil surface to support the walls and the roof. The wind forces are resisted mainly by piled trestles. To check the behaviour, two vertical piles and a trestle were driven. Measurements were made of the horizontal force, the displacement, the inclination and of the earth pressure at 10 points along each pile by means of oil-filled gauges.

The two vertical piles were loaded, first at the soil surface and then 2·0 m above it. The loads applied to one pile were smaller than those applied to the other, but the former were of sufficient magnitude to note if both piles behaved similarly. Following this a support corresponding to a pavement was placed on the ground surface around the first mentioned pile and the final test was made with the load applied 2 m above the ground.

The test results are shown in Fig. 2 where the earth pressures, F in front and B behind the pile, are given, together with the corresponding displacements and slopes of the pile at the soil surface, and examples of the relation between the load and deflection at the three dial gauges on one pile (I:1). The theoretical deflection curves for a beam on an elastic foundation are also given in Fig. 2, but no allowance has been made in these curves for the apparently smaller deformation modulus at the soil surface.

The theory is based, as mentioned above, on the equation for a beam on an elastic foundation

$$EI.\frac{d^4y}{dx^4} = -\,kB.y \qquad \ldots.\ (3)$$

Many papers have been written about this equation ever since it was solved, and H. Zimmermann in 1888 made practical use of it. A book by HETÉNYI (1946) with a large number of different load cases and a paper by ZAYTZEFF (1956) where influence lines for different elementary cases are included may be mentioned. The writer (1954) has made a survey which treats the variation of shear force and beam length.

The spring modulus, k, is, of course, not necessarily constant along the pile. Several treatments of this have been made

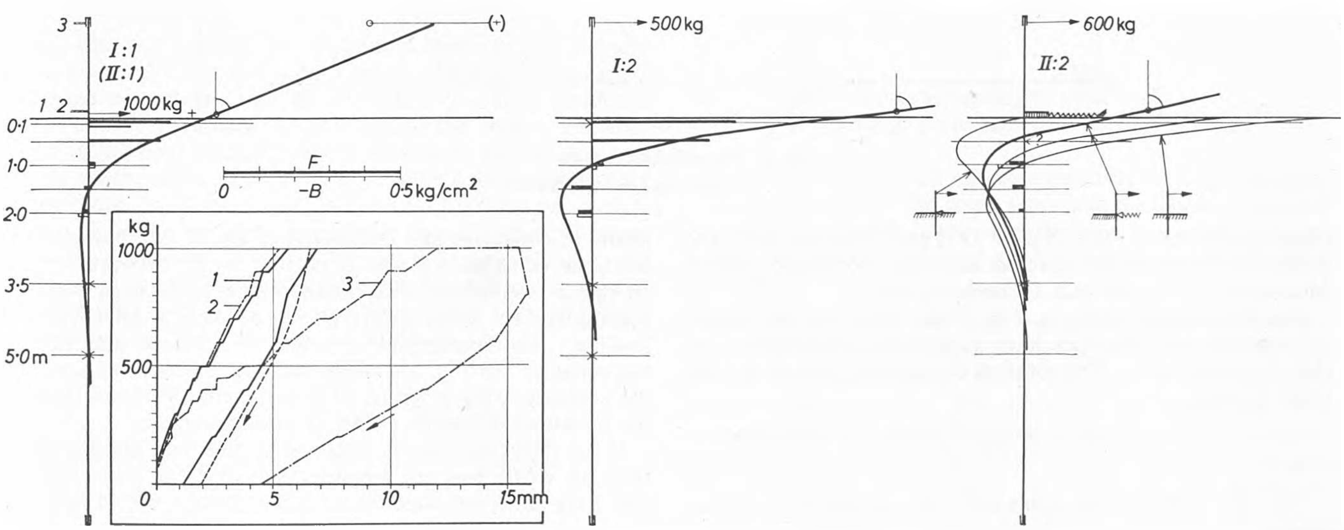

Fig. 2 (a) Measurement of horizontal forces, F, in front of, and B behind, test piles, and the corresponding displacements and inclinations at the soil surface—theoretical deflection curves; (b) relation between load and deflection at three points on one pile (I — 1)

