The Systematic Arching Theory Applied to the Stability Analysis of Embankments

Application de la Théorie de la Mise en Voûte Systématique à l'Analyse de la Stabilité des Remblais

by D. H. TROLLOPE, M.Sc., A.M.I.E.Aust., Senior Lecturer in Civil Engineering, University of Melbourne, Australia

Summary

A new theory describing the detailed stress distributions developed in wedges of granular materials resting on foundations of varying stiffness is briefly outlined.

The application of this theory to the design of earth and rock-fill dams is discussed in the light of available evidence concerning pressure measurements made on model and full size embankments. The evidence presented indicates that the theoretical predictions are substantiated by a wide range of field experience.

Two-dimensional stability analysis is then considered in terms of: (a) collapse over an assumed surface of failure; (b) a point-to-point stress analysis.

Finally the implications of the theoretical analyses as they affect the design and construction of embankments are discussed.

Sommaire

L'auteur esquisse une nouvelle théorie décrivant les détails de la distribution des contraintes dans des dièdres de matériaux pulvérulents, reposant sur des fondations ayant divers degrés de raideur.

L'application de cette théorie aux projets de barrages en terre ou en enrochements est discutée à la lumière des résultats utilisables concernant les mesures de pression effectuées sur des modèles réduits et sur des talus en vraie grandeur. Les résultats présentés indiquent que les prévisions théoriques sont réalisées par une grande diversité d'expériences pratiques.

Ensuite on étudie la stabilité à deux dimensions selon: (a) l'écroulement sur une surface de rupture prévue; (b) l'analyse des contraintes point par point.

Enfin on examine les conséquences des analyses théoriques, par rapport au projet et à la construction des remblais.

Introduction

A recent study of the stability of granular wedges has led to the development of a theoretical statement of the body-force stress distribution within such wedges.

The theory has been extensively tested by measuring the distribution of pressure across the base of model embankments composed of granular material. The results of this investigation showed close agreement between theory and experiment on a laboratory scale.

Confirmation of the theory on cohesive frictional materials presents a more formidable problem however, and recourse has to be made to measurements made on full scale embankments.

The aim of this paper is to show that, in addition to laboratory evidence, there is a wide range of field experience which confirms the application of the theory to the study of the behaviour of embankments.

Following this evidence the manner in which the theoretical solutions can be applied to the stability analysis of embankments is discussed.

The Systematic Arching Theory

It has been postulated (TROLLOPE, 1956) that the body-force stress distribution within granular masses can be evaluated from an analysis of the statical equilibrium of a systematically packed system of mono-sized, smooth, rigid spheres.

Under conditions of single (horizontal) surface restraint it has been shown that there is a unique arrangement which satisfies the necessary stability criteria. This arrangement is described as hexagonal-rhombohedral, which implies that the system is rhombohedrally packed and orientated in such a way that every (horizontal) layer parallel to the restraining surface is of rhombic form. The unique three-dimensional model is, therefore, a hexagonal pyramid.

For the two-dimensional wedge profile the model can be represented by mono-sized discs in place of spheres and the resulting arrangement for the limiting case of surface stability is shown in Fig. 1a, and for a more stable case in Fig. 1b. From these diagrams it will be seen that, in general, each disc is in tangent contact with six neighbours. The remaining requirement is that the diameter of each disc is such that it represents, statistically, the mean particle size of the prototype mass. In general this requirement is met if the individual diameter is very small compared with the size of the wedge.

Of the six contact forces on each disc, three will be determined from the known boundary conditions, two from equations of statics, so that the unit system has one statically indeterminate quantity. A limiting solution can be obtained by putting the horizontal forces equal to zero as in Fig. 1c.

Now consider the arrangement in Fig. 1d. If lq, mq, nq are the loading forces (known from boundary conditions) where l, m, n are integers, then it will be seen that the support force Y_2 cannot exceed $(l + 1)q$ otherwise the horizontal (X) support would have to supply a tensile restraint. The limiting values for Y_2 are therefore 0 and $(l + 1)q$ and for intermediate values

$$Y_2 = k[(l + 1)/(m + 1)]Y_1$$

where k is defined as the systematic arching factor and $0 \leqslant k \leqslant 1$.

