# Foundations of Structures—(a) General Subjects and Foundations other than Piled Foundations

## Fondations de Constructions — (a) Sujets Généraux et Fondations autres que Fondations sur Pieux

### General Report

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**Introduction**

Division 3a includes the theory and practice of foundations for buildings and other works (excluding piled foundations, and foundations for earth dams and pavements), bearing capacity, pressure distribution, consolidation and settlement, underpinning, laboratory and field studies of foundations.

It is only natural that this Division, which from a practical point of view is one of the most important ones, has inspired a very large number of contributions. No less than 44 papers have been accepted, and it is evident that in the limited space at our disposal only a few words can be said about each.

At the Zürich Conference in 1953 the present General Reporter was seriously worried by the fact that the main interest seemed to be centred in the collecting of miscellaneous case records, whereas the development of theories was discouraged as being premature.

We are of the definite opinion that real progress in soil mechanics—as in any other natural science—will only be possible through a combination of theories, tests and field experience. Theories are necessary to correlate the existing evidence, to direct further research and to enable extrapolations to new cases to be made. Tests are necessary to check the theories, to correct them if necessary and to study the influence of each individual variable separately. Field experience is, of course, the final proof of the reliability of a calculation method; it serves also to detect possible scale effects, and to determine whether theories based on the necessary simplifying assumptions are really applicable to the heterogeneity of actual soil conditions.

In view of this we have been pleased to notice that the present contributions to Division 3a are rather evenly divided between theories, tests and case records. Although they hardly contain any new and startling discoveries, a steady progress is being made towards a better understanding of the behaviour of soils and a greater assurance in the design of foundation structures.

We have chosen to divide the papers into the following main groups:

(1) *Bearing capacity*—(a) Theories (3 papers); (b) Tests (5 papers); (c) Case records (2 papers).

(2) *Stress distribution*—(a) Theories (6 papers); (b) Tests (2 papers).

(3) *Settlements*—(a) Theories (6 papers); (b) Case records (5 papers).

(4) *Special problems*—(a) Swelling clays (4 papers); (b) Collapsible soils (4 papers); (c) Effects of vibrations (2 papers); (d) Sand drains (2 papers); (e) Construction procedures (3 papers).

We shall now proceed to make a few comments on each of the individual papers belonging to the above-mentioned groups, and to combine with this survey some general remarks concerning the present state of our knowledge of the different subjects.

**Bearing Capacity**

*Theories*—Modern calculations of bearing capacity are based on the plasticity theory, usually combined with some simplifying assumptions. Speaking for simplicity about plane problems only, we can determine the bearing capacity when we know the shape and position of the critical rupture figure as well as the stress distribution along the different rupture lines.

However, the result will only be correct when the critical rupture figure is statically, as well as kinematically, possible. The first statement means that all equilibrium conditions should be fulfilled throughout the earth mass, and that in all rupture zones and rupture lines the shearing stresses should be equal to those defined by the failure condition, whereas they should be smaller than this outside the rupture zones and rupture lines. The second statement means that at least one set of the possible deformations and movements implied by the rupture figure should be compatible with a possible set of deformations and movements of the foundation structure.

Mathematically correct solutions have been obtained in only a few very simple cases (usually for $\phi = 0$ or $\gamma = 0$); therefore, most existing methods are based on different simplifying assumptions.

The most difficult part is the determination of the shape of the actual rupture figure; consequently, its general shape is usually assumed to be a straight line, a circle, a logarithmic spiral or a combination of these. Here is introduced an error on the unsafe side, because a rupture figure of another shape might possibly be more critical.

The next problem concerns the stress distribution along rupture lines. Actually, this should be no problem, because it has already been solved mathematically by Kötter (1903). However, Kötter's equation has been used very little, probably because it has been considered too complicated; moreover, a certain suspicion has been thrown on it by a paper by Coenen (1948) from which many engineers have got the impression that Coenen has disproved the validity of Kötter's equation. Actually, Coenen has only stated that if it is applied to a wrong rupture line the result will be wrong, which is hardly surprising. On the other hand, it may be assumed that if Kötter's equation is applied to an approximately correct rupture line the result must be approximately correct.

That Kötter's equation is actually an exceedingly valuable and convenient tool in the analysis of bearing capacity, earth pressure and stability problems has been shown by the General Reporter (BRINCH HANSEN, 1953). The method used for this purpose may be called the 'equilibrium method', because—apart from Kötter's equation—it employs only the equilibrium conditions for the different parts of the rupture figure. This is sufficient to determine both the position of the critical rupture figure and the corresponding value of the bearing capacity (respectively earth pressure or safety factor).

To avoid using Kötter's equation two other methods have been followed. One consists in assuming such a shape of the rupture line (generally a logarithmic spiral) that one equation of equilibrium (moment equation about the pole) can be indicated in which the unknown stresses in the rupture line do not enter at all. The critical rupture line is then determined by means of a minimum condition: a special example of this is the usual '$\phi = 0$ analysis'.

The other way consists in making a more or less arbitrary assumption enabling the corresponding distribution of stresses in the rupture line to be determined. To this group belong the well-known 'slice methods'. Also in these methods the critical rupture line is determined by the minimum principle. Consequently, both this method and the one mentioned above may be termed 'extremum methods'.

