Design Data for Partially Penetrating Relief Wells

Données de Calcul des Puits filtrants

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Summary

A revised nomograph for the design of partially penetrating relief wells has been developed for the case of an infinite line of equally spaced wells based upon the results of combined model and theoretical considerations. A simple means of accounting for the effects of leakage through slightly pervious top blankets is presented.

Serious errors develop in applying the results of an infinite line of wells to the design of a finite line because of the increased well discharges and uplift pressures midway between wells. These discrepancies increase excessively as the ends of the well line are approached and should be accounted for in a design. A means of determining these discrepancies is also presented.

Sommaire

Dans le cas d'une ligne infinie de puits également espacés, on a tracé une abaque modifié pour le calcul des puits filtrants sur la base des résultats d'études sur modèle et théoriques. Des méthodes simples permettent d'estimer les effets de pertes à travers des couvertures de faible perméabilité.

Des erreurs graves peuvent se développer lorsqu'on applique les résultats d'une ligne infinie de puits au calcul d'une ligne finie à cause des écoulements croissants des puits et des sous-pressions à mi-chemin entre les puits. Ces erreurs augmentent excessivement quand vers les extrémités de la ligne de puits on doit en tenir compte dans les calculs. Un moyen de déterminer ces erreurs est présenté.

The U.S. Army Corps of Engineers is responsible for the design and construction of a major portion of the flood control projects within the United States of America. Since 1938 the Corps has interested itself in the use of relief wells for reducing foundation seepage uplift pressures along the downstream toes of dams to prevent or control the occurrence of dangerous seepage boils. A line of wells below a dam ordinarily extends across the entire width of the pervious foundation, terminating at the impervious valley walls. A system terminating with impervious end boundaries, and hence without end effects, is said to be equivalent to one of infinite length.

Design curves were developed early for the case of infinite systems of equally spaced wells along a line parallel to the seepage source and extending distance W into a pervious foundation of thickness D covered by a top impervious blanket. A theoretical solution was available for completely penetrating wells, but recourse had to be made to electrical analogy models to develop data for partially penetrating wells.

When relief wells later came into use for controlling underseepage along levees, well systems were often terminated at points where thick top strata were found, but where the pervious stratum and the line source extended large distances beyond the end of the system. This type of system is said to be finite, and in contrast to the uniformity of an infinite system it is characterized by non-uniform well discharges and midpoint pressures between wells both of which are greater than in a corresponding infinite system, with an especially sharp increase at the ends of the line. Further, all project sites requiring wells have semi-pervious top blankets, rather than impervious, which may have important effects on the well operations. A civil works investigation programme was initiated by the Office of the Chief of Engineers and assigned to the Kansas City District to study these effects by means of electrical analogy models. A review of the basic model data used for the initial design curves coupled with a limited theoretical study indicated that the initial partial penetration design curves were somewhat in error. A re-study of these design curves was then included in the model studies.

The results of several initial models of finite systems showed that a very large number of models would be required for systematic coverage of the problem. By trial, it was found that ordinary computation was more productive than models involving only a few wells, and that by use of electronic computers the advantage of computations over models could be extended to any number of wells. Model investigation of single partial penetration wells was, however, a pre-requisite to adoption of the computational method.

At this same stage of the investigation it was found that model results for full penetration infinite lines, where an analytical check was possible, all showed a small but systematic error probably caused by self-induction of the model well wires. Models were then constructed representing a full penetration single central well supplied by a concentric circular source. Extremely close correspondence between model and analytic results for a single well, together with the need for single-well data for numerical summation of finite systems, led to the adoption of one-well models as the basis of all subsequent work.

A potential profile for a centrally located, partially penetrating well is shown in Fig. 1. Beyond a radial distance equal to approximately the thickness of the pervious foundation the

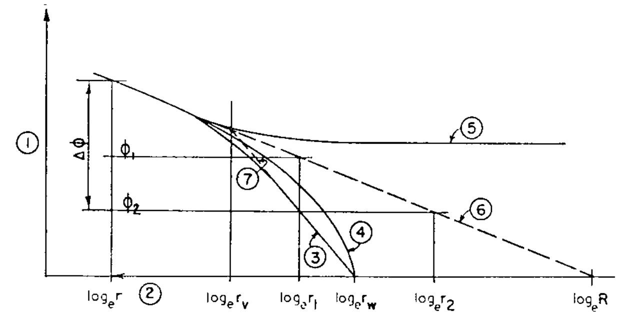

Fig. 1 Centrally partially penetrating well

1. Hydraulic potential; 2. Logarithm of radial distance; 3. Potential profile at top of pervious foundation; 4. Potential profile at elevation of well bottom; 5. Potential profile at base of pervious foundation; 6. Tangent to potential profile curve; 7. Tangent to potential profile curve (3)