The mechanism whereby Y_2 can be varied in the model is readily visualized. If any disc should suffer a slight downward movement it will completely remove the (Y_2) support from the discs in the layer immediately above whereas considerable movement is necessary before it ceases to afford horizontal (X) support to adjacent discs in the same layer.

This then provides the author's concept of systematic arching. In the model it is associated with deformation of the supporting surface (foundation).

The force analysis for each limiting case can be reduced to a simple summation of the appropriate q components, starting from the apex of the wedge.

From the geometry of the model the resulting force system can then be described in terms of stresses.

In Fig. 2 the limiting solutions for a wedge with side slopes at 30 degrees to the horizontal are shown as the no arching case $(k = 1)$ and the full arching case $(k = 0)$, and it follows that

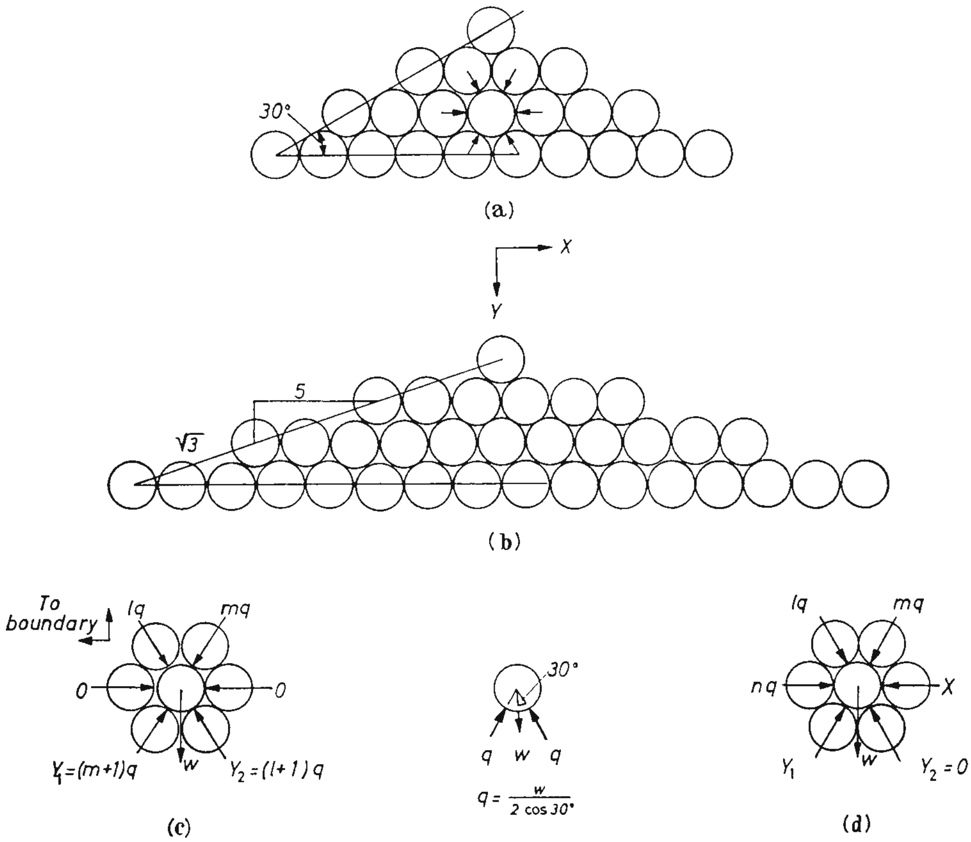

Fig. 1 Arrangement of the theoretical model
Arrangement du modèle théorique

for any wedge to which the model conditions apply the stress distribution must lie within the range of these solutions.

It has been found by experiment that the no arching case is obtained for a granular wedge on a rigid base, whereas arching behaviour is developed when the base is allowed to deflect downwards. Intermediate conditions are defined in terms of the arching factor (k) the appropriate value of k being dependent on the base deflection coefficient (Δ/L).

The no arching solution was also obtained by HUMMEL and FINNAN (1921) and a distribution similar to the full arching case has been predicted from mathematical argument (SAMSIOE, 1955).

Samsioe indicated that his solution was associated with large (plastic) deformations within the wedge, although he was unable to visualize the physical mechanism whereby these deformations were to be generated. The theory at present under discussion clearly indicates that this type of stress distribution can be developed as the result of foundation settlement and it will be shown below that the available evidence indicates that differential settlement within an embankment is also likely to develop arching characteristics.

