Mathematical Expression of the CBR (California Bearing Ratio) Relations

Expression Mathématique des Abaques CBR

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Summary

The U.S. Corps of Engineers adopted the CBR method of design of flexible pavements for airfields some years ago, and has developed and extended the method since that time. Fundamental to the system is the relation between CBR and thickness of protective, overlying layer for the many loadings which must be considered. These relations have been portrayed as families of curves for which each extension or extrapolation had to be verified and for which some measure of consistency was lost each time the curves had to be replotted. This paper presents mathematical developments leading to a general equation which represents the pattern of present CBR relations for airfield pavement design in the range of CBR values below about 12. This equation is

$$h_t = \sqrt{P\left[\frac{1}{8{\cdot}1\mathrm{CBR}} - \frac{1}{p\pi}\right]}$$

where h_t = thickness in in., P = total load in lb., and p = tyre pressure in lb. per sq. in.

Sommaire

Il y a plusieures années, le U.S. Corps of Engineers adopta pour le calcul des revêtements souples la méthode dite de l'indice portant californien qu'il a, depuis, développée et amplifiée. Le principe fondamental du système est d'exprimer les relations entre l'indice CBR et l'épaisseur à donner au revêtement pour les nombreuses charges à envisager. Ces relations sont representées par des ensembles de courbes pour lesquelles toute extrapolation doit être vérifiée et pour lesquelles le degré d'exactitude est réduit chaque fois qu'elles sont retracées. L'objet de cette communication est de présenter une série de calculs qui aboutissent à une équation générale décrivant la configuration actuelle des formules CBR pour le calcul des pistes d'aérodromes dans le domaine des CBR inférieurs à 12. L'équation est

$$h_t = \sqrt{P\left[\frac{1}{8{\cdot}1\mathrm{CBR}} - \frac{1}{p\pi}\right]}$$

ou h_t = épaisseur en pouces, P = charge totale en livres, p = pression des pneus en livres par pouce carré.

Background

The CBR test is a tool for evaluating materials in flexible pavement systems utilizing rational analytical procedures. The CBR values are considered to be indications of the resistance of subgrade and base course materials to the stresses that may be imposed upon them by repetitive wheel loads. The design curves used with the CBR method express relationships between loads, protective thickness of superior material, and CBR of the pertinent layer in the flexible pavement system.

The Corps of Engineers of the United States Army adopted the CBR system for design of airfield flexible pavements following an exhaustive study of available design methods, and in the years since this adoption has developed the method extensively. The original relations established by the Highway Department of the State of California were first extrapolated with respect to load magnitude based on the experience of a small group of well-qualified consultants. Later theoretical shear-stress relationships, allowable deformations, and relationships between relative sizes of loaded areas were used to develop the flexible pavement design curves needed for the new, large, more concentrated aircraft loadings. Subsequently, theoretical relationships such as shear stresses, vertical stresses and vertical deflections have been used to study the pattern of the CBR design curves. In this theoretical work a uniform circular load has been used and is considered to represent reasonably well a tyre loading for the purposes of CBR developments. Graphical relationships have also been used. These studies, together with service behaviour records, show that the CBR design curves can be divided into two parts. At the greater depths beneath the surface of flexible pavements, required strengths are governed primarily by the gross magnitudes of applied loads. These greater depths are the zone in which the smaller CBR values are encountered. On the other hand, at the lesser depths beneath the pavement surface, the intensity of applied loads is the primary governing factor in the determination of required strengths. These lesser depths are the zone in which the larger CBR values are usually encountered. This division of the CBR relations can be better understood or visualized by considering, first, a point at a shallow depth beneath the edge of a tyre and, second, a point at an appreciable depth beneath the centre of an airplane wheel or assembly of wheels. In the first instance, proximity to failure is controlled by stress conditions beneath the edge of the tyre, as determined by intensity of stress beneath the tyre relative to the unloaded area adjacent to the tyre. For this case, the width (size) of the contact area and, therefore, the magnitude of the gross load has little importance. In the second instance, proximity to failure is controlled by stress conditions deep beneath the centre of the wheel or assembly loading. For this case, the way in which the load is applied at the surface has only a minor effect on the stress, while the stress is related to the total load directly.