Un puit de décharge pénétrant partiel, au centre

1. Potentiel hydraulique; 2. Logarithme de la distance radiale; 3. Profil potentiel au sommet de la fondation perméable; 4. Profil potentiel au fond du puits; 5. Profil potentiel à la base de la fondation perméable; 6. Tangente à la courbe du profil potentiel; 7. Tangente à la courbe du profil potentiel (3)

flow is very nearly uniform and the potential profile is very nearly a straight line when plotted against the logarithm of the radial distance. At these distances the well acts as if it were a completely penetrating well of reduced or effective radius R. The difference in potential $\Delta\phi$, between radii r and r_1 is

$$\Delta\phi = f \log_e r/r_2 = f[\log_e r/r_1 + \log_e r_1/r_2] = f[\log_e r/r_1 + B]$$

where B is a correction factor equal to $\log_e r_1/r_2$ that depends both on the radial distance r_1 and depth z, and f is a constant. Values of the effective radius R may be obtained either from model data, as indicated in Fig. 1, or computed from the results given by MUSKAT (1937, p. 274, equation 6). It should be noted that Muskat's solution is presented in terms of well discharge and not the desired potential distribution. The equation for the logarithm of the ratio of the well radius r_w to R is

$$\log_e r_w/R = \left(\frac{D}{W} - 1\right) \log_e \frac{4D}{r_w} - \frac{1}{2\frac{W}{D}} \log_e F(\Gamma)$$

$$= \left(\frac{D}{W} - 1\right) \log_e \frac{D}{r_w} + \log_e G$$

where $F(\Gamma)$ is a gamma function given by Muskat. A comparison of the computed R with those obtained from the models showed good agreement.

The flux for a single well of an infinite system of wells spaced a distance a apart and located at distance L_s, from the effective seepage entrance into the foundation is

$$Q/(kHD) = 1/(L_s/a + \theta_a)$$

where

$$\theta_a = \frac{1}{2\pi}\left[\log_e \frac{a}{2\pi r_w} + \log_e r_w/R + 2\sum_{1}^{\infty} B_{na}\right]$$

Q is the well discharge, k is the foundation permeability, and H is the net hydraulic head on the system. This equation is obtained from MIDDLEBROOKS and JERVIS (1946, equation 13, p. 1334), based upon the replacement of the partially penetrating wells of radius r_w by fully penetrating wells of effective radius R. The terms $\log_e r_w/R$ and B_{na} are correction factors that are necessary when equivalent wells are used to account for the departure of the potential profile of a partially penetrating well from that of complete penetration. Values of $\log_e r_w/R$ may be obtained either from model test data or from the equation given above from MUSKAT (1937). Values of B_{na} are obtained from the test data, see Fig. 1.

The uplift, ϕ_m midway between wells at the base of the top blanket, measured from well pressure as zero, is

$$\phi_m = \frac{H\theta_m}{L_s/a + \theta_a} = \frac{Q\theta_m}{RD}$$

where

$$\theta_m = \theta_a + \frac{1}{2\pi}\left[\log_e 2 - 2\sum_{1}^{\infty} B_{(n+\frac{1}{2})a}\right]$$

Values of $B_{(n+\frac{1}{2})a}$ are obtained from the test data (see Fig. 1).

From the analytic expressions for θ_a and θ_m, it is seen that they are both linear functions of $\log_e a/(2\pi r_w)$ in the case of full penetration, in which the term $\log_e r_w/R$ and the summations in terms of B are equal to zero. It is also seen that for partial penetration, θ_a and θ_m would be linear functions of either $\log_e a/r_w$ or $\log_e r_w/R$ if the terms in B were zero, and that the relation approaches linearity as these terms approach negligible magnitudes as compared with $\log_e r_w/R$. This condition is met in practice if D/a is equal to or less than 2, and the departure from a linear relation is small even when D/a is of the order of 3 or 4. For simplicity, the nomograph of Fig. 2 is therefore based on the very close approximation that the θ curves are straight and parallel in all cases.

By substitution of the expression for $\log_e r_w/R$ in the equation for θ_a, and separating the result into terms involving $\log_e a/r_w$ and functions of D/a and W/D, it is found that θ_a for partial penetration may be obtained by multiplying the full penetration value of θ_a by D/w and adding a term which is constant if the ratios D/a and W/d are held constant. A similar relationship exists for θ_m. Once these constants terms are determined for any convenient value of a r_w, the construction of the nomograph becomes a simple problem, see Fig. 2.