Whereas the pure extremum method (as exemplified by the $\phi = 0$ analysis) does not introduce any errors apart from those involved in the chosen shape of the rupture lines, errors of unknown and often considerable magnitude are involved by the additional assumptions made in the different slice methods and other approximate methods.

This has been demonstrated by BISHOP (1955), who found the conventional slice method to be considerably on the safe side, whereas he succeeded in developing a more reliable method by changing the basic assumption concerning the slice forces.

Similarly, in a paper to this Conference Raes (3a/33) has investigated the approximate calculation method of Andersen and has found it to be very inaccurate and usually considerably on the safe side.

The most extensive contributions to bearing capacity theory have without doubt been supplied by MEYERHOF. He started with vertically and centrally loaded foundations (1951), continued with eccentric and inclined loads (1953) and has now extended his theories to cover also foundations on slopes (3a/26). The results should be very useful, although they are based on approximate rupture figures which are probably neither statically nor kinematically possible.

It is a deplorable fact that, as soon as both the conditions $\gamma \neq 0$ and $\phi \neq 0$ apply, nobody has as yet been able to indicate a figure of rupture which can be proved to be both statically and kinematically possible. This applies even to the simple case of a centrally and vertically loaded strip on the surface of cohesionless, unloaded sand. Only LUNDGREN and MORTENSEN (1953) have indicated a rupture figure which can be proved to be statically possible, but even they have not investigated its kinematical possibility; consequently, the exact value of $N_\gamma$ is still unknown!

Considering the difficulties described in making two-dimensional bearing capacity calculations, it is no wonder that three-dimensional theories are very rare and complicated. One such is, however, contributed by Kopácsy (3a/17) who tries to determine the shape of a doubly curved slip surface by means of a minimum principle.

*Tests*—By means of model tests the bearing capacity theories can be checked. Test results should always be plotted in terms of dimensionless quantities, as this will facilitate the detection of possible scale effects.

Bearing capacity tests are often made with 'dry, cohesionless sand', but when different sizes of the bearing plates are used a scale effect is usually found which may be interpreted as a 'cohesion', probably due to unavoidable soil moisture. Reliable values of the bearing capacity factors $N$ can only be obtained when the necessary corrections have been made for this and other possible scale effects.

Even then it seems difficult to obtain reasonable agreement between tests and theories. In our laboratory in Copenhagen tests have been made by the Assistant Reporter with circular, rough model foundations on 'dry' sand. At the same time the friction angle was determined by means of triaxial tests, and $N_\gamma$ and $N_q$ calculated from the conventional formulae. However, even after having corrected the loading test results for the above-mentioned scale effects, the actual values of $N_\gamma$ and $N_q$ were found to be several times those calculated. So far we have not been able to explain this discrepancy, but the influence of the intermediate principal stress may provide a partial explanation.

In a paper to this Conference Tcheng (3a/41) has studied, by means of model tests, the bearing capacity of a foundation on a shallow sand layer resting on a deep clay deposit. He found that when the thickness of the upper layer is less than 1·5 times the foundation width, the rupture lines in the upper layer are vertical.

In a very interesting paper, Berezantzev and Yaroshenko (3a/5) have dealt with foundations on sand at different depths, and have specially investigated the combination of plastic deformations (slidings) and elastic ones (compaction) in the different zones. The results of their model tests agree reasonably well with theoretical calculation methods.

The special problem of foundations subjected mainly to overturning moments has been investigated by Lazard (3a/19) by means of 200 full-scale tests on mast foundations. He has compared the results with two empirical formulae, the constants of which can be fixed so that they give the measured results with an error of usually not more than − 30 or + 40 per cent. Strangely enough the formulae contain no quantity characterizing the strength of the soil.

Roscoe (3a/35) has also investigated foundations subjected to moments from the superstructure. He has studied free foundations and foundations tied at ground level, both by means of model tests and calculations. The latter are based on classical earth pressure theory and are therefore rather crude; a better approximation could probably have been obtained by using the general equilibrium method. Roscoe's main result is that tied foundations should be preferred because they can withstand twice as great moments as free foundations.

An unusually interesting full-scale test is reported by Kérisel (3a/15) who has studied the load distribution for a 15 m deep pier or pile of 1·4 m diameter. A comparison with penetrometer tests shows clearly how necessary it is to be careful in the interpretation of model and field test results.

*Case records*—Two cases of actual failure have been reported. Bjerrum and Overland (3a/6) describe an oil tank which possessed a satisfactory safety margin against total failure but failed locally due to the lower shear strength of the upper clay layers. Raedschelders and Wallays (3a/32) report on a building near an excavation where a local failure started but was stopped by placing a counterweight filling.

**Stress Distribution**

*Theories*—The distribution of the contact pressure between soil and foundation may be determined either by plasticity theory (corresponding to the state of failure) or by elasticity theory (corresponding to working conditions). In the latter case the calculation is often simplified by introducing a 'coefficient of subgrade reaction' which, unfortunately, is seldom a constant for actual soils. Nevertheless, the method may give usable results (TERZAGHI, 1955). When dealing with clay, it must be remembered that due to the consolidation process and plastic flow the contact pressure distribution will change with time.