(a) Mesure des forces horizontales, F, à l'avant, et B, en arrière, des pieux d'essai, et les déplacements et inclinaisons à la surface du sol qui y correspondent — courbes déformations théoriques; (b) rapport entre la charge et la déformation en trois endroits d'un seul pieu (I — 1)

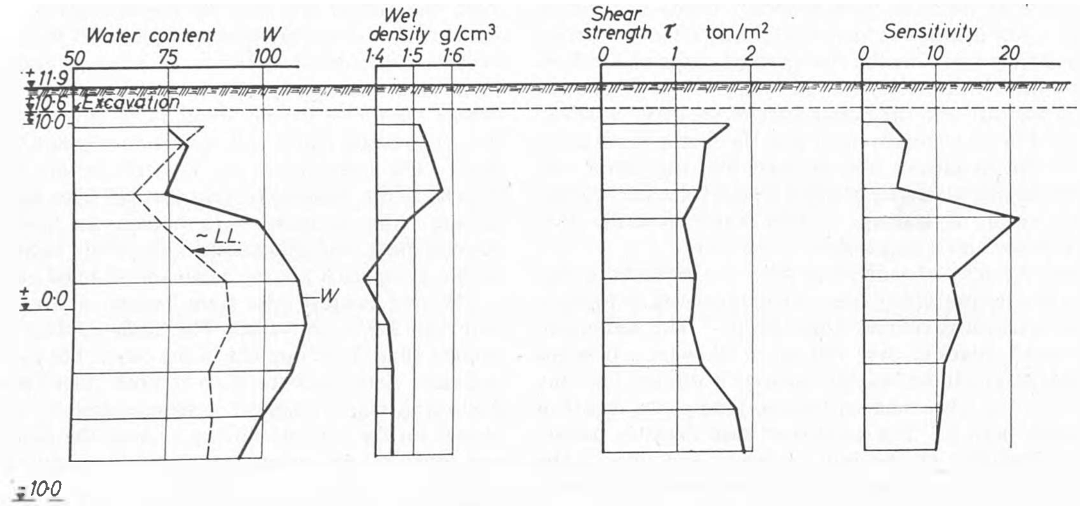

Fig. 3 Properties of the clay
Propriétés de l'argile

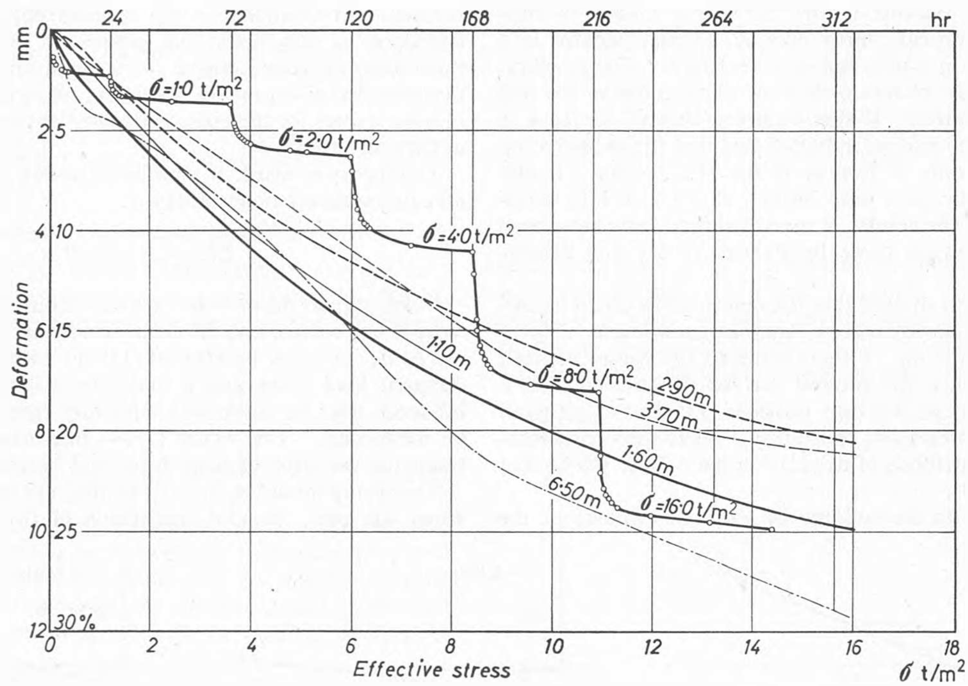

Fig. 4 Consolidation tests
Essais de consolidation

(Miche in 1926 and 1930, Titze in 1932 and Rifaat in 1935), and a method of solution was given at an earlier Conference on Soil Mechanics by PALMER and THOMPSON (1948).