The seepage up through a semi-pervious top blanket reduces the well discharge and the uplift between wells. Advantage should be taken of these conditions in a well system design. A second model series of partially penetrating single-well was constructed, simulating a slightly pervious top blanket by a mixture of copper powder, paraffin, and bentonite. Unfortunately, the resistance of the simulated blanket increased slowly with time, necessitating frequent check tests with full penetration, and in most cases limiting the observations to a determination of the intersection radius, r_v, and the relative

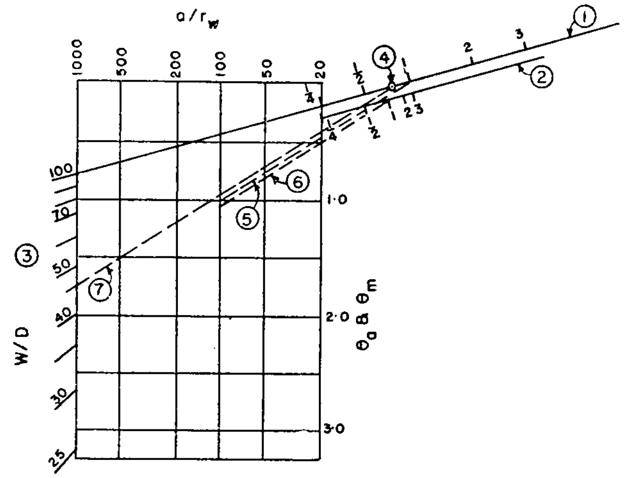

Fig. 2 Nomograph

1 D/a line for θ_a; 2. D/a line for θ_m; 3. Per cent well penetration; 4. Pole—Assume $a/r_w = 100$, $\theta_a = 1{\cdot}0$, $D/a = 1$; 5. Line from 1 on (1) to $a/r_w = 100$, $\theta_a = 1{\cdot}0$; 6. Line parallel to (5) through 1 on (2)—Read off $\theta_m = 1{\cdot}05$ at $a/r_w = 100$; 7. Line parallel to (5) through (4)—Read off 46 at (3)

Nomograph

1. Ligne D/a pour déterminer θ_a; 2. Ligne D/a pour déterminer θ_m; 3. Pourcentage de la pénétration de puits; 4. Pôle—prenez $a/r_w = 100$, $\theta_a = 1{\cdot}0$, $D/a = 1$; 5. Ligne de 1 à (1) à $a/r_w = 100$, $\theta_a = 1{\cdot}0$; 6. Ligne parallèle à (5) par 1 à (2) lisez $\theta_m = 1{\cdot}05$ à $a/r_w = 100$; 7. Ligne parallele à (5) par (4)—Lisez 46 à (3)

slope factor, M, both of which are analogous to their counterparts for the impervious blanket case, as defined on Fig. 1. The results showed that the partial penetration correction, as measured by the parameters r_v and M, are identical whether the top blanket is totally impervious or semi-pervious.

BARRON (1948) suggested that the effect of a semi-pervious top blanket could be accounted for by using a landside impervious top blanket of an effective length L_e determined as proposed by BENNETT (1945). Further analytical studies for a restricted case confirmed the above proposal for infinite lines, and also indicate that the complicated series of Barron could be replaced by simpler expressions, as indicated below, with an error of less than 1 per cent for values of K_e or L_s equal to or greater than 1·5. The expressions for Q and ϕ_m are similar to those given above except that θ_a is multiplied by a factor $(1 + L_s/L_e)$.

For finite lines, a model was arranged to permit rapid installation or removal of an insulating membrane over one half the area of the leaky top blanket, and installation of a line source parallel to one side of a square model tank. The side with the insulation and line source simulated the river and levee side; the non-insulated side represents the landside semi-pervious top blanket. It was originally planned to measure carefully the potential distribution along the centre line through the well, parallel to the source, as an accurate basis for summation of finite lines from a single well. This objective could not be accomplished with satisfactory accuracy because of the previously mentioned variable conductivity of the top blanket and the consequent need for frequent re-checks on the analytically known case represented when line source and insulator were removed. Prior to the model tests, analytic considerations indicated the strong probability that the semi-pervious blanket half could be replaced by an impervious blanket and a line source at a distance $L'_e = 0{\cdot}88L_e$ in which L'_e is the effective distance for a single-well system and L_e the previously defined effective distance for an infinite system. The model confirmed the accuracy of the predicted relation, and indicated no great difference between the model potential distribution and that computed mathematically when L'_e is used. Since the latter