The stress distribution at different depths below the foundation is usually determined by means of Boussinesq's formulae, which are based on the theory of elasticity. It is a well-known fact that even at working loads the actual soil does not behave as a perfectly elastic material. However, it is generally assumed that the errors involved in this are not very important, at least not in comparison with other sources of errors involved in the calculation of settlements, of which the determination of stress distribution is usually a part. For the same reason too much mathematical refinement in the calculation would be out of place.

In a paper to this Conference de Beer (3a/3) has investigated the contact pressure distribution under rectangular, rigid footings by assuming the soil to possess a modulus of elasticity which is either constant or increasing linearly with depth. He finds that although the pressure distribution in the transverse direction is far from uniform this does not affect the longitudinal distribution materially. Lousberg (3a/20) has continued this study by considering an eccentrically loaded foundation beam which may be infinitely rigid or possess a definite rigidity. However, he determines the longitudinal distribution only.

That the stress distribution under a continuous footing is influenced not only by the rigidity of the soil and the footing but also by the rigidity of the superstructure is emphasized by Krsmanovitch (3a/18), who advocates that the three parts (superstructure, foundation and soil) must be treated as a whole. The same problem is considered by Grasshoff (3a/9) who, by means of numerical examples, finds that the degree of fixity of the columns in the footing is actually more important than the rigidity of the superstructure.

A contribution of direct practical value has been supplied by Osterberg (3a/28), who gives an influence chart for determining the vertical stresses due to embankment loadings (trapezoidal surface loads).

In a very mathematical paper, Koning (3a/16) has extended Boussinesq's theory to the case of an anisotropic, elastic solid. Although soils are actually anisotropic it is difficult to see how this can be considered in practice as it would require the determination of five elastic constants instead of the usual two.

*Tests*—An interesting paper is supplied by l'Herminier, Bachelier and Soeiro (3a/10) who, by means of vibrating wire gauges, have measured the contact pressures below a 36 × 29 × 3·8 m raft foundation resting on 7 m of gravel on top of plastic clay. The stress distribution proved to be intermediate between a uniform distribution and the distribution determined by elasticity theory.

Recordon (3a/34) has made a great number of plate bearing tests on different soils in order to determine the coefficient of subgrade reaction. He finds this quantity to be inversely proportional to the diameter of the plate. This is actually an experimental verification of a theoretical model law.

**Settlements**

*Theories*—It is by now generally agreed that for the settlement of any foundation on clay three different components must in principle be considered: the immediate settlement, the consolidation settlement and the secondary settlement:

$$\rho = \rho_i + \rho_c + \rho_s$$

The immediate settlement takes place without any volume change and is due mainly to shear stresses. It is usually calculated on the basis of the theory of elasticity, with Poisson's ratio equal to 0·5 and with a modulus of elasticity determined by triaxial or unconfined compression tests. This is the main weakness of the method, because the determination of $E$ proves to be extremely sensitive to sample disturbance and testing method (see paper by Simons (3a/38)).

The consolidation settlement is due to the slow extrusion of pore water from the loaded clay. The final consolidation settlement is always calculated on the basis of oedometer tests, often corrected, e.g., as indicated by Terzaghi and Peck (for normally consolidated clays) or by SCHMERTMANN (1953) (for lightly over-consolidated clays). For heavily over-consolidated clays no generally established satisfactory method of correction exists as yet, although attempts are being made (see later).

The rate of settlement is usually calculated according to Terzaghi's one-dimensional theory of consolidation, although two- and three-dimensional theories are being developed now. At any given time $t$ the total settlement should be:

$$\rho_t = \rho_i + U_t\rho_c$$

where the degree of consolidation $U$ is a function of the so-called time factor $T$.

Concerning the secondary settlement very little is known. It goes on after the excess pore water pressures have vanished and is probably due to a 'creeping' of the grain structure under load, analogous to the creep of concrete. Fortunately this effect is in most cases of little practical importance and is therefore usually left out of consideration. An exception is provided by soils with a high content of organic matter, in which a considerable secondary settlement can occur probably due to slow chemical processes (see also paper by Ishii, Shinohara, Tateishi and Kurata (3a/11).

In another paper de Josselin de Jong (3a/13) investigates by means of stress functions the three-dimensional consolidation in a couple of simple cases with axial symmetry. It is interesting to notice that in the case of a circular surface foundation the immediate settlement should never be less than the consolidation settlement.

The same result is found by Gibson and McNamee (3a/8) who have undertaken the useful task of determining, by means of Biot's theory of three-dimensional consolidation, the settlement of the corner of a rectangular surface load. As the law of superposition is valid this can easily be extended to determine the settlement at any point of a surface loaded by a foundation of any shape.

In the first part of his paper, Mandel (3a/21) investigates the three-dimensional consolidation of a thick clay deposit, covered by a permeable layer which is loaded by a point load or a circular foundation. In the second part he studies the secondary settlement under constant load, assuming the strain to be a function of the time and proportional to the difference between the actual stress and a threshold value.

Da Silveira (3a/37) has investigated the special case of consolidation under varying loads such as may occur under a flexible footing. He has also checked his calculations by means of model tests and has found good agreement.

Palmer and Brown (3a/29) analyse the frequently occurring case where settlements have progressed for some time when the investigations are started. It is then necessary to compute backwards as well as forwards in time, and a method for this is indicated. A comparison between calculated and actual settlements is given for a certain case; a further check could have been obtained by making direct pore pressure measurements.

