# INTERNATIONAL SOCIETY FOR SOIL MECHANICS AND GEOTECHNICAL ENGINEERING 



This paper was downloaded from the Online Library of the International Society for Soil Mechanics and Geotechnical Engineering (ISSMGE). The library is available here:
https://www.issmge.org/publications/online-library
This is an open-access database that archives thousands of papers published under the Auspices of the ISSMGE and maintained by the Innovation and Development Committee of ISSMGE.

# Earth Pressures and Bearing Capacity Calculations by Generalized Procedure of Slices 

Les Poussées et la Force Portante des Sols-Calculs et Méthode Généralisée des Tranches

by N. Janbu, D.Sc., Norwegian Geotechnical Institute, Trondheim Laboratory, Norway

## Summary

A generalized procedure of slices for composite slip surfaces of any shape (Janbu, 1954) has herein been expanded to include earth pressure and bearing capacity calculations.
As examples of application, formulae and numerical values have been obtained for earth pressure coefficients and bearing capacity factors. Moreover, Terzaghi's formula for strip footings has been modified for inclusion of inclined loads. By independent control stability analyses, accuracy of the obtained formulae is indicated.
The factor of safety with respect to shear strength is recommended, whether the problem be concerned with earth pressure, bearing capacity or slope stability. Therefore a separate study of the relationship between the proposed and the orthodox safety factors is included, from which appropriate numerical values may be selected in accordance with past experience.

## Working Formulae

The principal considerations leading to the working formulae are briefly reviewed below. For this purpose reference is made to Fig. 1, showing key sketches, primary notations and equilibrium conditions.

Definition of safety factor-Let the shearing resistance $\tau_{f}$ along an element of a potential slip surface be given in terms of effective normal stress $\sigma^{\prime}$ by the empirical equation,



## Sommaire

Dans cette étude un procédé généralisé par tranches pour surfaces de glissement composites d'une forme quelconque (JANBU, 1954) a été étendu pour y inclure les calculs de la pression du sol et de la force portante.

Comme exemples d'application on a obtenu des formules et des valeurs numériques des coefficients de pression du sol et des facteurs de force portante. De plus, la formule de Terzaghi pour fondations sur semelles a été modifiée pour y introduire les charges inclinées. L'exactitude des formules obtenues se trouve indiquée par des analyses indépendantes de contrôle de la stabilité.

On recommande un facteur de sécurité concernante la résistance au cisaillement, qu'il s'agisse de problèmes portant sur la pression du sol, sa force portante ou de la stabilité de la pente. C'est pourquoi l'auteur fournit une étude spéciale sur le rapport entre les facteurs de sûreté proposés et les facteurs classiques, permettant de choisir des valeurs numériques convenables répondant aux expériences antérieures.

Along this element the shear stress $\tau$ necessary for equilibrium can be expressed as a certain portion of $\tau_{f}$, say

$$
\begin{equation*}
\tau=\tau_{j} / F=c_{e}+\sigma^{\prime} \tan \phi_{e} \tag{2}
\end{equation*}
$$

in which $c_{e}=c^{\prime} / F, \tan \phi_{e}=\tan \phi^{\prime} / F$, and $F$ is the familiar factor of safety with respect to shear strength.

Normal stress and shear stress-Condition (a) in Fig. 1 leads Equations of equilibrium for each slice

$$
\begin{align*}
& \text { Vertical: } \mathrm{d} W+\mathrm{d} P+\mathrm{d} T=\mathrm{d} S \sin \alpha+\mathrm{d} N \cos \alpha \quad \text { (a) } \\
& \text { Horizontal: } \mathrm{d} E-\mathrm{d} Q=-\mathrm{d} S \cos \alpha+\mathrm{d} N \sin \alpha  \tag{b}\\
& \left.\begin{array}{l}
\text { Morment }, \\
\text { about } M
\end{array}\right\}: T \mathrm{~d} x+E \mathrm{~d} y_{t}-\mathrm{d} E h_{t}+\mathrm{dQz}=0  \tag{c}\\
& \text { Equilibrium of whole sliding mass, consider free body } \\
& \text { Vertical }\left\{\begin{array}{c}
\int_{0}^{b}(\mathrm{~d} S \sin a+\mathrm{d} N \cos \alpha)=\int_{0}^{b}(\mathrm{~d} W+\mathrm{d} P)+T_{b} \\
\text { i.e. } \int_{0}^{b} \mathrm{~d} T=T_{b}
\end{array}\right.  \tag{al}\\
& \text { Horizontal }\left\{\begin{array}{c}
\int_{0}^{b}(-\mathrm{d} S \cos \alpha+\mathrm{d} N \sin \alpha)=E_{b}-Q \\
\text { i.e. } \int_{0}^{b} \mathrm{~d} E=E_{b}
\end{array}\right. \tag{bl}
\end{align*}
$$

