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A new Soil Constant and its Applications

Une nouvelle constante du sol et son application

by M. P. P. dos Santos, Chief Engineer, Laboratorio de Ensaios de Materiais e Mecanica do Solo (C.P. 268), Lourenço Marques, Portuguese East Africa

Summary

The use of a new soil constant, related to the sand fraction of the grain-size distribution curve, has proved itself a valuable tool so far as some classical soil mechanics problems are concerned. This paper presents preliminary results related to the correlation between Atterberg limits and the grain-size distribution curve, the mixture of soils, the identification and classification of soils, and the specifications for granular stabilisation.

Sommaire

Dans cette communication sont présentés les premiers résultats de recherches portant sur l'application d'une nouvelle constante des sols, liée au diagramme granulométrique. Cette constante a permis le traitement de quelques problèmes classiques de la mécanique des sols, tels que la corrélation entre les limites d'Atterberg et la granulométrie, le mélange des sols, l'identification et la classification des différentes classes de sols et les spécifications concernant la stabilisation des sols.

Introduction

In connection with research in progress on some soil stabilisation problems, a new soil constant was introduced by the author (*dos Santos*, 1950, 1951). This constant is related to the grain-size distribution curve through the expression:

$$a = \frac{\Sigma y}{100\, n} \qquad (1)$$

where y = the ordinates (percentages passing) of the grain-size distribution curve, corresponding to No. 200, 100, 52, 25, 14 and 7 B.S. sieves (or the equivalent A.S.T.M. sieves);

n = the number of such ordinates (six).

It may be easily proved that the a-constant is proportional to the area limited by the grain-size distribution curve, the $y = 0$ axis and the No. 200 and 7 sieves ordinates (sand fraction of the soil). *Abrams* (1918), in the concept of fineness modulus for concrete agregates, and *Turnbull* (1948) used the area over the curve, with different boundaries. *Burmeister* (1938) used the area under the curve, also with different boundaries.

The quotient:

$$t = \frac{x}{a} \qquad (2)$$

of the percentage (divided by 100) passing No. 200 sieve, by the a-constant plays an important part in the following results.

Correlation between the Liquid Limit and the Plasticity Index, and the Grain-Size Distribution Curve

A number of attempts were made in the past in order to correlate the liquid limit and the plasticity index of soils with some characteristics of their particle-size distribution curve. The work of *Cooling* and *Skempton* (1946), *Clare* (1948) and *Skempton* (1948) pointed out the multiform type of correlation existing between the Atterberg limits and the clay fraction in soils. As a matter of fact, besides the size of the particles, there are other pertinent factors having a bearing on this problem; namely, the mineralogy of the clay fraction, the organic content, the shape of the grains, etc. The importance of research work covering these points does not need to be emphasised; *Terzaghi* and *Peck* (1948) state that "the investigation of statistical relations between the Atterberg limits and the other properties of cohesive soils constitutes one of the most promising fields for research in soil physics". At our laboratory, 353 samples of soils of different origin, location and type were examined. At this stage of investigation, no attempt was made to separate the influence of the factors listed above; nevertheless, the results obtained seem worthy of careful consideration. Figs. 1 and 2 show LL and PI ploted against $t = \frac{x}{a}$. Using the least square method, a curve of the form

$$LL \text{ or } PI = b_1 + b_2 t + b_3 t^2 \quad \ldots \qquad (3)$$

has been fitted to the experimental data. The determination of constants lead to the following equations:

$$LL = 0.067 + 0.303\, t + 0.309\, t^2 \quad \ldots \qquad (4)$$

$$PI = -0.026 + 0.097\,t + 0.240\,t^2 \qquad (5)$$

It follows that

$$PL = 0.093 + 0.206\,t + 0.069\,t^2 \quad \ldots \ldots \ldots \quad (6)$$

A statistical study of the frequency of discrepancies between theoretical and experimental values is summarized in Figs. 3 and 4, for *LL* and *PI*, respectively. It may be pointed out that 87.7% of the *LL* determinations show a difference <10 from the values given by (4); in the case of the *PI*, 84.1% show a difference <6 from the values given by (5).