Concerning settlements on sand, de Beer and Martens (3a/4) have described a calculation method in which the compression index is determined by cone penetration tests (equal to 1·5 times the bearing capacity factor). The possible heterogeneity of the sand is taken into account by considering the variations of at least three cone tests. The calculated settlements for the abutments of six bridges have been compared with the measured settlements with the result that the calculated upper limit is always less than twice the actual settlement.

*Case records*—A comparatively great number of case records, in which actual settlements are compared with calculated values, have been published recently, mainly by COOLING and GIBSON (1955), MACDONALD and SKEMPTON (1955), PECK and UYANIK (1955) and SKEMPTON, PECK and MACDONALD (1955).

The general result seems to be that for normally consolidated and lightly over-consolidated clays the existing methods allow a calculation of the probable settlements with errors usually not exceeding − 30 and + 50 per cent. However, this presumes that care is taken to obtain samples as undisturbed as possible for the oedometer tests.

Whereas this must be considered satisfactory for practical purposes, the situation is radically different where heavily over-consolidated clays are concerned. If the primary branch of the oedometer curve is used the calculated settlements will be considerably greater than the observed ones, e.g. for Danish glacial clays 5 to 10 times greater. Two possible ways of making the observed and calculated settlements agree are being tried at present: one way is to establish semi-empirical reduction coefficients which may depend on the pore pressure coefficients (Skempton and Bjerrum) or other properties of the clay; another is being used successfully in Denmark for the above-mentioned glacial clays. These are so heavily pre-compressed that any new loading must be a re-compression; consequently, the re-compression branch of the oedometer curve is used, which proves to ensure a fair agreement between calculated and observed settlements, at least for the Danish glacial clays.

The reliability of the conventional methods for normally consolidated clays has been investigated by Simons (3a/38), who describes two damaged buildings in Norway. As the values of $E$ varied in the ratio 1:10, depending on the testing method, the immediate settlement could only be indicated very roughly. The final consolidation settlements were calculated with fairly good accuracy, but the observed rate of settlement was higher than calculated, probably due to three-dimensional consolidation.

In a paper by Youssef, Sabry and Tewfik (3a/44) two cases of severe damage to hospitals in Egypt are described. In one case the cause was swelling of a stiff clay, in the other consolidation of a soft clay. Both buildings were constructed without preceding soil investigations.

Nichiporovich (3a/27) has studied the settlements of 16 large hydraulic structures, founded mainly on pre-consolidated clays. He calculated the final settlements by means of an elasticity theory with $E$ determined from oedometer or loading tests. The observed settlements were, however, considerably smaller, and took place much quicker than they should according to conventional consolidation theory.

A much better agreement was obtained by Egorov, Kuzmin and Popov (3a/7) in their study of the settlements of seven multi-storey buildings in Moscow. The settlements were kept within allowable limits by means of floating foundations.

In a very interesting paper Polshin and Tokar (3a/31) examine first the accuracy of four methods of settlement calculation by comparison with 93 observations; secondly, they give the results of an extensive study concerning allowable deformations of different structures characterized by slope, relative deflection and average settlement.

A rather similar investigation of great practical value has been made by SKEMPTON and MACDONALD (1956) who, as settlement criteria, consider the maximum settlement, the differential settlements and the angular distortions.

**Special Problems**

*Swelling clays*—Both swelling and shrinking clays are very common in Canada, and the experience gained is discussed by Baracos and Bozozuk (3a/2). Such clays are characterized by a high liquid limit and plasticity index as well as a high content of active clay minerals. A correlation of ground movements and seasonal climatic changes has been established by the authors.

A study of the design problems in connection with swelling clays has been undertaken by Salas and Serratosa (3a/36). They advocate laboratory tests to determine the swelling pressure and the free swelling under constant load: on the basis of such tests the movements of a foundation can be predicted approximately. They also discuss different constructive measures and indicate an empirical method for the calculation of reinforcement in footings on swelling clays.

In most clays a certain swelling takes place when the clay is relieved of part of its former load, e.g. by an excavation. A number of such cases are described in the above-mentioned papers by Nichiporovich (3a/27) and Egorov, Kuzmin and Popov (3a/7).

A rather unusual case is reported by Mayer and Habib (3a/23). It concerns the swelling of a silty soil with a high content of chalk due to accidental infiltration of phosphoric acid into the soil.

Finally, Pietkowski (3a/30) has studied the danger of frost heaving of a skating-rink foundation. The heat transfer is calculated by means of a new numerical method based on Fourier's law.

*Collapsible soils*—It is a well-known fact that wind-blown deposits of fine-grained materials in a loose, non-saturated state may suddenly collapse if they are inundated while being loaded. This danger exists, for instance, in loess and similar soils, and if excessive settlements are to be avoided the soil must be stabilized by mechanical or chemical means.

One such case is described by Karafiáth (3a/14); in this instance the loess above ground water level was compacted by means of pile driving. The settlements of the structure on the compacted soil were calculated on the basis of a modulus of compressibility determined by loading tests; they proved to be greater than the observed ones. For the rate of settlement a three-dimensional numerical calculation was made, taking into consideration the anisotropy of the compacted soil.