Abbreviations used in text:

$$
p=\frac{d W}{d x}+\frac{d P}{d x}=\gamma \cdot z+q, \quad t=\frac{d T}{d x}
$$

Fig. 1 Illustration of the elementary conditions of equilibrium for the generalized procedure of slices Illustration des conditions élémentaires d'équilibre pour le procédé généralisé par tranches
to the following formulae for effective normal stress $\sigma^{\prime}=$ $\mathrm{d} N / \mathrm{d} l-u$,

$$
\begin{equation*}
\sigma^{\prime}=(p+t-u)-\tau \tan \alpha \tag{3}
\end{equation*}
$$

Equations 2 and 3 yield,

$$
\begin{equation*}
\tau=\frac{c_{e}+(p+t-u) \tan \phi_{e}}{1+\tan \alpha \tan \phi_{e}} \tag{4}
\end{equation*}
$$

Internal forces and boundary conditions-When eliminating $\mathrm{d} N$ from conditions (a) and (b) one finds,

$$
\begin{equation*}
\mathrm{d} E=\mathrm{d} Q+(p+t) \tan \alpha \mathrm{d} x-\tau \cos ^{-2} \alpha \mathrm{~d} x \ldots \tag{5}
\end{equation*}
$$

The horizontal force $E$ at any vertical section is obtained by integrating equation 5 from 0 to $x$.

The corresponding shear force $T$ is expressed in terms of $E$ and $Q$, on dividing condition (c) by $\mathrm{d} x$,

$$
\begin{equation*}
T=-E \tan \alpha_{t}+h_{f} \frac{\mathrm{~d} E}{\mathrm{~d} x}-z \frac{\mathrm{~d} Q}{\mathrm{~d} x} \tag{6}
\end{equation*}
$$

If a resultant force at a boundary face, $x=b$, is represented by the components $E_{b}$ and $T_{b}$, the overall, directional equilibrium of the free body requires

$$
\begin{equation*}
\int_{0}^{b} \mathrm{~d} T=T_{b}, \quad \int_{0}^{b} \mathrm{~d} E=E_{b} \tag{7}
\end{equation*}
$$

Overall moment equilibrium is satisfied by equation 6.
Stability criterion-For the proposed procedure it is most convenient to use the overall condition of equilibrium in horizontal direction as stability criterion, equations 5 and 7

$$
\begin{equation*}
Q+\int_{0}^{b}(p+t) \tan \alpha \mathrm{d} x-\int_{0}^{b} \tau \cos ^{-2} \alpha \mathrm{~d} x=E_{b} \tag{8}
\end{equation*}
$$

In the derivation no simplifying assumption has been made regarding the shape of the potential slip surface. The criterion is therefore applicable to any specified or chosen surface, whether the problem is to determine $F$ (e.g. slope stability analysis) or to determine earth pressure or bearing pressure corresponding to a specified $F$.

For a specified value of $F$ the stability criterion is more complete when we substitute for $\tau$ the value given by equation 4. Observing that $\tan \alpha \mathrm{d} x=\mathrm{d} y$, equations 4 and 8 yield,

$$
\begin{align*}
E_{b}- & \int_{0}^{b} u \mathrm{~d} y=Q+\int_{0}^{b}[(p+t-u) \\
& \left.+c_{e} \cot \phi_{e}\right] \tan \left(\alpha-\phi_{e}\right) \mathrm{d} x-\int_{0}^{b} c_{e} \cot \phi_{e} \mathrm{~d} y \ldots \tag{8a}
\end{align*}
$$

For the purpose of determining $F$ a working formula is obtained by introducing $\tau_{f}=F \tau$ into equations 4 and 8 and solving the latter with respect to $F$. For finite differences,

$$
F=\frac{\Sigma \tau_{f} \cos ^{-2} \alpha \Delta x}{Q-E_{b}+\Sigma(p+t) \tan \alpha \Delta x}
$$

where

$$
\begin{equation*}
\tau_{f}=\frac{c^{\prime}+(p+t-u) \tan \phi^{\prime}}{1+\tan \alpha \tan \phi^{\prime} / F} \tag{8b}
\end{equation*}
$$

(When $\tau_{f}$ is introduced into the well known moment equation for slip circle analyses one obtains the formula derived by Bishop, 1954.)

