Influence of Vibration and Seepage on Stability of Cohesionless Soil

Influence des Vibrations et de l'Eau de Percolation sur la Stabilité des Massifs Pulverulents

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Summary

The aim of this article is to study the prevention of piping under weirs founded on cohesionless sandy soil. The stability of the sand as attested by vibration combined with seepage is studied by two methods. First, by dimensional analysis equation 11 is determined, giving seven dimensionless arguments which it is necessary to study in model tests. The model laws, equations 12 to 18, are derived and the simplified equation 19 is found. Secondly, the dynamic pressure head analysis is made and the depth of foundation is given by equation 21. The dynamic pressure head expressing the influence of vibration is determined by equations 22 and 23. An example shows the use of this method.

Sommaire

L'objet de cette communication est d'étudier les moyens d'empêcher la formation de renards sous les barrages fondés sur sols sableux pulvérulents. La stabilité du sable sous l'effet des vibrations et de l'eau de percolation, est étudiée par deux méthodes. La première, basée sur une analyse des dimensions de l'équation 11, fournit sept coefficients sans dimensions, qui sont nécessaires pour l'étude des essais sur modèles; les lois de similitudes (équations 12 à 18) sont dégagées, et l'équation simplifiée 19 est trouvée. La seconde analyse mathématiquement la charge dynamique de l'eau; et la profondeur à donner à la fondation est obtenue par la formule 21. La charge dynamique résultant des vibrations est donnée par les formules 22 et 23. Un exemple d'utilisation de cette méthode est donné.

Statement of Problem

The stability of cohesionless sandy subsoil forming the foundation of a weir is a problem of vital importance in all arid regions where water for the irrigation of arable land is taken from rivers flowing on great sand deposits, e.g. in Egypt, India and central U.S.S.R. This problem was studied by the author in the case of instability in shear along a cylindrical failure surface—BAŽANT (1953). Analysis and tests proved that the failure of stability known as 'piping' can be prevented by the proper depth of foundation at the toe of the structure. Graphs are given for the simple determination of this depth.

The author's tests showed a satisfactory agreement between the calculated depth and the reality for pure cohesionless sand. Sometimes the depth used in practice can be smaller, because of either the presence of a small percentage of clay producing cohesion in the sand or a sand density greater than the assumed value of 1·8. Even a slight cohesion enables us to use a depth which is 50 to 70 per cent of the theoretical one given by the author—BAŽANT (1953).

No tests have been made to study the behaviour of sand exposed to vibration combined with seepage—such as exists in the foundation of an overflow weir—and to verify the author's suggestion that the effect of vibration may be represented by the apparent angle of friction at vibration. In this article two other ways, suggested by the author, of solving the vibration problem are studied. First, the possibility of model testing is considered and the dimensional analysis of the stability of sand when vibration is combined with seepage is given. Secondly an analysis using the dynamic pressure head is proposed.

Dimensional Analysis of Vibration

Analytical solutions of the problem being unknown, the solution can be achieved by model tests. To ensure the reliability of model tests it is necessary to find out the mechanical similitude of tests and reality. The dimensional analysis serves for this purpose—MURPHY (1952).

Let us assume that the stability of saturated sand exposed to vibration combined with seepage (Fig. 1) may be expressed by the function

$$f(d, 2b, h_p, A, \bar{n}, t, d_s, d_a, k) = 0 \qquad \dots. (1)$$

where d denotes the depth of foundation at the toe of the structure, $2b$ the width of the structure, h_p the head at the beginning of piping, A the amplitude of vibration of soil at the surface, $\bar{n}$ the frequency of vibration of soil, t the time elapsed from the beginning of vibration, d_s depth of sand deposits, d_a active depth of sand, giving the depth which is affected by the vibration and which is governed by damping, and k the coefficient of permeability of sand.

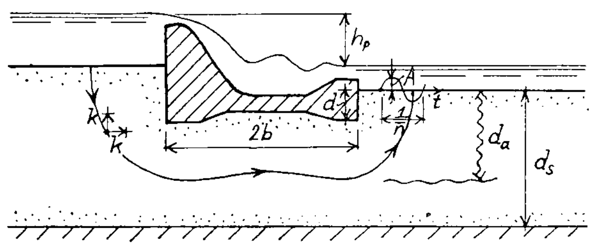

Fig. 1 Arguments influencing the stability at vibration combined with seepage

Facteurs influençant la stabilité des sols soumis à la fois à des vibrations et à une percolation d'eau