Laboratory Studies

Typical results of an investigation carried out in the author's laboratory are shown in Fig. 3. In these tests, wedges of granular material, with side slopes at the angle of repose, were built up on a steel plate (102 in. × 45 in. × 3/16 in.) the deflection of which was controlled by air pressure applied to

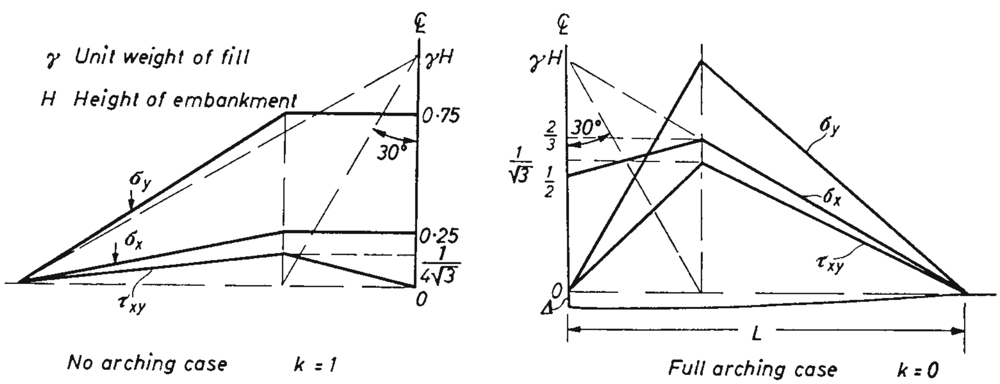

Fig. 2 Theoretical stress distributions on the base of a granular wedge
Distributions théoriques des contraintes sur la base d'un coin pulvérulent

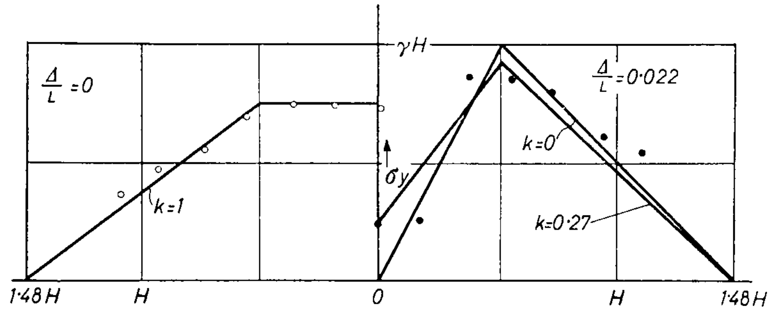

Fig. 3 Vertical pressure measurements on the base of model embankment
Mesures de la pression verticale sur la base d'un remblai modèle

the underside. The pressures were obtained directly using pressure cells and indirectly from bending strain measurements on the plate.

HUMMEL and FINNAN (1921) obtained results similar to the no arching case from direct measurement (using pressure cells) of the base pressure distribution on wedges supported by a stout wooden platform.

HOUGH (1938) reports the results of an extensive series of tests in which model dams were built of lead shot resting first on weak gelatin and then on a soft clay foundation. The models were built up until severe foundation deformations were developed. In Fig. 4 the shaded area shows the final shape of

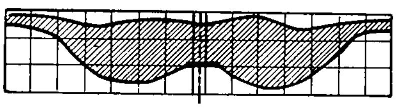

Fig. 4 Settlement of a model dam (after Hough)
Tassement d'un remblai modèle réduit (d'après Hough)

the lead shot model after it had been allowed to deform from its original triangular outline. It will be observed that the resulting heart-shaped settlement profile clearly mirrors the double peaked pressure distributions predicted by the full arching solution.

The evidence submitted above confirms to a significant extent the application of the systematic arching theory to wedges of granular materials and in the case of the lead shot model it is shown that the characteristic arching stress distribution is retained even with very large (relative) base deformation.