Calculation procedure-The working formulae contain the quantity $t$, which is statically indeterminate as long as the actual stress conditions are not explored. However, by assuming a reasonable position of the line of thrust, accurate values of the corresponding internal forces, $E$ and $T$, are obtained from equations 5 and 6 by means of a successive approximation procedure.

First, initial values $E_{0}$ and $T_{0}$ are calculated for the condition $t=0$. From $T_{0}$ one obtains $t_{0}=\mathrm{d} T_{0} / \mathrm{d} x$, which when introduced into equations 5 and 6 leads to improved values $E_{1}$ and $T_{1}$, and so on. In most cases the convergence is very rapid.

Often, it is possible to obtain directly accurate results without successive approximation, particularly when $T$ can be estimated at the outset of the calculation. In such cases it is recommended to use equation 6 as control of whether or not the estimated $T$ is reasonable. Both procedures are used herein.

The application of the working formulae require a consistent use of signs. It is therefore important to notice the positive directions of all external and internal forces defined in Fig. 1.

## Application to Earth Pressure Calculation

The calculations below are restricted to dry sand, i.e. $\boldsymbol{c}^{\prime}=0$, $u=0$ and $\phi^{\prime}=\phi$. Since it is possible to deal with active and passive earth pressure simultaneously, the double subscript, ap is applied. Moreover, when double signs occur, the upper and lower signs refer to active and passive pressure, respectively.

Principal derivations-Reference is made to Fig. 2a. Introducing $c^{\prime}=0, u=0, Q=0$ and $E_{b}=P_{a p}$ into equation 8 a and observing that $\tau$ is negative for the passive case, one obtains for the horizontal component,

$$
\begin{equation*}
P_{a p}=\int_{0}^{b}(p+t) \tan \left(\alpha \mp \phi_{e}\right) \mathrm{d} x \tag{9}
\end{equation*}
$$

The vertical component at the contact face $b-b$ is expressed as,

$$
\begin{equation*}
T_{a p}=\mp P_{a p} \tan \delta_{e} \tag{10}
\end{equation*}
$$

in which $\tan \delta_{e}=\tan \delta / F$, where $\tan \delta$ represents wall friction (see Fig. 3). Introducing equations 10 and 5 into 6 , the boundary condition in vertical direction along $b-b$ is established
$\mp \tan \delta_{e}=-\tan \alpha_{f w}+\frac{H_{t}}{P_{a p}}\left(\gamma H+q+t_{w}\right) \tan \left(\alpha_{w} \mp \phi_{e}\right) \ldots$
The symbols are defined in Fig. 2 and $q=\mathrm{d} P / \mathrm{d} x$
For inclined walls equations 9 and 11 are first applied to an imaginary vertical section $b-b$ through the top (for inward) or the bottom of the wall (for outward inclinations). The pressure components on the actual contact face are obtained by means of a force polygon for the triangular slice located between the contact face and section $b-b$.

Plane surfaces-When applying plane slip surfaces rising at an angle $\alpha=\alpha_{w}$ with the horizontal, equations 9 and 10 yield for vertical wall and horizontal terrain surface,

$$
\begin{equation*}
P_{a p}=\frac{\cot \alpha \tan \left(\alpha \mp \phi_{e}\right)}{1 \pm r \tan \phi_{e} \tan \left(\alpha \mp \phi_{e}\right)}\left(\frac{1}{2} \gamma H^{2}+q H\right) \tag{12}
\end{equation*}
$$

The abbreviation $r$ represents the 'roughness ratio',

$$
\begin{equation*}
r=\frac{\tan \delta_{e}}{\tan \phi_{e}}=\frac{\tan \delta}{\tan \phi} \tag{13}
\end{equation*}
$$