The theoretical curves in Fig. 2 are based on the simple assumptions that the piles have a constant cross-section and that k is a constant. The solution of equation 3 for an infinite beam is then:

$$y = e^{-x/L}(a.\cos x/L + b.\sin x/L) \text{ with } L = (4EI/kB)^{\frac{1}{4}} \quad \ldots\ (4)$$

Thus for a pile length equal to $3L$ the agreement is almost perfect, but at shorter lengths correction terms must be added. At a length less than $1{\cdot}5L$, one should consider the pile to be infinitely stiff; between those limits there is, of course, a transition curve. For the test piles with a soil length of 8 m, L equal to about 1·0 was obtained, and thus the pile length was $8L$, which is why the approximation is very good. For the measurements of deflection and inclination of the 20 cm diameter test piles the constant kB was calculated to lie between 50 and 10 kg/cm², the higher value corresponding to the early stages of loading and the lower applying to large loads or after repeated loading. The corresponding values of L are 80 and 120 cm respectively. In Fig. 2 a mean value of 100 cm was used but the abscissae were enlarged so that the curves passed through the measured deflection points at the soil surface.

If the deflection curves obtained in this way are compared with the earth pressure measurements and the scales used in Fig. 2 are taken into account we obtain from $p = ky$ a value for $kB = pB/y = 0{\cdot}333 \times 20/0{\cdot}5 = 13{\cdot}3$ kg/cm². This value is less than would correspond to $L = 100$ cm and it has been considered probable that either the earth pressure gauges read low when in the ground or that the pressure distribution is not uniform over the projection of the surface of the pile.

The properties of the clay and the results of consolidation tests are shown in Figs. 3 and 4. In addition, deformations and failure loads for the clay were determined for differently shaped objects drawn horizontally through the clay. From this it was possible to get some comparison with the value of kB as determined directly from the pile tests.

A more direct method for the translation of values obtained from one pile test to another is, however, probably based on the following theoretical reasoning, even if it is for an ideal case. Starting from Boussinesq's formulae for an elastic body, GRANHOLM (1929) has derived the approximate value of kB for a loaded area with the length of the half-wave given in equation 3 as:

$$kB = \frac{4\pi G_c}{1 + 2\log_e \frac{2\pi L}{B}} = \frac{8\pi G_c}{\log_e \frac{EI}{B^4} + 10{\cdot}74 - \log_e kB} \quad \ldots\ldots (5a)$$

From a paper by BIOT (1937) dealing with a beam on a semi-infinitive elastic body the following formula can be deduced:

$$kB = \frac{8\pi G_c / \sim 1{\cdot}1.(1-\nu)}{\log_e \frac{EI}{B^4} + 7{\cdot}85 - \log_e kB} \quad \ldots\ldots (5b)$$

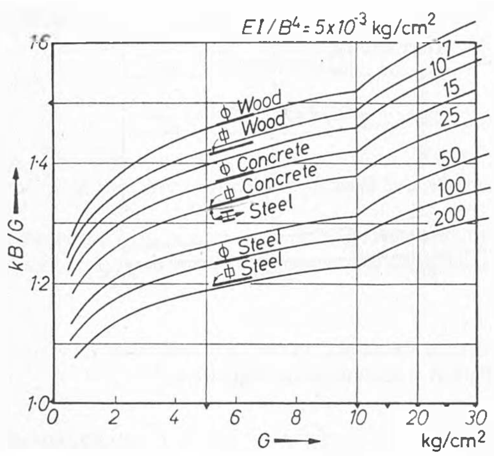

Fig. 5 Comparison of theoretical kB values for various piles according to equation 5a

Comparaison des valeurs théoriques de kB pour plusieurs pieux, après l'équation 5a

This equation applies to a sinusoidal loading on an ideally elastic material which can take tension. In deriving equation 5b it has been assumed that half the load on the beam is resisted by the elastic half-space on each side of the beam. In other words it has been assumed that a loading in the centre of an infinite space can be replaced by two elastic half-spaces without the deformation at the common boundary having any significance. This is somewhat more exact than Granholm's treatment, since the latter is based only on the mean buckling due to a rectangularly distributed load, while Biot takes into account the sinusoidal form of the load and the adjacent opposing loads.