Field Studies

GILBOY (1933) has reported the settlement of a highway embankment as illustrated in Fig. 5 where the silt stratum under

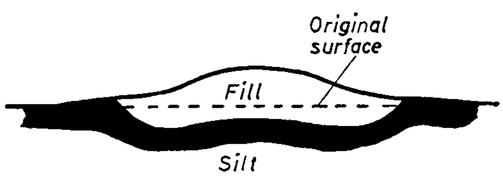

Fig. 5 Settlement of a highway embankment (after Gilboy)
Tassement d'un remblai de route (d'après Gilboy)

the embankment was originally at the surface and sensibly horizontal. Again the development of arching characteristics, as shown in the settlement profile, is marked.

The most significant information available at the time of writing is that obtained from measurements made on four dams in America (TAYLOR, 1947); the results of this large-scale investigation are reproduced in Fig. 6. At the time when the measurements were made the construction of these dams (with the exception of Arkabutla) was not complete; hence the nominal overburden pressure curves in these diagrams represent the values which should have been recorded if the vertical pressure distribution at the measuring stations was merely a function of the height of the fill at each gauge point. In the original discussion Taylor concluded that erratic gauge behaviour rendered the results suspect. He assumed however that the pressure measurements should closely follow the nominal overburden pressure curves in each case.

It is the author's view that, despite obvious gauge deficiencies, the full range of arching behaviour is reflected in these results.

It may be concluded therefore that there is a wide range of experience on actual embankments which indicates the general development of stress distributions similar to those predicted by the systematic arching theory.

Relationship of Embankment Type to Arching Development

Before proceeding to a discussion of the likelihood of arching development in actual embankments it should be pointed out that the full arching case represents an idealized theoretical solution which cannot be fully attained in practice. However, provided the material of which an embankment is constructed can be assumed to satisfy the no-tension law, then the internal stress regime must lie between the limiting solutions of the type shown in Fig. 2. In the author's opinion an arching factor of the order of $k = 0{\cdot}25$ is likely to represent the most severe arching development in actual structures but much more evidence of the type shown in Fig. 6 is required before a design value for k can be accepted with confidence.

Hence at the present time it is desirable to restrict our considerations to those of the limiting cases only. In this context it is convenient to identify three major types of embankment as shown in Fig. 7 and to consider these types in relation to the deformation necessary to develop arching.

Rock-fill embankments (*type a*)—The results of laboratory model tests on granular wedges can be safely extrapolated to cases of this type. The type of stress distribution is mainly dependent on the compressibility of the foundation.

Thus the two extreme cases may be recognized: (1) rigid (rock) foundation—no arching case; (2) compressible foundation—full arching case.

Rock-fill with impervious core embankments (*type b*)—In this type of embankment there is a tendency for the rolled-fill core to be, relatively, highly compressible when compared with the surrounding rock-fill; thus there will be a tendency for the rock-fill to 'squeeze' the core. This tendency will be reflected in a slight general movement of the rock-fill downwards and towards the centre of the embankment. The resulting deformation is similar in nature to the foundation movement required to develop arching in a rock-fill embankment. It appears therefore that the most likely stress regime in embankments of this type b is represented by the full arching case. This con-