The critical earth pressure coefficient in equation 12 is found from the requirement $\mathrm{d} P_{a p} / \mathrm{d} \alpha=0$, which leads to,

$$
\begin{align*}
K_{a p} & =\frac{\cot ^{2} \alpha_{a p}}{1+r} \\
\cot \alpha_{a p} & =\frac{\left\{(1+r)\left(1+\tan ^{2} \phi_{e}\right)\right\}^{\frac{1}{2}} \mp(1+r) \tan \phi_{e}}{1-r \tan ^{2} \phi_{e}} \tag{14}
\end{align*}
$$

Herein, $\alpha_{a p}$ is the angle between the critical plane and the horizontal.

Particularly, for smooth walls ( $r=0$ ), the following classical results are derived from equation 14 ,
$K_{a p}=\tan ^{2}\left(45^{\circ} \mp \frac{1}{2} \phi_{e}\right), \quad$ for $\quad \alpha_{a p}=45^{\circ} \pm \frac{1}{2} \phi_{e} \ldots$
Furthermore, for $r=-1$ one obtains

$$
\begin{equation*}
K_{a p}=\cos ^{2} \phi_{e}, \quad \text { for } \quad \alpha_{a p}=90^{\circ} \tag{14b}
\end{equation*}
$$

corresponding to a potential slip along the wall.
Composite surfaces-For the initial investigation, the conditions $t=0$ and $q=0$ are considered. Moreover, the line
(a)

(b)

Active

$R=H e^{\omega \tan \phi_{e}} \quad \frac{L}{H}=\sin \left(45^{\circ}+\frac{1}{2} \phi_{e}\right) e^{\left(\frac{\pi}{4}+\frac{1}{2} \phi_{e}\right) \tan \phi_{e}}$

Fig. 2 Key sketches for earth pressure calculations Schèmas pour le calcul de la pression du sol


Key sketch and working formulae for active case:


Fig. 3 Earth pressure coefficients in sand for horizontal terrain and vertical walls of different roughness Coefficients de pression du sol dans du sable pour terrain horizontal et murs verticaux de rugosités différentes
of thrust is drawn through the lower third points of each slice. Accordingly, equations 9 and 11 can be written as follows:

$$
\begin{gather*}
K_{a p}=\frac{2}{H^{2}} \int_{0}^{b} z \tan \left(\alpha \mp \phi_{e}\right) \mathrm{d} x  \tag{15}\\
\mp \tan \delta_{e}=-\frac{2}{3} \tan \alpha_{w}+\frac{2}{3 K_{a p}} \tan \left(\alpha_{w} \mp \phi_{e}\right) \ldots \tag{15a}
\end{gather*}
$$

For a given value of $\phi_{e}$ corresponding values of $K_{a p}$ and $\tan \delta_{e}$ can be obtained for any chosen slip surface by means of equations 15 and 15a. Since both equations must be satisfied simultaneously, the condition $t=0$ (in the soil) is equivalent to a concentrated shear force along the wall. The consequences of this simplification are investigated separately by a numerical example.

If the potential slip surfaces are represented by combinations of straight lines and logarithmic spirals with their centres at the top of the wall, such as shown in Fig. 2b, it is seen that $\alpha_{w}= \pm \phi_{e}$ and $z$ tan ( $\alpha \mp \phi_{e}$ ) equals $x$ and $2 L-x$ in intervals 0 to $L$, and $L$ to $2 L$, respectively. Hence, from equations 15 and 15 a ,
$K_{a p}=2\left(\frac{L}{H}\right)^{2}=2 \sin ^{2}\left(45^{\circ} \mp \frac{1}{2} \phi_{e}\right) \exp \left(\phi_{e} \mp \frac{\pi}{2}\right) \tan \phi_{e} \ldots$.
for,

$$
\begin{equation*}
\tan \delta_{e}=\frac{2}{3} \tan \phi_{e}, \quad \text { or } \quad r=\frac{2}{3} \tag{16a}
\end{equation*}
$$

For logarithmic spirals with their centres on line $M-M^{\prime}$ (to both sides of the top of the wall) the following formulae were derived

$$
\begin{equation*}
K_{a p}=2\left(\frac{L}{H}\right)^{2}+\frac{L}{H} \tan \omega_{0} \tag{17}
\end{equation*}
$$

for,

$$
\begin{equation*}
\pm \tan \delta_{e}=-\frac{2}{3} \tan \left(\omega_{0} \pm \phi_{e}\right)+\frac{2}{3 K_{p}} \tan \omega_{0} \tag{17a}
\end{equation*}
$$

The positive value of $\omega_{0}$ is defined in Fig. 2 b , otherwise the symbols are the same as above. The last term in equation 17 is slightly simplified without significant loss in accuracy.