JAMPEL (1949) gives a somewhat smaller value

$$kB = \frac{8\pi E_c / 3}{\log_e \frac{EI}{B^4} + 7{\cdot}96 - \log_e kB} \quad \ldots\ldots (5c)$$

All the formulae are thus rather similar, but they have of course no exact meaning, as they contain the values E_c and G_c, which are only hypothetical for clay. They ought, however, to show a tendency towards a solution. The solution for equation 5a is given in Fig. 5. This gives a comparison of kB values for various types of piles, and for a clay with a specified value of the hypothetical modulus G_c. The variations are small compared with the uncertainties of kB.

The pile trestle was also subjected to horizontal forces, and the same measurements were made as for the vertical piles. The results are shown in Fig. 6. It will be seen that there are axial forces in the piles in addition to the lateral forces. Those lateral forces are so large that there is a risk of breakage when the piles are long, before the axial bearing capacity is fully developed.

From the experiments it has thus been found

(a) that piles in soft clay can be considered as beams on an elastic foundation in respect of the horizontal forces,
(b) that the bearing capacity of vertical piles against horizontal loads is considerable, and
(c) that inclined piles in trestles subjected to horizontal loading develop bending.

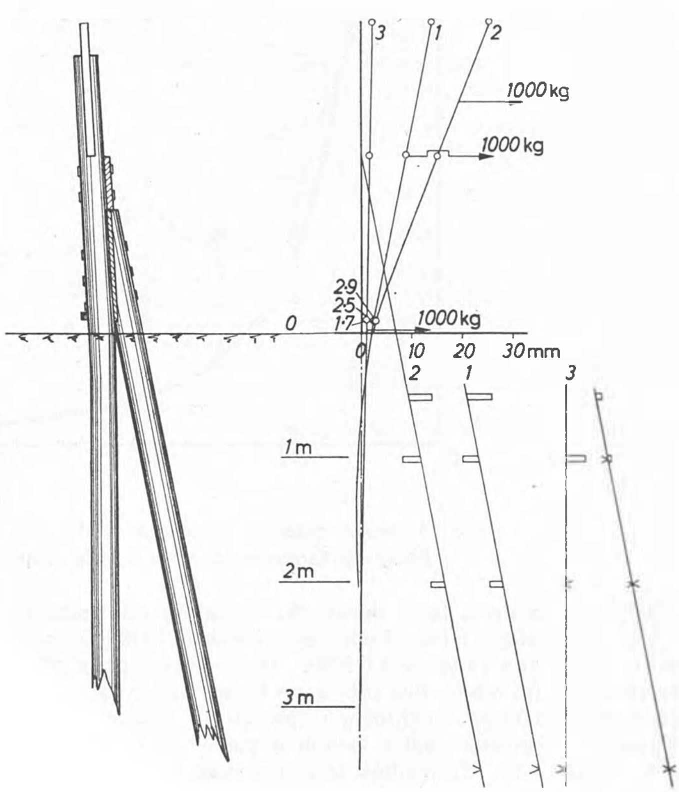

Fig. 6 Horizontal loading of a pile trestle

Charge horizontale d'un chevalet de pieux

This should be taken into consideration in design as it may cause breakage of the piles, especially long ones. Calculation of this double function of the piles is sometimes necessary, but it can be treated approximately as a simple static problem.

The recorded tests are from short-term loads on the piles. Tests with long-term loading are going on at the time of submitting this report (May, 1956) and will, it is hoped, give complementary results. For instance, the trestle has been subjected to a horizontal force for 1 month, and the calculated failure load based on axial forces alone is 50 per cent greater than the actual failure load. Nothing else has happened, however, except that the pile displacement at the soil surface has increased from 1 to 2 cm.

Buckling of Piles

When piles are driven through soft clay to rock they may buckle under load.