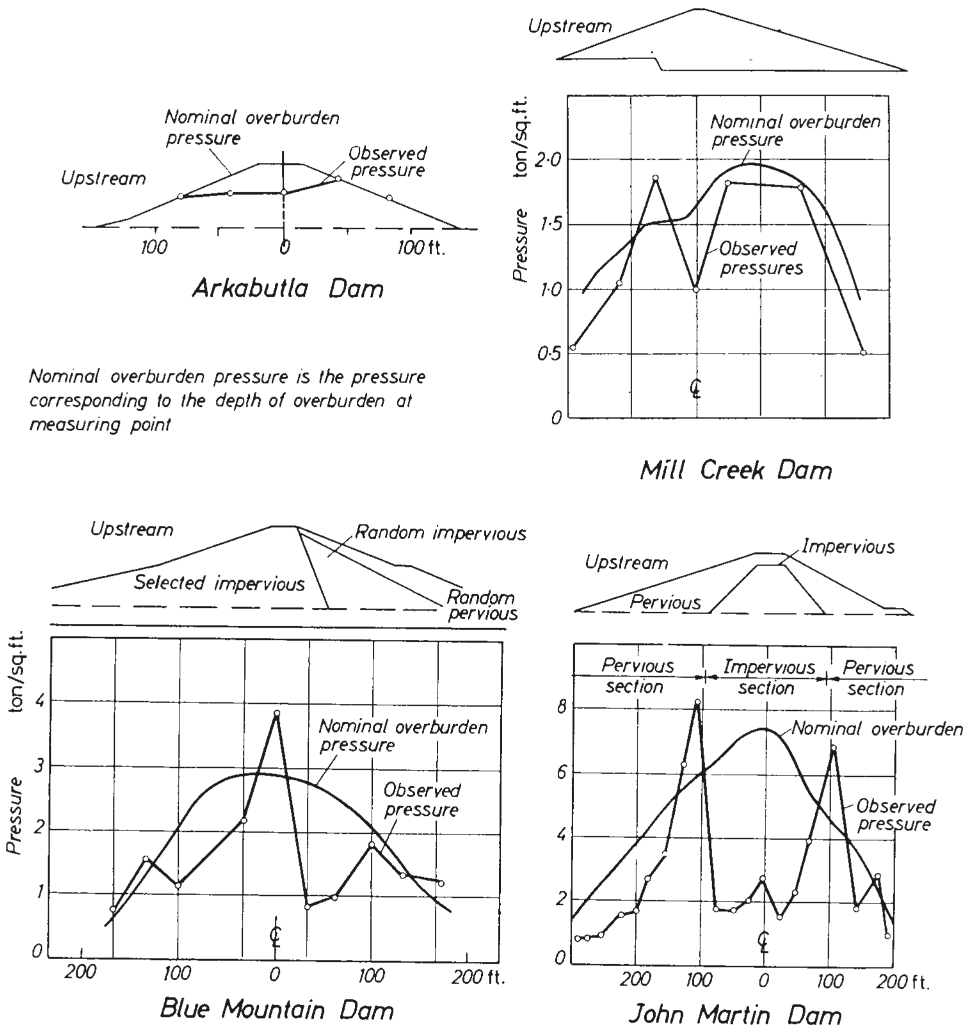

Fig. 6 Pressure cell measurements on four dams (after Taylor)
Mesures de pression effectuées avec des cellules sur quatre remblais (d'après Taylor)

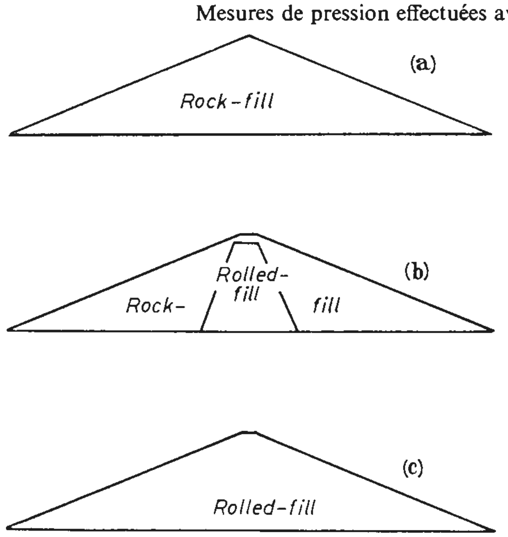

Fig. 7 Three basic types of embankment
Trois types fondamentaux de remblais

clusion is borne out by the clear indication in Fig. 6 of arching development being related to the location of the core.

Rolled-fill embankments (*type c*)—This case is the most complicated of the three types inasmuch as it is not immediately obvious that the behaviour attributed to granular materials can be extrapolated to cases involving cohesive granular rolled-fill. However it is to be anticipated that with typical rolled-fill, settlement within the embankment will occur and this deformation is compatible with that required to develop arching. Hence, if the material cannot withstand significant tensile stresses it is likely that arching type behaviour will occur.

The results of the measurements made on Arkabutla and Blue Mountain Dams (Fig. 6) appear to support this viewpoint. It is of particular importance that further field evidence should be obtained in cases of this type.

At the present state of knowledge, however, any analysis of a rolled-fill embankment should be based on the assumption of full arching development.

Two-dimensional Stability Analysis of Embankments

Within the limitations of the assumption of plane strain behaviour of embankments, two methods of stability analysis are recognized (Bennett, 1951).

The first of these methods entails the further assumption of the shape of the potential surface of failure and the safety of the embankment is measured by the ratio

$$\frac{\Sigma \text{ available shear resistance}}{\Sigma \text{ applied shear stress}}$$

over the assumed surface of failure*.