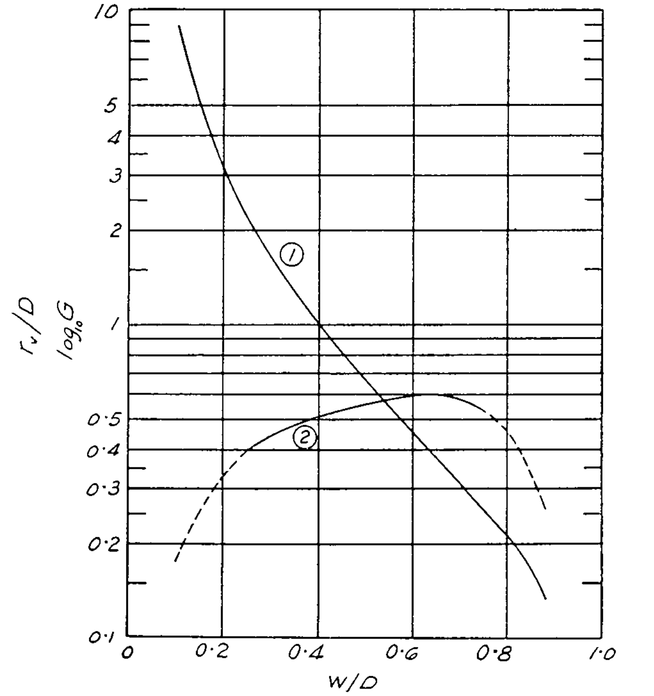

Fig. 3 1. Curve $\log_{10} G$ *versus* W/D; 2. Curve of r_v/D *versus* W/D
1. La courbe de $\log_{10} G$ en fonction de W/D; 2. La courbe de r_v/D en fonction de W/D

has the enormous advantage of being readily computable it was decided to use the mathematical expression in preference to further model studies of questionable accuracy. This decision was made despite the fact that the true landside exit distance varies in some unknown manner from L'_e for one well to L_e for an infinite line. In order to reduce the number of variables from six to five, a synthetic pair of distances L''_s and L''_e was introduced, related to L_s and L'_e as follows

$$L''_e = \infty\,;\; L''_s = \frac{L_s + L_e}{\pi}\sin\frac{\pi L_s}{L_s + L'_e}$$

This synthetic system produces the same head-discharge relation as the real system, and their pressure distributions are identical near the well although they diverge somewhat at great distances. The error introduced by this substitution has been found to amount to 7 per cent or less at the ends of systems of 31 wells. It is negligible for small groups of wells, and would be very appreciable for an infinite line.

The two approximations just described reduce the semi-pervious landside blanket case to that of a totally impervious blanket, in which only the synthetic source distance L''_s need be considered. The application of a correction for partial penetration to this relatively simple system is in principle exactly identical whether the system be finite or infinite. In practice, however, it was desirable to express the difference between potentials for full and partial penetration in the form of an empirical equation from which corrections could be computed for any combination of W/D and D/a. A systematic measurement and analysis of the experimentally determined intersection radius, r_v, as shown in Fig. 1, and slope difference M, led to the following empirical relation

$$M = \frac{\log_e r_w/R}{\log_e r_v/r_w} = \frac{(D/W - 1)\log_e D/r_w + \log_e G}{\log_e D/r_w + \log_e r_v/D}$$

Numerical values of $\log_{10} G$ are shown in Fig. 3. The model studies showed r_v/D to be very nearly independent of D/r_w, and for simplicity in numerical work it was taken to be a function

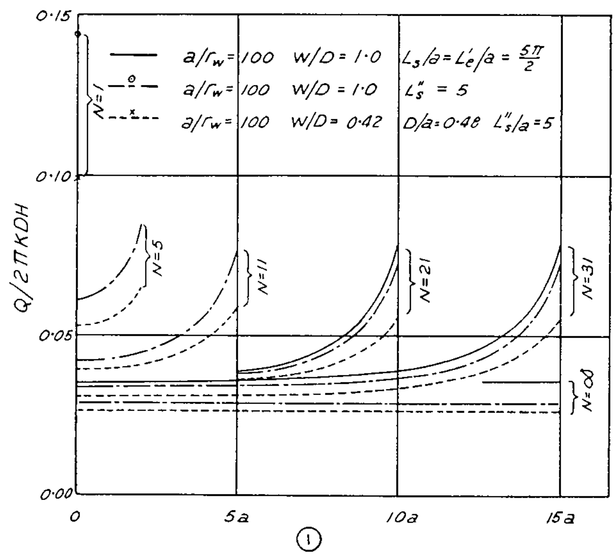

Fig. 4 Variation of discharge, finite line; 1. Distance of well from centre of system
La variation du déchargement: la ligne finie; 1. La distance du centre du système

of W/D only. The experimentally determined relation between W/D and r_v/D is shown in Fig. 3.