A more comprehensive discussion of foundations on loess has been given by Abelev and Askalonov (3a/1). The first describes the compaction by means of tampers, piles and explosions in bore holes. The second discusses stabilization by chemical injections.

In South Africa a fine, red, silty sand has given similar trouble. Jennings and Knight (3a/12) have developed a special laboratory technique for recognizing such soils and have indicated a method for predicting the settlements. A. A. B. Williams (3a/42) has extended this study by investigating the shear strength of these soils at natural moisture content by loading after inundation and by inundating after loading. Considerable differences were found, and bearing capacity calculations according to Terzaghi's formula could only be made to agree with loading tests when the lowest friction angle was used.

*Effects of vibrations*—The problems encountered in this group belong to the most difficult in soil mechanics. An analysis of test results and their extrapolation to full-scale foundations requires a knowledge of the deformation properties of the soil and of the stress variations caused by the vibrations, for which no satisfactory theories exist as yet.

Mencl and Kazda (3a/25) have studied the bearing capacity of vibrating model foundations on sand. They have found the interesting result that the effect of the vibrations decreases with increasing surface load outside the foundation and ends by vanishing completely for a certain load.

Maslov (3a/22) considers the stability of a mass of saturated sand subjected to vibrations. These will tend to compact the sand, thereby causing excess pore water pressures and an ascending flow of water, the gradients of which must reduce the effective weight and the shear strength of the sand. The theory developed is believed to constitute a rational approach to the problem and a useful basis for further research.

*Sand drains*—Two papers deal with the calculations for vertical sand drains. Takagi (3a/40) has first supplemented Barron's graphs, taking into account a greater range of the ratio of the diameter of the equivalent circle to the diameter of the sand pile. Further, he has indicated a method for calculating the degree of consolidation by gradually increasing load.

Ishii, Shinohara, Tateishi and Kurata (3a/11) start with a discussion of the error sources connected with conventional oedometer tests. The disturbing influences are secondary consolidation, side friction and partial remoulding; this is, of course, pertinent to all consolidation computations. The results are applied to four sand drain projects with amazing agreement between calculated and observed settlements.

*Construction procedures*—In this group Steuerman and Murphy (3a/39) describe the foundation works for a new subaqueous tunnel resting on sand and silt. For protection of the portals two artificial islands had to be built, one on a natural sand deposit, the other by substituting the existing silt by sand. In order to avoid excessive settlements due to vibrations the sand in both islands was compacted by means of vibroflotation.

G. M. J. Williams (3a/43) gives an account of the problems encountered in building tall office blocks with deep basements at a site in London where several subway tunnels pass. Some of the buildings must have deep foundations, consisting of concrete cylinders cast *in situ*, in order to avoid excessive differential settlements or damage to the tunnels.

Finally, de Mello and Geotecnica (3a/24) report on two very unusual foundation works for multi-storey buildings in São Paulo. One of them required excavation to 20 m below ground level and 10 m below ground water. The design of the structures was very much influenced by the foundation problems, which were solved by unconventional and ingenious methods.

**Conclusions**

In the design of foundations two problems are of paramount importance. One concerns the estimation of the settlements and the other the determination of the safety against failure.

Present settlement calculations determine the final settlements on the basis of oedometer tests (consolidation settlements) respectively compression tests (immediate settlements) and a stress distribution corresponding to the theory of elasticity. The rate of settlement is computed by means of Terzaghi's one-dimensional theory of consolidation or, in rare cases, by two- or three-dimensional extensions of it.

All this is based on a number of simplifying assumptions which are crude approximations at the best. Actual soils are not homogeneous, isotropic or elastic, samples for laboratory tests are not undisturbed, laboratory techniques involve several sources of error, scale effects may exist, and uncharted effects as the secondary consolidation will introduce further deviations. Therefore, it is actually surprising that the described method, which is hardly more than semi-empirical, is usually able to predict the observed settlements within a reasonable margin, at least for normally consolidated and lightly over-consolidated clays. On the other hand, for heavily over-consolidated clays the deviations between calculated and observed settlements are generally excessive, and no fully satisfactory method has been devised as yet to remedy this.

Concerning bearing capacity the situation is somewhat different. The mathematical theories of plasticity have been fairly well developed, and although most of the calculations are approximate only, because the assumed figures of rupture are seldom both statically and kinematically possible, the results cannot be very much wrong. Moreover, in the state of failure considered in the theories of plasticity, actual soils do very nearly exhibit the properties assumed in these theories.

It is rather disconcerting, therefore, to find that, at least in some series of experiments, excessive deviations may exist between calculated and observed bearing capacities, the latter being far greater than the former. The explanation is still obscure but it is possible that the whole basic concept of shear strength will have to be revised.

A paradox of another kind is that most building codes still retain the fiction of 'allowable foundation pressures' dependent on the type of soil only, in spite of the now well-known fact that the bearing capacity depends also on the size, shape and depth of the foundation, as well as on the inclination and eccentricity of the foundation load.

A good deal of theoretical and experimental work has already been done to evaluate the above-mentioned effects, but what a practical engineer needs is a simple formula which takes all the pertinent influences into account in an approximately correct way. Two semi-empirical formulae of this kind have, in fact, been proposed by the General Reporter (BRINCH HANSEN, 1955) and it may, perhaps, be of interest to cite them here.