We use for model tests the same sand as exists in reality at the site of the proposed structure, and so in the analysis we can neglect the further variables of the problem which become constant. They are: ϕ angle of internal friction of sand, γ_t the unit weight of combined soil and water, γ_w the unit weight of water and V the velocity of elastic wave in sand. To simplify the problem, we neglect further the surface loading p, the weight of an apron or riprap. Equation 1 we reduce to the form

$$f(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7) = 0 \qquad \dots. (2)$$

where π_{1-7} denote the dimensionless arguments which are generally expressed by

$$\pi = d^x (2b)^y h_p{}^z A^u \bar{n}^v t^w d_s{}^a d_a{}^c k^e$$

On inserting the fundamental units, length $[L]$ and time $[T]$, we get

$$[1] = [L]^x [L]^y [L]^z [L]^u [T^{-1}]^v [T]^w [L]^a [L]^c [LT^{-1}]^e$$

from which it follows

$$[1] = [L]^{x+y+z+u+a+c+e}[T]^{-v+w-e}$$

This equation is valid only when exponents vanish:

$$x + y + z + u + a + c + e = 0$$
$$- v + w - e = 0 \quad \ldots. (3)$$

In equation 3 there are nine unknowns and we can choose seven independently in seven combinations and thus reduce the number of independent variables by two, which is the number of fundamental units. The chosen combinations and dimensionless arguments are:

(1) $x = 1, y = -1, z = u = w = a = c = 0;$

$$\pi_1 = \frac{d}{2b} \quad \ldots. (4)$$

(2) $x = -1, z = 1, y = u = w = a = c = 0;$

$$\pi_2 = \frac{h_p}{d} \quad \ldots. (5)$$

(3) $y = -1, w = 1, x = z = u = v = a = 0;$

$$\pi_3 = \frac{kt}{2b} \quad \ldots. (6)$$

(4) $u = v = 1, e = -1, x = y = z = a = 0;$

$$\pi_4 = \frac{A\bar{n}}{k} \quad \ldots. (7)$$

(5) $u = 1, x = -1, y = z = a = v = w = 0;$

$$\pi_5 = \frac{A}{d} \quad \ldots. (8)$$

(6) $u = 1, a = -1, x = y = z = v = w = 0;$

$$\pi_6 = \frac{A}{d_s} \quad \ldots. (9)$$

(7) $a = 1, c = -1, x = y = u = v = w = 0;$

$$\pi_7 = \frac{d_s}{d_a} \quad \ldots. (10)$$

The result of dimensional analysis is that stability under vibration combined with seepage is seen to be governed by the function f with seven dimensionless arguments:

$$f\left(\frac{d}{2b}, \frac{h_p}{d}, \frac{kt}{2b}, \frac{A\bar{n}}{k}, \frac{A}{d}, \frac{A}{d_s}, \frac{d_s}{d_a}\right) = 0 \quad \ldots. (11)$$

Equation 11, replacing equation 1, determines the arguments which may be necessary to the complete solution of the model tests. The unknown function f in equation 11 is to be found by model tests and it gives a satisfactory description of the stability under vibration combined with seepage, assuming that all the essential arguments have been introduced into equation1.

Equation 11 can be used for model tests if it is possible to realize the model laws—as many as the dimensionless arguments, i.e. seven. They are as follows:

(1) We choose the scale of reduction of length λ. We insert it into equation 11 and apply the condition that similitude exists when dimensionless arguments of model and reality are equal. We get then the model law

$$\frac{d_1}{d_2} = \frac{b_1}{b_2} = \frac{1}{\lambda} \quad \ldots. (12)$$

where index 1 denotes the model and index 2 the reality.

(2) From equations 5 and 12 it follows

$$\frac{h_{p1}}{h_{p2}} = \frac{1}{\lambda} \quad \ldots. (13)$$

(3) Using equations 10 and 12 we get

$$\frac{d_{s1}}{d_{s2}} = \frac{d_{a1}}{d_{a2}} = \frac{1}{\lambda} \quad \ldots. (14)$$

(4) For the scale of reduction of time we choose τ. This gives the law

$$\frac{t_1}{t_2} = \frac{1}{\tau} \quad \ldots. (15)$$

(5) Equations 6 and 15 give

$$k_1 = \frac{\tau}{\lambda} k_2 \quad \ldots. (16)$$

(6) From equations 8 and 12 we find

$$\frac{A_1}{A_2} = \frac{1}{\lambda} \quad \ldots. (17)$$

(7) Finally equations 7, 16 and 17 give

$$\frac{\bar{n}_2}{\bar{n}_1} = \frac{1}{\tau} \quad \ldots. (18)$$

The model laws require the geometric similarity $1:\lambda$ of length of structure d and width $2b$, of water head h_p and depth of sand d_s; this is easy to realize. So it is with the reduction of time $1:\tau$.

The coefficient of permeability of sand k should be changed in the ratio $1:\lambda/\tau$. It was assumed that the same sand would be used, in which case $k_1 = k_2$; values compatible with equation 16 when $\lambda = \tau$, a condition which must be fulfilled. The frequency $\bar{n}$ must be increased in the ratio $1:\tau$ and the amplitude decreased in the ratio $1:\lambda$.