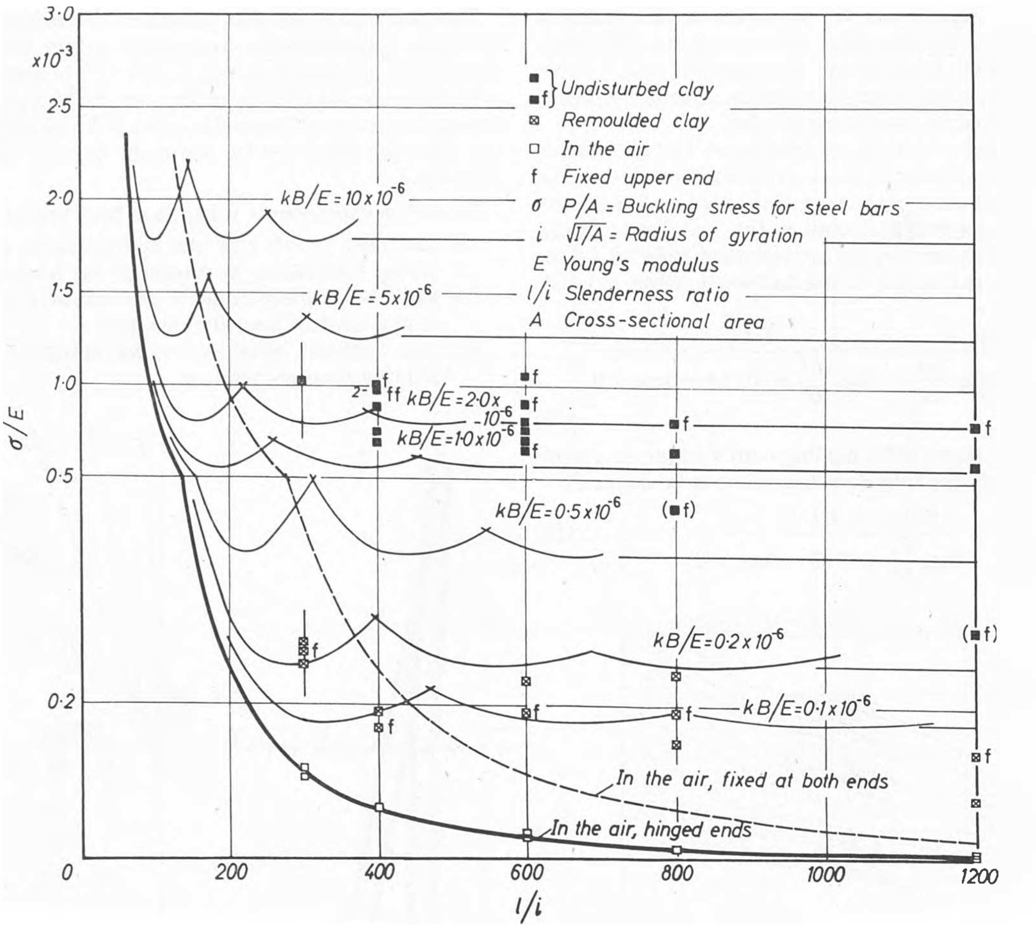

Fig. 7 Measured buckling stresses on model piles compared with theoretical values from equation 7
Efforts de flambage des pieux modèle comparés avec l'effort théorique selon l'équation 7

Earlier treatments have shown that buckling will seldom occur with normal types of piles, see FORSSELL (1918), GRANHOLM (1929) and CUMMINGS (1938). However, a type of pile is now used for which this rule is no longer valid, viz., steel piles with a compact cross-section (circular or square). The reason for choosing such a section is partly to reduce the risk of corrosion. In loading tests on such piles buckling has occurred before the bearing capacity was reached.