For clay ($\phi = 0$) the formula, which is an extension of SKEMPTON's (1951) reads:

$$\frac{Q}{A} = 5c\left(1 + 0{\cdot}2\frac{B}{L}\right)\left(1 + 0{\cdot}2\frac{D}{B}\right)\left(1 - 1{\cdot}3\frac{H}{V}\right) + q$$

with the limitations: $B \leqslant L$, $D \leqslant 2{\cdot}5B$ and $H \leqslant 0{\cdot}4V$.

For sand ($c = 0$) the following similar formula is proposed:

$$\frac{Q}{A} = \tfrac{1}{2}\gamma BN_\gamma\left(1 - 0{\cdot}3\frac{B}{L}\right)\left(1 - 1{\cdot}5\frac{H}{V}\right)^2 + qN_q\left(1 + 0{\cdot}2\frac{B}{L}\right)\left(1 + 0{\cdot}1\frac{D}{B}\right)\left(1 - 1{\cdot}5\frac{H}{V}\right)$$

with the limitations: $B \leqslant L$, $D \leqslant 15B$ and $H \leqslant V\tan\phi$.

$Q$ is the vertical bearing capacity of the foundation, $V$ and $H$ the vertical and horizontal components of the actual foundation load; $A$ is the so-called useful area, i.e. that part of the total foundation area which is centrally loaded by the foundation load; $B$ and $L$ are the greatest width and length of the area $A$, and $D$ its smallest depth below the earth surface; $q$ is the minimum effective overburden pressure at foundation level, and $\gamma$ the effective unit weight of the earth below this level; finally, $N_\gamma$ and $N_q$ are the usual bearing capacity factors for a centrally and vertically loaded strip on the surface, and $c$ is the undrained shear strength of the clay.

We should like to add a few comments concerning the way in which a safety factor is introduced in the design. When the ultimate bearing capacity is calculated by means of the plasticity theory or found by tests, the allowable foundation load is usually obtained by dividing the ultimate bearing capacity by a safety factor.

However, concerning stability analyses there is by now an almost universal agreement that the most rational way to introduce a safety factor consists in reducing the actual shear strength of the soil by dividing $c$ and $\tan\phi$ with suitable safety factors which may—or may not—be identical.

In developing this general earth pressure theory (BRINCH HANSEN, 1953) the General Reporter found that here also the safety factor must be introduced in the manner described above in order to make rational design methods possible. We are convinced that this will be generally accepted as soon as earth pressure calculations proceed beyond the semi-empirical stage represented by the classical theories and their current applications.

Then, it would seem a natural and logical extension to introduce the safety factor in the same way in bearing capacity calculations too, i.e. by using reduced values of $c$ and $\tan\phi$. For clay the safety factor on $c$ may be practically the same as that used for foundation pressures hitherto, but for sand it will be found necessary to use a much smaller safety factor on $\tan\phi$, due to the sharp increase of the bearing capacity factors $N_\gamma$ and $N_q$ with $\phi$.

Moreover, it will probably be found—as suggested by the General Reporter (BRINCH HANSEN, 1956)—that a general and logical system can only be obtained when certain safety factors are applied to the loads too. The principle is to consider a nominal state of failure in which the actual loads are multiplied by certain (partial) safety factors, whereas the actual strengths of the soils and of the building materials are divided by other (partial) safety factors. In this nominal state of failure equilibrium should exist.

The following values of the partial factors of safety have been proposed in Denmark: dead load: $F_g = 1{\cdot}0$, live load: $F_p = 1{\cdot}5$, water pressure: $F_w = 1{\cdot}2$, cohesion: $F_c = 1{\cdot}5 - 2{\cdot}0$, friction: $F_\mu = 1{\cdot}2$. The structural members may be designed for the forces and moments found in the nominal state of failure, but with 'allowable' stresses 50 per cent higher than those normally used.

### Summary and Proposals for Discussion

The most important subjects in this Division are bearing capacity, stress distribution and settlements.

(1) *Bearing capacity*—Bearing capacity theories have been fairly well developed, mainly by Meyerhof, who has given his results in a considerable number of graphs. For practical use a few simple formulae, such as those indicated earlier in this Report, might be more convenient in spite of being somewhat less accurate.

Unfortunately, model tests have, at least in some cases, shown such excessive deviations between calculated and observed bearing capacities that the explanation can hardly be faulty theory alone. From a practical point of view it is reassuring, however, that the observed bearing capacities are usually greater than the calculated ones.

A special problem concerns the most suitable way of introducing safety factors into a limit design method for foundations. In our opinion a general and logical system can only be devised by multiplying the actual loads by certain (partial) safety factors, and at the same time dividing the actual strengths of the soils and building material by other (partial) safety factors. The dimensions of the structure should then be chosen so that equilibrium will exist in a nominal state of failure, in which the nominal loads and strengths are as defined above.

Of the contributions to this Division, 3 deal with bearing capacity theories, 5 with model or full-scale tests, whereas 2 are case records.

*Proposals for discussion*—(a) The development and use of semi-empirical formulae for bearing capacity calculations.

(b) Model tests, their agreement with calculations, and their application to full-scale bearing capacity problems.

(c) The way of introducing safety factors into a limit design method for foundations.