* Vectorial summation is implied.

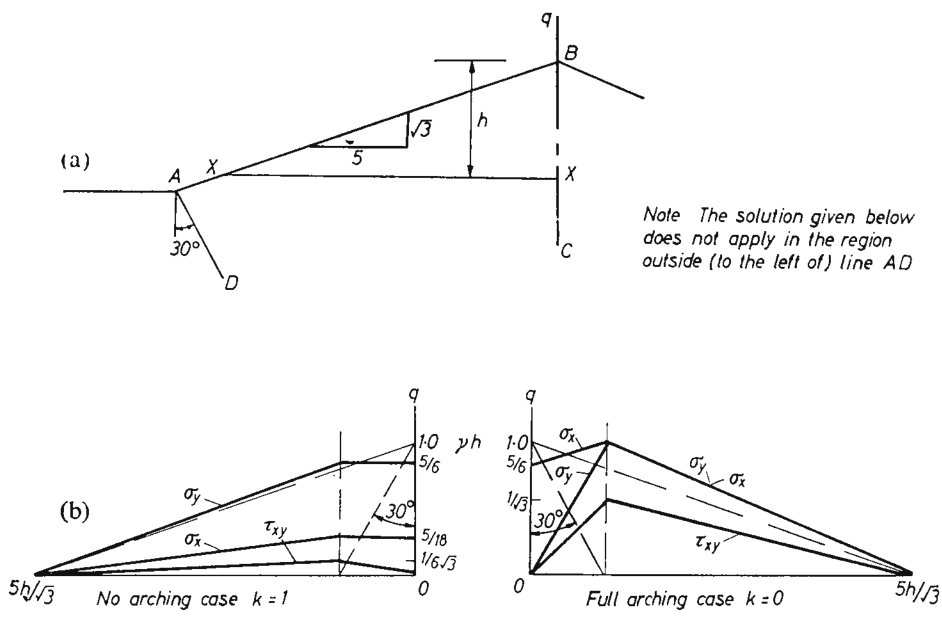

Fig. 8 Theoretical solution for an embankment with side slopes approximately 3:1
Solution théorique pour un remblai incliné approximativement à 3:1

The second method seeks to evaluate the ratio

$$\frac{\text{available shear strength}}{\text{applied shear stress}}$$

for all parts of the embankment by means of a point-to-point stress analysis.

In current literature there is a tendency to use the term 'factor of safety' indiscriminately to cover both the above ratios. It is the author's view that these differing measures of safety should be separately identified. Hence the following terminology is used in the present paper

$$\frac{\Sigma \text{ available shear resistance}}{\Sigma \text{ applied shear stress}}$$

over the assumed surface of failure ≡ Stability Factor (S_F)

$$\frac{\text{available shear strength}}{\text{applied shear stress}}$$

at any point ≡ Factor of Safety (F).

Assumed surface of failure analysis—The limiting theoretical solutions for the special case of Fig. 8a are given in Fig. 8b. These solutions can be represented in terms of principal stress trajectory patterns and the associated potential slip lines. In Fig. 9 the no arching and full arching cases are represented in this manner.

It will be noted that, theoretically, there is no possibility of failure along a continuous surface within the embankment if the stress system satisfies the no arching condition. Hence the shear strength of the foundation is critical in this case.

On the other hand failure within the embankment is possible when full arching conditions are developed. The resulting surface of potential failure is shown in Fig. 9b. The state of stress is completely defined and the equilibrium of the zone ABDE can be readily assessed if the shear strength parameters for the embankment material are known.

A similar but less rigorous analysis has been proposed by Reinius (1955) following the work of Samsioe (1955).