The developments presented in this report are concerned with the CBR range below about 12 (greater depths) for which they establish a firm mathematical pattern interrelating the various pertinent factors. Though the spread in CBR values is small for these greater depths, it includes the predominant portion of the general pattern of CBR *versus* required overlying thickness relations. The relations in the range of large CBR values can be quite satisfactorily established by interpolation between the pattern established for the lower CBR range and minimum required pavement thicknesses for given base strengths. Work is being done on the development of mathematical expressions for the CBR relations in the large CBR range which shows considerable promise, but specific results are not yet available.

Mathematical Developments

Prior work (Corps of Engineers, Waterways Experiment Station, 1951, and FERGUS, 1950) has already established a relation between design thickness, wheel load and a constant, which has been shown to be dependent on the CBR, for single-wheel loads having the same contact pressure (tyre pressure)*. This relation is expressed as follows:

$$h_t = K\sqrt{P} \qquad \ldots . \ (1)$$

where: h_t = thickness requirement to protect a given CBR material; P = wheel load; K = a constant (shown to be dependent on the CBR).

The formula is arrived at in the following manner: Relations of the theory of elasticity for a uniformly loaded circular area (Corps of Engineers, Waterways Experiment Station, 1953) can be made to show that for a given intensity of surface load the stresses beneath total loads of different magnitudes will be equal at homologous points. It is reasonable to assume that the strength or CBR required will be the same at depths at which the stresses are identical. Therefore, it can be considered that for a given CBR and intensity of load, the depth of cover or thickness of protective layer for any magnitude of load must be such that the ratio of the thickness, h_t, to the radius of contact area, a, is a constant, C, or

$$\frac{h_t}{a} = C \qquad \ldots . \ (2)$$

The relation between load, P, load intensity, p, and radius of contact area, a, is:

$$P = p\pi a^2 \quad \text{or} \quad a = \sqrt{P} \times \frac{1}{\sqrt{p\pi}} \qquad \ldots . \ (3)$$

Combining equations 2 and 3 gives:

$$h_t = \frac{C}{\sqrt{p\pi}} \times \sqrt{P} \qquad \ldots . \ (4)$$

Now $C/(p\pi)^{\frac{1}{2}}$ being a constant can be designated as K. Substituting K for $C/(p\pi)^{\frac{1}{2}}$ results in equation 1,

$$h_t = K\sqrt{P} \qquad \ldots . \ (1)$$

This is the expression presented in earlier work (Corps of Engineers, Waterways Experiment Station, 1951, and FERGUS, 1950).

Use of the above relation permits the development of CBR curves for increasingly larger wheel loads from existing CBR curves. The relation has been shown to be valid for CBR values below about 15. In the initial development of the CBR relations, tyre pressure was not considered. Tyre pressures used originally in developing service behaviour for airplane loadings ranged from 60 to slightly above 100 lb./sq. in. When preparation of design curves for 200 lb./sq. in. tyres became necessary, the relationships already established were assumed to be valid for tyre pressures up to 100 lb./sq. in. It was found that the simple relation of equal stress at homologous points would not serve for extrapolating the design curves from 100 to 200 lb./sq. in. tyre pressures, but that the relations of the theory of elasticity for deflection at any depth beneath the centre of a uniform circular load could be used (Corps of Engineers, Waterways Experiment Station, 1951). The pertinent relation is:

$$\rho = 1{\cdot}5\frac{p}{E} \times \frac{a^2}{\sqrt{a^2 + h_t^2}} \qquad \ldots . \ (5)$$

where: ρ = deflection; E = modulus of elasticity. p, a and h_t are the same as in equations 2 and 3.