The general simplified theory makes the problem clear. With the vertical force P acting, equation 3 becomes:

$$EI.\frac{d^4y}{dx^4} + P.\frac{d^2y}{dx^2} + kB.y = 0 \qquad \dots. \ (6)$$

The limiting solution is an undamped sine wave from which is derived

$$P = n^2.\frac{\pi^2.EI}{l^2} + \frac{1}{n^2}.\frac{kB.l^2}{\pi^2} \qquad \dots. \ (7)$$

This equation gives a group of curves for each value of kB and the various numbers n (see Fig. 7 for circular piles). The minimum buckling load for a particular value of kB is

$$P_{min} = 2(kB.EI)^{\frac{1}{2}} \qquad \dots. \ (8a)$$

The buckling waves correspond to a somewhat shorter wavelength than the damped curve of equation 4, i.e. $l = \pi L/2^{\frac{1}{2}}$ instead of the earlier $l = \pi L$, and this makes some modification to the relation between G and kB/G in Fig. 5. The changes are, however, very small, only a 1 to 2 per cent increase in the values of kB/G, and the values determined from lateral loading tests ought thus to be applicable. The buckling tests give considerably smaller values, for example, kB = 3 to 4 kg/cm² in buckling

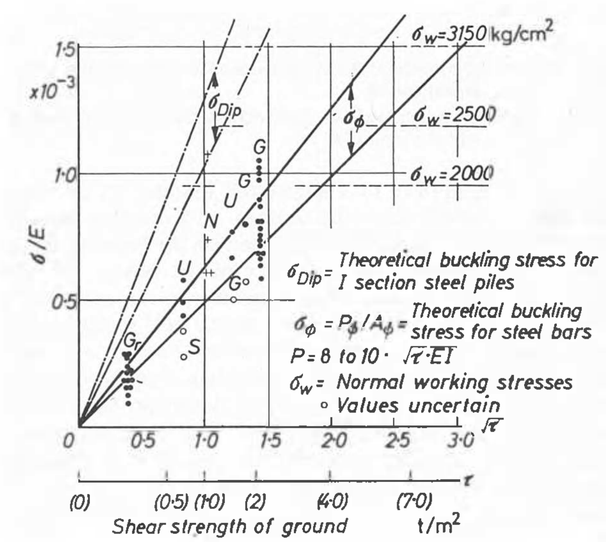

Fig. 8 Measured buckling stresses on steel rod piles in clay as function of shear strength (τ). Theoretical lines for round steel piles (σ_{ϕ}) and for I section piles (σ_{Dip}) according to equation 8b. Test results from piles at: G—Goteburg, U—Uddevalla, S—Sjolevad, N—Lademoen Church, Norway, and Gr—Model tests in remoulded Goteburg clay

Effort de flambage théorique des pieux en acier rond en fonction de la résistance au cisaillement. Ligne théorique pour pieux en I selon l'équation 8b. Résultats d'essais sur pieux à: G — Goteburg, U — Uddevalla, S — Sjolevad, N — L'église de Lademoen, Norge, et Gr — essais sur modèles dans une argile remaniée de Goteburg compared with 50 to 10 kg/cm^2 in lateral loading. Probably it is not until lateral loading causes failure of the clay that the kB values decrease to the buckling load values.

The writer has made a large number of model tests on piles of various materials (steel, brass, bronze and wood) of various shapes and dimensions. The results on circular steel piles are drawn in Fig. 7. It will be seen that there is a good grouping of the points near the lines for the two actual kB values. These tests have, in the main, only confirmed that equation 8a is of practical value, even though the assumption that the clay is an ideal elastic material is not fulfilled.

An extremely interesting result is, however, the one shown in Fig. 8. The results from several failure tests on steel piles of compact cross-section have been collected and related to the clay strength. The large groups of dots are mainly model pile tests, but the other points are from piles of 25 to 40 cm diameter on construction jobs. The figure, an outline of which the author showed in 1956, indicates a straight-line relation between σ_ϕ and $\sqrt{\tau}$. One or two values appear to be less, but the points marked with circles indicate that there is some uncertainty in the τ value or that the test load was extremely eccentric.

The equations of the straight lines are:

$$\sigma_{Dip} = 8 \text{ to } 10(\tau EI)^{\frac{1}{2}}$$

and

$$\sigma_{\phi} = 8 \text{ to } 10(\tau EI)^{\frac{1}{2}} \quad \ldots\ldots (8b)$$

where σ_{Dip} is the average buckling stress for piles of I section and σ_ϕ is the average buckling stress for piles of round section.

Tests for comparison between short-term tests and long-term tests have so far been too few to indicate any definite tendency.

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