There remains the law of equation 14 requiring the change of d_a. This change cannot be made because, as the sand is the same, the damping does not alter. Hence the model is only approximately correct. It is probable that the sand will be vibrating through the whole depth of the model. Therefore tests of weirs will require greater depth, but the difference will not be great as is shown by a comparison between equations 22 and 23. On the other hand, in the case of earthquake vibration, the active depth $d_a = \infty$ and the model test is strictly valid.

The increase in frequency $\bar{n}$ by the ratio $1:\tau$ and the decrease in amplitude by the ratio $1:\lambda$ cannot be large because large changes would alter the behaviour of the sand. If it is assumed that $\lambda = \tau \doteq 10$ then the models will be of large dimensions.

The number of variables in equation 11 being great, we wish to derive some simplified solution. Let us assume the dimensions of the structure have been chosen to make $d/2b = \text{const.}$ Supposing that the sand fails when maximum h_p is reached, we can neglect the effect of time so that the argument $(kt)/(2b)$ vanishes. Finally we can take $(d_s)/(d_a) = 1$ and $A/d = A/d_s$ because the whole depth of sand in the model is vibrating. The simplified function of stability under vibration combined with seepage is then given by

$$f\left(\frac{h_p}{d}, \frac{A\bar{n}}{k}, \frac{A}{d}\right) = 0 \quad \ldots. (19)$$

which can be plotted with coordinates $(h_p)/d$, $(A\bar{n})/k$ and with A/d as the system of curves. The unknown function in equation 19 must be found by model tests. This equation shows that it is necessary to study both the amplitude A and the frequency $\bar{n}$ even in the simplest case.

Dynamic Pressure Head Analysis

The problem of stability under vibration combined with seepage can be solved as in the static case of the author's theory (Bažant, 1953) if the influence of vibration is introduced into the neutral stress as the dynamic pressure head h_d which is added to pressure head h_r at the toe. This is valid assuming that the pattern of seepage is not altered by vibration.

From the author's analysis (Bažant, 1953, Vol. 2, p. 201,

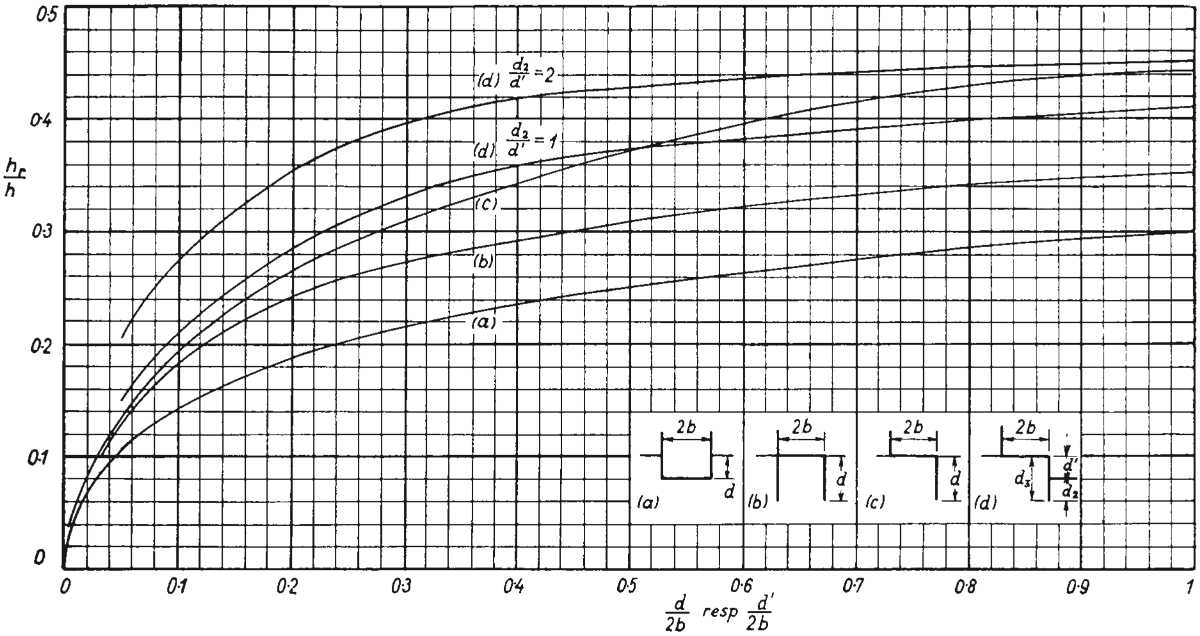

Fig. 2 Chart of relative pressure head h_r/h at toe (compiled after Khosla, Bose and Malhotra)
Diagramme de la pression relative h_r/h (d'après Khosla, Bose et Malhotra)