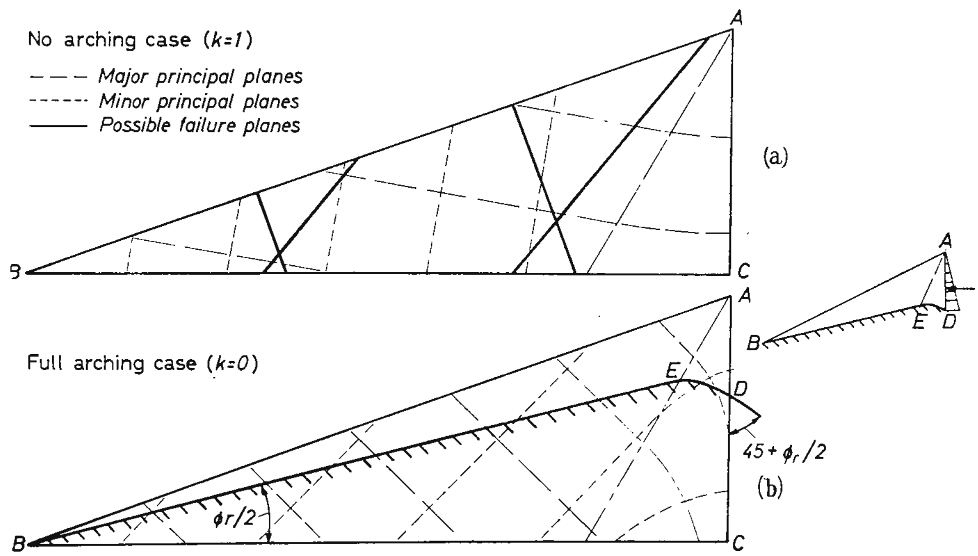

Fig. 9 Theoretical principal stress trajectory pattern for an embankment with side slopes approximately 3:1
Reseaux théoriques des trajectoires des contraintes principales pour un remblai incliné approximativement à 3:1

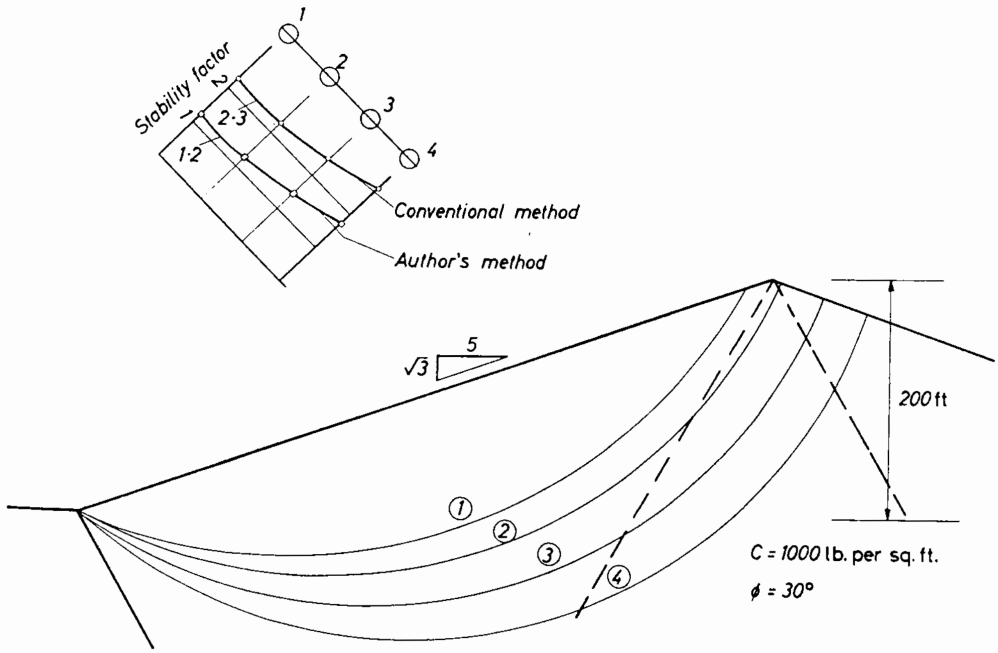

Fig. 10 Stability analysis of homogeneous embankment with side slopes approximately 3:1
Analyse de la stabilité d'un remblai homogène incliné approximativement à 3:1

It must be recognized however that there may be possible modes of collapse other than that represented by the above analysis. In this latter case no allowance is made for the re-distribution of stress that would result from plastic yield within the embankment or foundation.

A possible alternative method of analysis is to assume, in common with the conventional slip-circle methods, that rotational collapse will occur along a circular arc.

In Fig. 10 a comparison is made between the stability factor as calculated by the conventional method (May and Brahtz, 1936) and as derived from assuming that the distribution of stresses along the circular arc is completely defined by the full arching condition.