Values of $K_{a}$ and $K_{p}$-Numerical values of the earth pressure coefficients have been evaluated, and they are plotted versus $\tan \phi_{e}$ for different $r$, Fig. 3. For positive $r$, equations 16 and 17 were applied. For negative $r$, equation 14 was used since control investigations by means of composite slip surfaces indicated sufficient accuracy, except perhaps near $r=-1$.

An all round comparison between plane and composite slip surfaces verified previous findings. The differences are small in the active zone, while, in the passive zone, the plane surfaces lead to errors on the unsafe side for positive $r$. Moreover, these errors increased rapidly with increasing $\phi_{e}$ and $r$. Particularly, for $\phi_{e}=\delta_{e}=45$ degrees the plane surface yields $K_{p}=\infty$, while for composite slip surfaces values between 22 and 25 have been obtained. For further comparison reference is made to Caquot and Kérisel (1949), and Brinch Hansen (1953).

The orthodox formulae for $P_{a}$ and $P_{p}$, given in Fig. 3, are slightly incorrect for composite slip surfaces, since the coefficients before the $\gamma$ - and the $q$-terms are not identical for such surfaces. However, the numerical example below appears to indicate that this simplification is justified in practice.

Accuracy investigations-The data given in Fig. 4 lead to the following passive earth pressure component, according to Fig. 3, and using $F=1.50$.

$$
P_{p}=176 \mathrm{t} / \mathrm{m}, \quad T_{p}=67.8 \mathrm{t} / \mathrm{m}
$$

Let it now be assumed that the calculated components are actually acting on the wall. For comparison the corresponding factor of safety can be explored by means of stability analysis, carried out independently of the formula calculations. For this purpose equation 8 b is applicable.

The four slip surfaces used in the analysis are logarithmic spirals with their centres on line $M-M^{\prime}$, combined with straight lines inclined at 45 degrees $-\frac{1}{2} \phi_{e}$ with the horizontal.

In the first part of the analysis, the distribution of $T$ was assumed, according to the formula, Fig. 4,

$$
\begin{equation*}
T=\xi^{n} T_{p} \tag{18}
\end{equation*}
$$

For surface No. 1 different values of $n$ were used. The safety factors calculated by equation 8 b are assembled in a table in Fig. 4, from which it is indicated that $F$ approaches 1.50 for increasing $n$, which is in accordance with the assumption of a concentrated shear force at the wall.

In the second part of the investigation, $T$ was calculated by step-wise approximation, using $h_{t} \simeq 0.4 z$ in equation 6. It was then disclosed that the calculated and assumed $T$ were


Fig. 4 Control stability analyses for determining the accuracy of the formula calculation of passive earth pressure
Analyses de stabilité de contrôle pour la détermination de l'exactitude de la formule de calcul de la pression passive du sol
almost identical for $n=2$ (see plot of internal forces in Fig. 4).
For the four surfaces the calculated values of $F$ are plotted versus the point of intersection at ground level. A minimum value of $F=1.54$ is indicated.

Since the difference between the applied $F=1.50$ (for concentrated $T$ ) and the calculated minimum $F=1.54$ (for distributed $T$ ) is so small, it is believed that the only consequence of the simplifications made is that the locations of the corresponding critical spirals are different.

In this connection it may also be of interest to observe the fairly wide zone within which $F$ varies between fairly narrow limits, say between 1.54 and 1.57 .