The correlation factor of the curve fitting has been found to be $R = 0.988$ for the *LL*, and $R = 0.983$ for the *PI*. (The correlation factor has been taken as defined by $R^2 = 1 - \frac{U}{n\sigma^2}$,

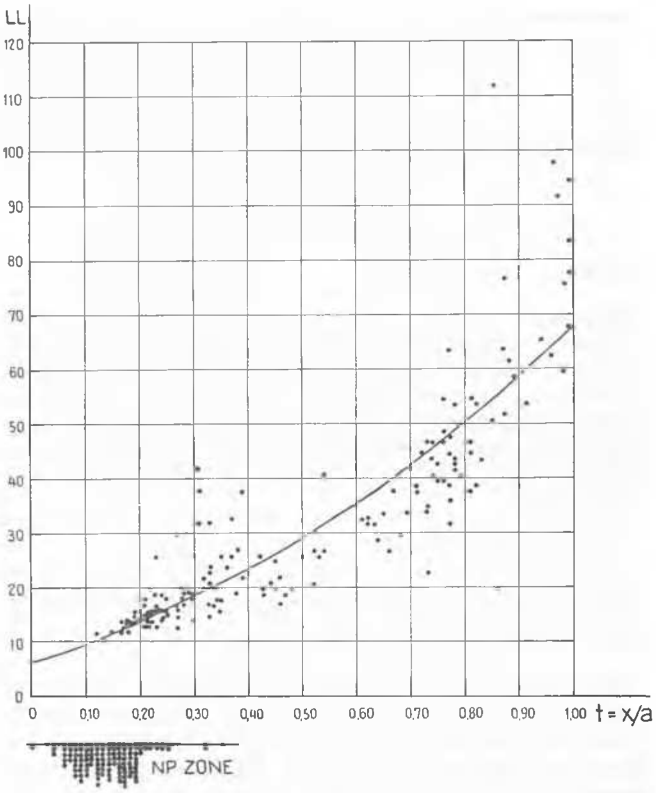

Fig. 1 Relation Between Liquid Limit and *t*
Relation entre la limite de liquidité et *t*

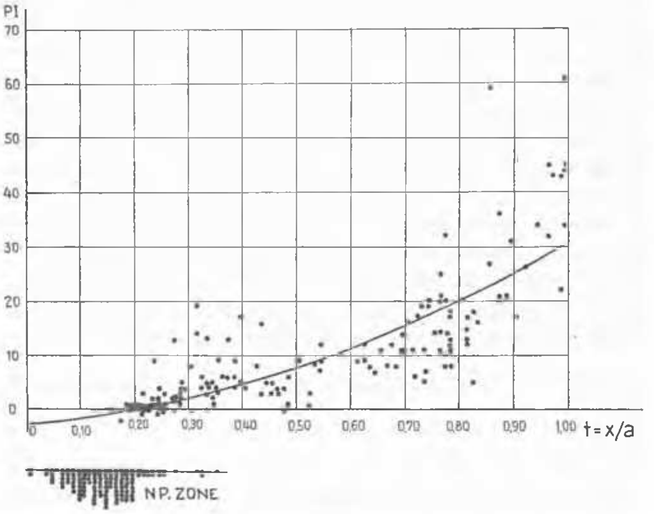

Fig. 2 Relation Between Plasticity Index and *t*
Relation entre l'indice de plasticité et *t*

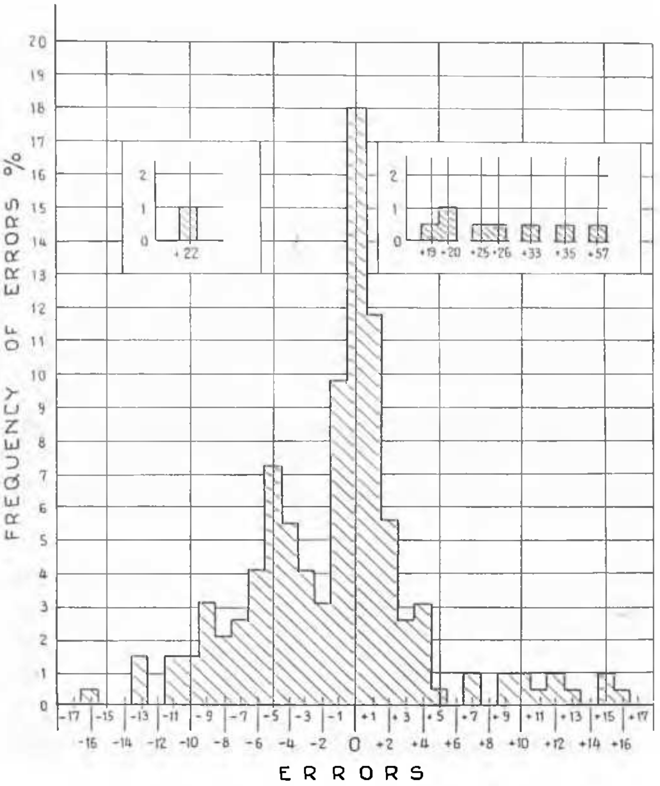

Fig. 3 Frequency of Discrepancies Between Theoretical and Experimental Values of the Liquid Limit
Fréquence des différences entre les valeurs théoriques et expérimentales de la limite de liquidité