(2) *Stress distribution*—The stress distribution in the soil under a foundation may be determined either by plasticity theory or by elasticity theory. In the latter case Boussinesq's formulae can be used, or the calculation can be based on a coefficient of subgrade reaction. Due to the necessary assumptions these methods will, of course, only give approximately correct results, but the errors are seldom of any practical importance, as the results are mostly used in connection with a settlement calculation implying still greater sources of error.

Of the contributions to this Division, 6 deal with pressure distribution theories and 2 with model or full-scale tests.

*Proposals for discussion*—(a) The influence of the rigidity of the superstructure on the pressure distribution.

(b) The determination of the soil constants necessary for calculating the pressure distribution.

(3) *Settlements*—For clay it is necessary to distinguish between immediate settlement, consolidation settlement and secondary settlement.

The immediate settlement is usually calculated on the basis of the theory of elasticity, but the usual determination of the modulus of elasticity proves to be rather inaccurate. The consolidation settlement, which is usually the most important part, is calculated on the basis of elastic stress distribution, Terzaghi's theory and oedometer tests. About the secondary settlement very little is known yet; it is usually disregarded.

Due to a great number of sources of error the results of such settlement calculations cannot be expected to agree too well with observations. Nevertheless, a reasonably good agreement is usually obtained for normally consolidated and lightly over-consolidated clays, whereas for highly over-consolidated clays excessive deviations are found. The observed rates of settlement are for most clays found to be greater than calculated, probably due to two- or three-dimensional consolidation.

A special problem concerns the determination of the allowable settlements or deformations of different types of superstructures, and the necessary safety against actual damage. So far this problem has mainly been treated statistically.

Of the contributions to this Division, 6 deal with theories for settlement calculation, whereas 5 are case records.

*Proposals for discussion*—(a) Current methods of settlement calculation and their application to full-scale structures.

(b) Theories of two- or three-dimensional consolidation and other means of bridging the present gap between observed and calculated settlements, especially for over-consolidated clays.

(c) Allowable deformations of superstructures of different types.

(4) *Special problems*—Of the remaining contributions to this Division, 4 deal with swelling or shrinking clays, 4 concern themselves with loess and other collapsible soils, 2 discuss the effects of vibrations in sand, 2 report on the calculation of vertical sand drains, and the last 3 describe special construction procedures. Discussions can be made on all these subjects.


### Résumé et Sujets Proposés à la Discussion

Les sujets les plus importants de cette Division, sont la force portante, la répartition des contraintes et les tassements.

(1) *Force portante*—Les théories sur la force portant ont été très développées par Meyerhof en particulier, qui a traduit ses résultats en un nombre considérable de graphiques. Dans la pratique, quelques formules simples telles que celles étudiées plus haut dans ce rapport, peuvent se révéler plus intéressantes bien que moins précises.

Malheureusement, des essais sur modèles réduits ont montré, au moins dans quelques cas, des écarts tellement importants entre les forces portantes calculées et observées, que l'explication en peut difficilement être trouvée dans la seule insuffisance de la théorie. D'un point de vue pratique, il est néanmoins rassurant de noter que les forces portantes observées sont généralement plus grandes que celles que donne le calcul.

Un problème particulier réside dans le choix des coefficients de sécurité à adopter dans la détermination de la charge limite des fondations. A notre avis, la seule méthode logique et de portée générale consiste à multiplier les charges réelles par certains coefficients de sécurité (coefficients partiels) et en même temps à diviser les résistances réelles des sols et matériaux de

construction par d'autres coefficients de sécurité (également partiels). Les dimensions de la construction doivent alors être calculées afin qu'il y ait équilibre dans un état de rupture fictif entre charges et résistances nominales définies ainsi:

Parmi les communications relatives à cette Division, 3 traitent des théories de la force portante, 5 se réfèrent à des essais sur modèles réduits ou en vraie grandeur, tandis que 2 exposent des cas particuliers.

*Sujets proposés pour la discussion* — (a) Développement et emploi de formules semi-empiriques pour le calcul de la force portante. (b) Essais sur modèles réduits, concordance avec le calcul; application aux problèmes concernant la force portante en vraie grandeur. (c) Processus d'application de coefficients de sécurité dans une méthode de calcul limite des fondations.

(2) *Répartition des contraintes* — La répartition des contraintes dans le sol sous une fondation peut se déterminer soit en théorie plastique, soit en théorie élastique. Dans le dernier cas, on peut, soit employer les formules de Boussinesq, soit introduire dans les calculs, un module de réaction de la couche de fondation.

Etant données les hypothèses qu'appellent ces méthodes, celles-ci ne peuvent donner évidemment, que des résultats approchés; mais les erreurs commises ont rarement une importance pratique, puisque ces résultats sont le plus souvent utilisés conjointement avec un calcul de tassement qui implique des sources d'erreurs, bien plus importantes encore.

Parmi les communications concernant cette Division, 6 traitent des théories relatives à la distribution des pressions, et 2 concernent des essais sur modèles réduits ou en vraie grandeur.

*Sujets proposés pour la discussion* — (a) Influence de la rigidité de la superstructure sur la répartition des pressions. (b) Détermination des constantes du sol nécessaires au calcul de la répartition des pressions.

(3) *Tassement* — Pour les argiles, il est nécessaire de distinguer entre tassement immédiat, tassement dû à la consolidation, et tassement secondaire.