## Application to Bearing Capacity Calculations

Herein, equation 8a is applied for the calculation of allowable bearing pressures on infinitely long strip footings under inclined


Fig. 5 Key sketches for bearing capacity calculations
Schémas pour les calculs de la force portante
loads. The calculations are restricted to centric loads and $u=0$.
Principal derivation-The bearing capacity of shallow strip footings under inclined loads can be determined, at least approximately, by the following modified equation, Fig. 5.

$$
\begin{equation*}
\frac{P_{v}+N_{h} P_{h}}{B}=\frac{1}{2} N_{\gamma} \gamma B+N_{q} \gamma D+N_{c} c_{e} \tag{19}
\end{equation*}
$$

wherein

$$
P_{h} \leqslant P_{\nu} \tan \phi_{e}
$$

On substituting for $p$, according to Fig. 5a, and comparing equations 8 a and 19 term for term, one finds for $T=0$,

$$
\begin{gather*}
N_{h}=\frac{B}{I}, \quad N_{\gamma}=-\frac{2}{B I} \int_{0}^{b} z \tan \left(\alpha-\phi_{e}\right) \mathrm{d} x \\
N_{q}=-\frac{1}{I} \int_{B}^{b} \tan \left(\alpha-\phi_{e}\right) \mathrm{d} x  \tag{20}\\
N_{c}=\frac{\cot \phi_{e}}{I}\left[d-\int_{0}^{b} \tan \left(\alpha-\phi_{e}\right) \mathrm{d} x\right]
\end{gather*}
$$

Herein, the abbreviation $I$ represents the integral

$$
\begin{equation*}
I=\int_{0}^{B} \tan \left(\alpha-\phi_{e}\right) \mathrm{d} x \tag{20a}
\end{equation*}
$$

The condition $T=0$ leads to conservative $N$-values. However, by assuming a reasonable distribution of $T$ for each term ( $P_{h}, \gamma, \gamma D$ and $c_{e}$ ) improved $N$-values are obtained. Moreover, equation 6 is applicable for control of the assumed resultant $T$.

Shallow footing on horizontal terrain-For the simple case illustrated in Fig. 5b the bearing capacity factors can be obtained directly from the condition of equilibrium in horizontal direction. For $c=0$

$$
\begin{equation*}
\frac{P_{v}}{B}+\frac{P_{h}}{H K_{a}}=\frac{1}{2}\left(\frac{K_{p}}{K_{a}}-1\right) \gamma H+\frac{K_{p}}{K_{a}} \gamma D \tag{21}
\end{equation*}
$$

On comparing equations 19 and 21, and introducing for $K_{a}$ and $K_{p}$ according to equation 16 , one finds
$N_{h}=\frac{B}{H K_{a}}=\sec \left(45^{\circ}+\frac{1}{2} \phi_{e}\right) \exp \left(\frac{\pi}{4}-\frac{1}{2} \phi_{e}\right) \tan \phi_{e} \ldots$
$N_{q}=\frac{K_{p}}{K_{a}}=\tan ^{2}\left(45^{\circ}+\frac{1}{2} \phi_{e}\right) \exp \pi \tan \phi_{e}$
$N_{\gamma}=\left(\frac{K_{p}}{K_{a}}-1\right) \frac{H}{B}=\frac{1}{2} N_{h}\left(N_{q}-1\right)$
Furthermore, since $d=0$, the following relationship is derived from equation 20

$$
N_{c}=\left(N_{q}-1\right) \cot \phi_{e}
$$

The values obtained for $N_{c}$ and $N_{q}$ are in agreement with the Prandtl theory for smooth base. Moreover, a comparison
between the values of $N_{\gamma}$ calculated from equation 22 c and the corresponding values obtained by Terzaghi (1943) show that the differences are small. The formula for $N_{h}$, however, lacks any basis for direct comparison, since it is used herein for the first time. For an indirect comparison reference is made to Meyerhof (1953).

The bearing capacity factors are plotted versus $\tan \phi_{c}$ in Fig. 6.

Accuracy investigations-When utilizing the data given in Fig. 7, a formula calculation (Fig. 6) for vertical load yields,

$$
P_{v}=56 \mathrm{t} / \mathrm{m}, \quad(\text { for } F=1 \cdot 5)
$$

For the sake of comparison the stability of this footing is explored independently by means of equation $8 b$. Three


Fig. 6 Bearing capacity numbers for shallow strip footings acted upon by centric loads of different inclinations
Valeurs de force portante pour semelles peu profondes sous l'action de charges centriques d'inclinaisons différentes
different distributions of the contact pressure are considered. For each case both calculated and assumed distributions of $T$ are used, including the initial step $T=0$. The result of the investigation is assembled in a table in Fig. 7.