The results of this analysis show that the stability factor calculated by the author's method (1·2) is significantly lower than that calculated from the conventional method (2·3). It is recognized that under full arching conditions the assumed state of stress represents a theoretical limit which can be approached but not fully attained in practice. Thus this method leads to a conservative estimate of the stability factor with the assumed mode of rotational collapse. The example serves, however, as an indication that under circumstances where arching may be developed the conventional analysis can give an over-estimate of the overall margin of safety of an embankment.

A further mode of collapse which may have to be considered is that represented by the sliding block analysis, wherein failure occurs in a weak foundation stratum. The stability factor for this case may be readily evaluated from the known shear conditions along the base of the embankment.

Factor of safety analysis—If the shear strength of the embankment material is defined in terms of the fundamental soil constants c_r, ϕ_r (Skempton and Bishop, 1954) then the factor of safety at any point may be evaluated from the following equation:

$$F = \frac{2c_r + [\{(\sigma_x + \sigma_y) - ((\sigma_y - \sigma_x)^2 + 4\tau_{xy}^2)^{\frac{1}{2}} . \sin\phi_r\} - u]\tan\phi_r}{((\sigma_y - \sigma_x)^2 + 4\tau_{xy}^2)^{\frac{1}{2}} . \cos\phi_r} \qquad \dots . (1)$$

where u denotes pore pressure.

Brahtz (1936) proposed a similar expression which differs from equation 1 only in that the mean normal stress and the mean shear stress are used in place of the normal and shear stresses on the planes inclined at (45 degrees + ($\phi_r/2$)) to the major principal plane.

It has been shown (Trollope, 1956) that a solution for the detailed stress distribution developed by Brahtz can be modified to give solutions which agree closely with those given by the systematic arching theory. Brahtz's solution depends on the assumption that the ratio $\sigma_x:\sigma_y$ at the centre line is a constant (K)—termed the 'compaction factor'—for which he suggested the value $50/\gamma$. If however K is regarded as a variable arching factor it is found that the no arching case is closely represented by the solution obtained with $K = 0{\cdot}33$ and the full arching case by the solution obtained with $K = \infty$.

Brahtz extended his analysis to include an assessment of the pore pressure distribution that would be required to develop failure conditions ($F = 1$) throughout the embankment.

This is an interesting and important use of the point-to-point stress analysis. A further use of the method is also found in the fact that it will assist in determining the shape of the potential failure surface in a composite embankment-foundation arrangement.

Care must be exercised in interpreting the results, however, as in nearly all cases there will be regions within the embankment where the calculated factor of safety will be less than unity. Allowance must therefore be made for the re-distribution of stresses within these regions.

Conclusions

It is shown that there is good agreement between the solutions for the stress distribution within embankments as predicted by the systematic arching theory and a wide range of practical experience.

These solutions range from the most stable no arching case to the full arching case under which collapse due to instability is most likely to occur.

The development of arching characteristics within an embankment is found to be associated with deformation of the foundation and it is suggested that settlement within the embankment itself leads to the formation of an arching stress pattern.

The application of the theory to the problem of two-dimensional stability analyses is considered in two ways. The first application is one in which stability is assessed in terms of the equilibrium conditions over an assumed surface of failure. It is shown that the shape of this surface differs considerably from the conventional circular arc if failure within the embankment can occur. Further if the mode of collapse is assumed to be over a circular arc the conventional method of analysis can give a serious over-estimate of the margin of safety.

The second method entails a point-to-point stress analysis and leads to an expression for the factor of safety at all points within the embankment. This approach is deficient however in that no allowance can be made for plastic yield of the material. It is probable that in an economic design for an embankment some zones of plastic yield will occur. The usefulness of this method of analysis lies therefore in that it will assist the prediction of the shape of the potential failure surface.

A significant implication of the theoretical work is that control of deformation will contribute to the stability of an embankment. This is particularly important in the case of a rock-fill dam where considerable economies in design are possible if the assumption of a no arching stress distribution is warranted.

The most vital immediate problem is to obtain further information on the actual pressure distribution in full scale embankments. In this context it is to be hoped that, in future, pressure cell installations will be included in all major embankments. The economies that are possible as a result of adequate empirical information on the extent of the arching development far outweigh the cost of obtaining this information.

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