Le tassement immédiat est généralement calculé d'après la théorie de l'élasticité, mais la détermination classique du module d'élasticité se révèle assez inexacte.

Le tassement dû à la consolidation, généralement le plus important, se détermine d'après une répartition élastique des contraintes, et en fonction de la théorie de Terzaghi, et d'essais œdométriques. En ce qui concerne le tassement secondaire, très peu de choses sont aujourd'hui connues; il est d'ailleurs, en général, négligé.

Par suite du grand nombre de causes d'erreurs, on ne peut espérer que les résultats de tels calculs de tassement puissent cadrer vraiment bien avec les observations faites. Toutefois, on obtient généralement une assez bonne concordance en ce qui concerne les argiles normalement consolidées ou légèrement surconsolidées; au contraire, pour des argiles nettement surconsolidées, on enregistre d'importants écarts. Les vitesses de tassement observées se sont révélées, pour la plupart des argiles, supérieures aux résultats du calcul, sans doute à cause d'une consolidation bi ou tridimensionnelle.

Un problème particulier concerne la détermination des tassements ou déformations admissibles pour divers types de superstructures, et la marge de sécurité nécessaire pour éviter des incidents. Jusqu'à maintenant, ce problème a surtout été traité par des voies statistiques.

Parmi les communications se rapportant à cette Division, 6 traitent des théories pour le calcul du tassement, tandis que 5 se réfèrent à des observations particulières.

*Sujets proposés pour la discussion* — (a) Méthodes courantes pour le calcul des tassements; leur application aux constructions en vraie grandeur. (b) Théories sur la consolidation à 2 ou 3 dimensions, et autres moyens de réduire l'écart existant actuellement entre tassements observés et tassements calculés, en particulier dans le cas des argiles surconsolidées. (c) Déformations admissibles pour des superstructures de divers types.

(4) *Problèmes particuliers* — Parmi les autres communications se rattachant à cette Division, 4 traitent des argiles gonflantes ou sujettes au retrait; 4 concernent les loess et autres sols susceptibles de s'affaisser; 2 étudient les effets des vibrations dans le sable, 2 se rapportent au calcul des drains en sable verticaux. Enfin, les 3 dernières décrivent des procédés particuliers de construction. Tous ces sujets peuvent faire l'objet de discussions.

## References


BISHOP, A. W. (1955). The use of the slip circle in the stability analysis of slopes. *Géotechnique*, **5**, March

BRINCH HANSEN, J. (1953). *Earth Pressure Calculation.* Copenhagen; Danish Technical Press

— (1955). Simpel Beregning af Fundamenters Bæreevne (Simple calculation of the bearing capacity of foundations). *Ingeniøren No. 4*, Jan. (In Danish with an English summary)

— (1956). Brudstadie beregning og Partialsikkerheder i Geoteknikken (Limit design and safety factors in soil mechanics). *Ingeniøren No. 18*, May or *Bulletin No. 1* of the *Danish Geotechnical Institute.* (In Danish with an English summary)

COENEN, P. A. (1948). Fundamental equations in the theory of limit equilibrium. *Proc. 2nd International Conference on Soil Mechanics and Foundation Engineering*, Vol. 7

COOLING, L. F. and GIBSON, R. E. (1955). Settlement studies on structures in England. *Proc. Conf. Calc. and Obs. Stresses and Displacements*, Inst. Civ. Eng.

KÖTTER, F. (1903). Die Bestimmung des Druckes an gekrümmten Gleitflächen. *Sitzungsbericht Kön. Preuss. Akad. der Wissenschaften*, Berlin, 1903

LUNDGREN, H. and MORTENSEN, K. (1953). Determination by the theory of plasticity of the bearing capacity of continuous footings on sand. *Proc. 3rd International Conference on Soil Mechanics and Foundation Engineering*, Vol. 1

MACDONALD, D. H. and SKEMPTON, A. W. (1955). A survey of comparisons between calculated and observed settlements of structures on clay. *Proc. Conf. Calc. and Obs. Stresses and Displacements.* Inst. Civ. Eng.

MEYERHOF, G. G. (1951). The ultimate bearing capacity of foundations. *Géotechnique*, **2**, Dec.

— (1953). The bearing capacity of foundations under eccentric and inclined loads. *Proc. 3rd International Conference on Soil Mechanics and Foundation Engineering*, Vol. 1

PECK, R. B. and UYANIK, M. E. (1955). Observed and computed settlements of structures in Chicago. *Univ. Ill. Eng. Exp. Station, Bull.* 429

SCHMERTMANN, J. H. (1953). Estimating the true consolidation behaviour of clay from laboratory test results. *Proc. Amer. Soc. Civ. Eng.*, No. 311, Oct.

SKEMPTON, A. W. (1951). The bearing capacity of clays. *Proc. Build. Research Congress.* London

— and MACDONALD, D. H. (1956). The allowable settlements of buildings. *Proc. Inst. Civ. Eng.*

— PECK, R. B. and MACDONALD, D. H. (1955). Settlement analysis of six structures in Chicago and London. *Proc. Inst. Civ. Eng.*, July

TERZAGHI, K. (1955). Evaluation of coefficients of subgrade reaction. *Géotechnique*, **5**, Dec.