For cases (b) and (c), with a maximum contact pressure under the centre of the footing, the obtained values of $F$ are close to $1 \cdot 50$, as used in the formula calculation of $P_{v}$. A further comparison appears to indicate that a rigid footing (case a) is slightly safer than a flexible footing (cases bor c) for otherwise equal conditions.

Nearby trial surfaces above and below the 'Prandtl surface' indicated equal or slightly higher safety factors.

As another example an inclined load is considered (Fig. 8) utilizing the same soil constants as above. The formula calculation for $F=1.5$ yields, Fig. 6

$$
P_{v}=34 \mathrm{t} / \mathrm{m}, \quad P_{h}=9 \mathrm{t} / \mathrm{m}
$$

For control, separate stability analyses are carried out along 6 composite slip surfaces shown in Fig. 8.
Sofety factors oblained

| Load | For $T=0$ | Assumed | Catculated |
| :---: | :---: | :---: | :---: |
| distribulion | (Initial step) | $T$ | $T$ |
| (a) Uniform | 1.39 | 155 | 1.58 |
| (b) Theoretical | 1.375 | 153 | 1.54 |
| (c) Parabolic | 1.36 | 151 | 1.48 |




Fig. 7 Stability investigation of a strip footing on sand with vertical load and different contact pressure distributions
Recherches de stabilité d'une fondation à semelle sur sable avec charge verticale et différentes répartitions de la pression de contact.

The obtained values of $F$ are plotted versus the point of intersection between the slip surface and level $A-A^{\prime}$. From this plot it is indicated that $F_{\text {min }}=1.47$, while $F=1.50$ was used in the formula calculation. It is also interesting to notice the wide zone within which the calculated $F$ varies between the narrow limits of 1.47 to 1.50 .

Altogether, the investigations above indicate that the formula obtained for shallow strip footings is accurate, even though the actual critical slip surface for inclined loads is not as deep seated as that used for the formula derivations.

## Concluding Remarks

The generalized procedure of slices, presented herein, has a wide field of application, since it is not limited to any particular shape of the potential slip surface, nor is it restricted to simple, uniform soil conditions, or to any particular category of those soil stability problems where potential slip surfaces are applicable.

Despite the circumstance that all equations of equilibrium are satisfied and the internal forces between the slices are included, the procedure is no more time consuming in practical appli-
cation than the available procedures for specified slip surfaces. Particularly, when the factor of safety is to be determined, it has been found through practical experience that irrespective of the shape of the slip surface, the proposed procedure is just as simple as the slip circle analyses.

The factor of safety with respect to shear strength (termed $F$ ) is herein suggested as the basis of design in all stability problems. For problems concerning slopes, cuts, embankments and earth dams, $F$ has already found widespread application, while in the


Fig. 8 Stability investigation for a strip footing on sand under an inclined load
Recherches de stabilité d'une fondation à semelle sur sable sous une charge inclinée
case of permissible earth or bearing pressures it has only recently been recommended (Brinch Hansen, 1953, 1955).

## References

Bishop, A. W. (1954). The use of the slip circle in the stability analysis of slopes. European Conf. on Stability of Earth Slopes, Vol. I. Stockholm
Caquot, A. and Kérisel, J. (1949). Tables de Butee, de Poussee et de Force Portante des Fondations. Paris; Gauthier-Villars
Hansen, J. Brinch (1953). Earth Pressure Calculation. Copenhagen; Danish Technical Press

- (1955). Simpel beregning af fundamenters bereevne. Ingeniören, Vol. 4. Copenhagen
JANBU, N. (1954). Application of composite slip surfaces for stability analyses. European Conf. on Stability of Earth Slopes, Discussion, Vol. III, Stockholm
— Bjerrum, L. and Kjarnsli, B. (1956). Veiledning ved lösning av fundamenteringsoppgaver. Norwegian Geotechnical Institute, Oslo
Lazard, A. (1955). Nouvelles remarques sur le calcul de la stabilité des talus en terre. Travaux, Paris
Meyerhof, G. G. (1953). The bearing capacity of foundations under eccentric and inclined loads. Proc. 3rd International Conference on Soil Mechanics and Foundation Engineering, Vol. 1, pp. 440, 445 Terzagh, K. (1943). Theoretical Soil Mechanics, Chaps. VII and VIII. New York; Wiley