By considering that an increase in tyre pressure for a given load would require an increase in protective thickness (depth) such that the theoretical deflections would be equal, CBR relations were developed for higher tyre pressures. These have since been validated by field test results (Corps of Engineers, Waterways Experiment Station, 1953).

Equation 5 can be combined with equation 3 to give the following:

$$\rho = 1{\cdot}5\frac{P}{\pi E} \times \frac{1}{(a^2 + h_t^2)^{\frac{1}{2}}} \quad \text{or} \quad a^2 + h_t^2 = \left(\frac{1{\cdot}5 \times P}{\rho\pi E}\right)^2 \qquad \ldots . \ (6)$$

For a given material E is constant, and if the total load is considered to be constant and tyre pressure permitted to vary, the right-hand side of the last expression for equation 6 is constant when the deflection is constant, and the following expression can be written:

$$a^2 + h_t^2 = C' \qquad \ldots . \ (7)$$

where a and h_t are as defined previously and C' is a constant.

It is now possible to examine the effect on thickness requirements of changes in gross load and of changes in tyre pressure. First, consider a change in gross load from P_a to P_b. From equation 1, the following can be written:

$$\frac{h_{ta}}{(P_a)^{\frac{1}{2}}} = K = \frac{h_{tb}}{(P_b)^{\frac{1}{2}}} \quad \text{or} \quad \frac{h_{tb}}{h_{ta}} = \left(\frac{P_b}{P_a}\right)^{\frac{1}{2}} \qquad \ldots . \ (8)$$

where $p_a = p_b$.

Now consider a change in tyre pressure from p_b to p_c with no change in gross load ($P_b = P_c$). From equation 7, the following can be written:

$$a_b^2 + h_{tb}^2 = C' = a_c^2 + h_{tc}^2 \quad \text{or} \quad h_{tc} = (h_{tb}^2 + a_b^2 - a_c^2)^{\frac{1}{2}} \qquad \ldots . \ (9)$$

where $P_b = P_c$.

Squaring equations 8 and 9 and combining gives:

$$h_{tc}^2 = h_{ta}^2\frac{P_b}{P_a} + a_b^2 - a_c^2 \qquad \ldots . \ (10)$$

This can be developed in the following manner to give an expression for the combined effect of variations in total load and tyre pressure:

$$h_{tc}^2 = h_{ta}^2\frac{P_b}{P_a} + \frac{\pi p_b a_b^2}{\pi p_b} - \frac{\pi p_c a_c^2}{\pi p_c}$$

but, $\pi p a^2 = P$, therefore:

$$h_{tc}^2 = h_{ta}^2\frac{P_b}{P_a} + \frac{P_b}{\pi p_b} - \frac{P_c}{\pi p_c}$$

and since, $p_a = p_b$ and $P_b = P_c$:

$$h_{tc}^2 = h_{ta}^2\frac{P_c}{P_a} + \frac{P_c}{\pi}\left(\frac{1}{p_a} - \frac{1}{p_c}\right)$$

This may be written:

$$h_{tc} = \left\{P_c\left[\frac{h_{ta}^2}{P_a} + \frac{1}{\pi}\left(\frac{1}{p_a} - \frac{1}{p_c}\right)\right]\right\}^{\frac{1}{2}} \qquad \ldots . \ (11)$$

It may also be written:

$$\frac{h_{ta}^2}{P_a} + \frac{1}{p_a\pi} = \frac{h_{tc}^2}{P_c} + \frac{1}{p_c\pi}$$

Since the subscripts represent arbitrary sets of values, the following more general expression can be written:

$$\frac{h_{ta}^2}{P_a} + \frac{1}{p_a\pi} = \frac{h_{tc}^2}{P_c} + \frac{1}{p_c\pi} = \frac{h_{te}^2}{P_e} + \frac{1}{p_e\pi} = \ldots = \frac{h_{ta}^2}{P_n} + \frac{1}{p_n\pi}$$

Or, since any set of values combined in this way must equal any other set so combined, it follows that the expression must equal a constant:

$$\frac{h_t^2}{P} + \frac{1}{p\pi} = D \qquad \ldots . \ (12)$$

where D is a constant.