equation 18 and Vol. 3, p. 221) we know that stability in the static case requires that $h_r/d = 1{\cdot}11$; which is valid for $\phi > 15$ degrees and $\gamma_t/\gamma_w = 1{\cdot}8$. Multiplying the left-hand side by the coefficient of permissible loading $K_1 = 1{\cdot}7$, for concrete foundation, or $K_1 = 2$, for sheet pile wall foundation, and adding the dynamic pressure head we obtain

$$K_1\frac{h_r + h_d}{d} = 1{\cdot}11 \qquad \ldots. \ (20)$$

which is the solution for the stability under vibration combined with seepage. Then, from Fig. 2, we get for the chosen value of $d/2b = K_5$ the coefficient

$$\frac{h_r}{h} = K_6$$

from which we derive

$$h_r = K_6 h$$

Inserting this into equation 20 we have

$$K_1\frac{K_6 h + h_d}{d} = 1{\cdot}11$$

and the required depth of foundation under vibration combined with seepage is given by

$$d = \frac{K_1}{1{\cdot}11}(K_6 h + h_d) \qquad \ldots. \ (21)$$

Finally from chosen $d/2b = K_5$ we get

$$2b = \frac{d}{K_5}$$

The dynamic pressure head h_d entering into equation 21 was studied by Maslov (1954) who measured the compaction of sand at the beginning of vibration which squeezes the water from pores. Owing to the high permeability of sand this squeezing of water takes place within a few seconds, e.g. in the test cylinder of 195 cm height it took no more than 45 sec. for sand having $D_{65} = 0{\cdot}25$ mm.

Maslov found the maximal dynamic pressure head at the beginning of vibration

$$h_d = \frac{v_n}{k}\left(d_s z - \frac{z^2}{2}\right) \qquad \ldots. \ (22)$$

which holds for constant vibration through the whole depth d_s of sand. In equation 22, v_n denotes the velocity of compaction during vibration, k the coefficient of permeability and z the depth of a point under the surface of submerged soil. For vibration decreasing linearly to zero at the active depth d_a it is possible to derive the equation

$$h_d = \frac{v_0}{k}\left[d_s z - \frac{z^2}{2d_a}(d_s + d_a) + \frac{z^3}{3d_a}\right] \qquad \ldots. \ (23)$$

where v_0 denotes the velocity of compaction at the surface.

The dynamic pressure head h_d entering equation 21 can be found for $z = d$. The depth d is chosen, and then h_d computed, inserted into equation 21 and adjusted by trial until the chosen d equals the computed one.

The velocity of vibration v_n was found to depend on the acceleration of vibration, grain size and density of sand. Equation 19 shows that the tests should be supplemented by measuring amplitude A too. The velocity v_n was measured indirectly, the sand being vibrated in a cylinder in which a piezometric tube was inserted. The head h_d was measured every 5 sec, the maximal head determined, and from it v_n computed by the use of equation 22.

The acceleration, which is smaller than a certain critical value ($\alpha_{er} < 25$ to 200 mm/s²), does not produce measurable compaction. The actual acceleration of sand under the weir lies in this range; but because in equation 20 the loosest sand is assumed with $\gamma_t/\gamma_w = 1{\cdot}8$ and porosity $n > 0{\cdot}4$, it is reasonable to expect that the test of this sand with actual acceleration will produce measurable compaction.

To show the possibilities of this method we will compute the necessary depth of the foundation of the Lloyd Dam on the Indus, studied previously by the author (Bažant, 1953, Vol. 3, p. 220). On inserting into equation 23 $v_0 = 0{\cdot}0001$ cm^{-1}, $k = 1{\cdot}10^{-1}$ cm s^{-1}, $d_s = 5000$ cm, $d_a = 1000$ cm, $z = 366$ cm, we get $h_d = 148$ cm. Further we know $K_6 h = 0{\cdot}16h$ and from equation 21 we get, for $h = 583$ cm, the required depth

$$d = \frac{2}{1{\cdot}11}(0{\cdot}16 \times 583 + 148) = 434 \text{ cm}$$

As actual depth is 366 cm, the agreement is satisfactory.

The dynamic pressure head analysis depends on the right determination by tests of the velocity of compaction v_n which must be measured for the loosest sand. The evaluation of the active depth of sand d_a is not so important as we can resort to equation 22. The amplitude A and frequency $\bar{n}$ of the system weir–soil is assumed to be known, it can be determined by measurement on existing structures or by theoretical analysis. Valuable information for analysis can be derived from the study of forced damped vibrations of engine foundations—*Symposium* (1954). Nevertheless, it will be necessary to elaborate the solution for the amplitude and frequency of the vibration of a weir resting upon sandy soil and vibrated by the water discharging over the spillway.

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