It was found that the potential difference between equivalent completely penetrating and partial penetration cases could be expressed by the following empirical equation, see Fig. 1, which fits very closely the top of stratum potential as measured from the models

$$(\phi_1 - \phi_2) = \frac{Q}{2\pi KD}\cdot\frac{M}{2}\log_e\left[\frac{\cosh\frac{2r}{r_v} + 1}{\cosh\frac{2r}{r_v} - 1}\right]$$

With the foregoing equations, it is possible to evaluate numerically the incremental changes in potential at any one well in a finite system due to the unknown Q's of all the wells in the system. Repetition of this process for each well leads to a system of simultaneous linear equations in the undetermined Q's. Because of symmetry the number of Q's and equations is $N/2$ or $(N + 1/2)$ depending on whether the number of wells, N, is even or odd.

The simultaneous equations have been written for combinations of limits. Solution of the equations is in progress, using a Univac computer. Several complete penetration cases are being solved for the condition $L_s = L_e$ for comparison with the nearly equivalent substitution employing

$$L''_s = \frac{2L_s}{\pi}; \quad L''_e = \infty$$

Fig. 4 shows the results of one such comparative computation, together with comparative results for full and partial penetration. Fig. 5 shows typical examples of the variation of Q, at centre and end wells, as a function of the parameter β, a convenient measure of the length of the line.

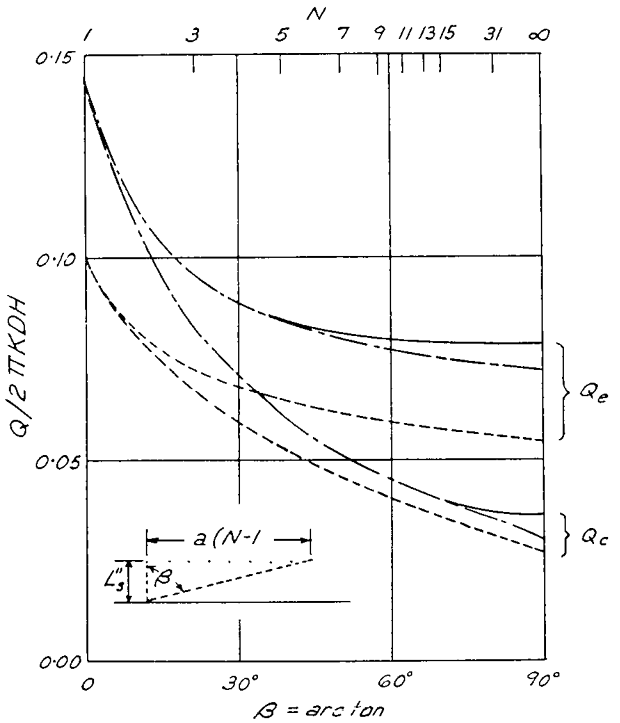

Fig. 5 Typical variation of Q_e and Q_c with N:

N = Number of wells in system; Qe = Discharge of end well—extrapolated for N equal to infinity; Qc = Discharge of centre well—computed for N equal to infinity

La variation typique de Q_e et de Q_c avec N:

N = Le nombre des puits du système; Qe = Décharge du puits d'extrémité—extrapolé pour N infini. Qc = Décharge du puits central calculé pour N infini.

It is not possible to introduce friction and velocity head losses into the linear equations since these losses are not linear with Q. The effect of these losses in physical systems will be to redistribute the Q's somewhat more uniformly than those obtained by computation.

Reduction of the mass of data resulting from numerical analysis to a form convenient for practical use is the major problem remaining for finite lines.

These investigations were initiated by the late T. A. Middlebrooks. Design of the equipment and model techniques was the work of K. V. Taylor, assisted by G. F. Rentschler and J. A. Swenson. J. W. Coate and J. A. Swenson performed the model studies with leaky blankets, and prepared the equations for solution. O. T. Steiner coded and solved the questions. The authors jointly supervised all activities and made the analytic investigations.

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