* For the purposes of these developments, contact pressure and tyre pressure are considered to be identical, and in the range of aircraft tyre pressures at present in use the errors introduced by such an assumption are small.

This expression is similar to equation 1 except that it will accommodate variations in tyre pressure as well as in gross load. The similarity of equations 1 and 12 is readily apparent, and their combination as follows provides a relation between the constants D and K. This relation is:

$$D = K^2 + \frac{1}{p\pi} \quad \ldots\ldots (13)$$

Since K was shown in earlier work (Corps of Engineers, Waterways Experiment Station, 1951, and FERGUS, 1950) to be dependent on the CBR, it follows that D is also dependent on CBR.

Values for the constant K were carefully developed as shown in FERGUS (1950). The values developed in that work and in Corps of Engineers, Waterways Experiment Station (1951), are shown in the following table. These are from single-wheel CBR curves for design of flexible airfield pavements.

Table

CBR	*Values of K*		*Values of K^2*		*Values of $D = K^2 + \frac{1}{p\pi}$*		*Values of $D \times$ CBR*	
	For 100-lb./sq. in. CBR *curves*	*For* 200-lb./sq. in. CBR *curves*	*For* 100-lb./sq. in. CBR *curves*	*For* 200-lb./sq. in. CBR *curves*	*For* 100-lb./sq. in. CBR *curves*	*For* 200-lb./sq. in. CBR *curves*	*For* 100-lb./sq. in. CBR *curves*	*For* 200-lb./sq. in. CBR *curves*
3	0·195	0·199	0·03803	0·03961	0·04121	0·04120	0·124	0·124
4	0·166	0·171	0·02755	0·02923	0·03073	0·03082	0·123	0·123
5	0·147	0·152	0·02161	0·02311	0·02479	0·02470	0·124	0·124
6	0·132	0·138	0·01742	0·01905	0·02060	0·02064	0·124	0·124
7	0·120	0·126	0·01440	0·01588	0·01758	0·01747	0·123	0·122
8	0·111	0·118	0·01232	0·01392	0·01550	0·01551	0·124	0·124
9	0·103	0·110	0·01060	0·01210	0·01378	0·01369	0·124	0·123
10	0·096	0·104	0·00921	0·01081	0·01239	0·01239	0·124	0·124
12	0·085	0·093	0·00723	0·00865	0·01041	0·01024	0·125	0·123
15	0·073	0·082	0·00533	0·00672	0·00851	0·00831	0·128	0·124
17	0·067	0·075	0·00449	0·00563	0·00767	0·00722	0·130	0·123
20	0·059	0·068	0·00348	0·00462	0·00666	0·00621	0·133	0·124

The table also develops values for the constant D from those for the constant K. Study of the values developed showed a relationship between D and the CBR. Their simple product was found to be substantially constant for CBR values below about 12. The last column in the table shows this. The resulting constant has an average value of 0·1236. Thus, the following relation can be written:

$$D \times \text{CBR} = 0{\cdot}1236 \quad \text{or} \quad D = \frac{0{\cdot}1236}{\text{CBR}} \quad \ldots\ldots (14)$$

This may also be written as:

$$D = \frac{1}{8{\cdot}1\text{CBR}}$$

This value for D can now be substituted in equation 12 and the result solved for h_t to give a general relation directly involving gross load in lb., load intensity in lb./sq. in., thickness in in. and CBR as follows:

$$h_t = \sqrt{P\left[\frac{1}{8{\cdot}1\text{CBR}} - \frac{1}{p\pi}\right]} \quad \ldots\ldots (15)$$

This expression is a representation of the CBR design relations for values of CBR less than 12.

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