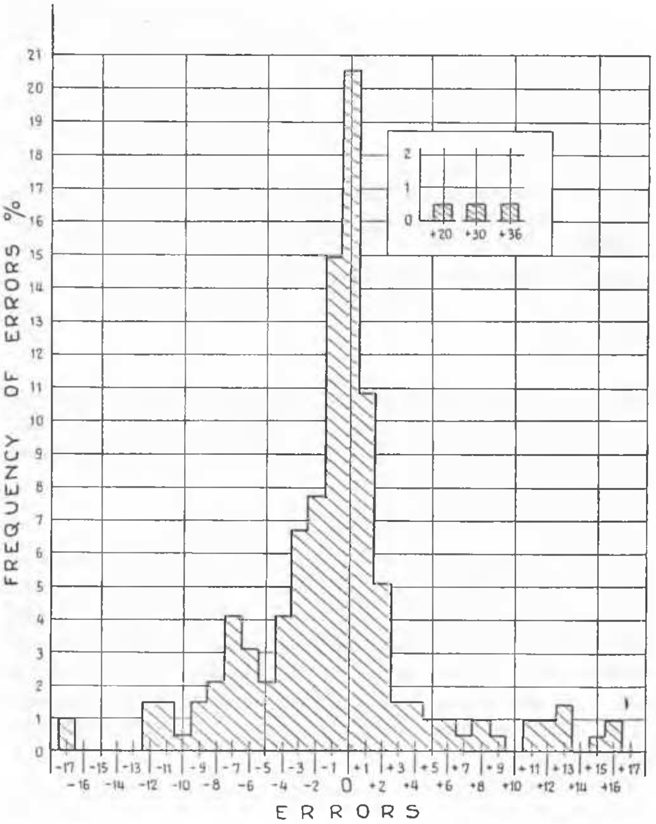

Fig. 4 Frequency of Discrepancies Between Theoretical and Experimental Values of the Plasticity Index
Fréquence des différences entre les valeurs théoriques et expérimentales de l'indice de plasticité

where U represents the sum of the squares of the residuals, n the number of observations and σ the standard deviation.) All the soils showing residuals larger than 10 for the *LL*, and than 6 for the *PI* were found to have a high organic content. Other pertinent factors have not so far been investigated, but the next step of the research programme will take them into account. It is not unlikely that the introduction of correction factors may account for the observed discrepancies.

Soil Mixtures

In the everyday work of a laboratory dealing with granular stabilisation of soils, the determination of *LL* and *PI* of mixtures of soils has a broad field of application.

The problem was solved in previous works by the author (*dos Santos*, 1950, 1951, in press), with an approximation believed to be sufficient for practical purposes. The *LL*, *PL*, *SL*, *PI* or *FME* of a mixture of two soils are given by

$$K = \frac{A_1 a_1 K_1 + A_2 a_2 K_2}{A_1 a_1 + A_2 a_2} \qquad (7)$$

where K = characteristic of the mixture;

K_1 and K_2 = corresponding characteristics of the soils to be mixed;

A_1 and A_2 = percentage by weight (divided by 100) of the soils in the mixture;

a_1 and a_2 = the a-constants, as given by equation (1).

The quadratic equation (7) is believed to represent a first approximation to the problem. As a matter of fact, it holds only if the correlations (3) assume a linear form. As this is not the case, we have

LL or *PI* =

$$= b_1 + b_2\frac{x}{a} + b_3\frac{x^2}{a^2} = b_1 + b_2\frac{A_1x_1 + A_2x_2}{A_1a_1 + A_2a_2} + b_3\frac{(A_1x_1 + A_2x_2)^2}{(A_1a_1 + A_2a_2)^2} =$$

$$= \frac{b_1(A_1a_1 + A_2a_2) + b_2(A_1x_1 + A_2x_2) + b_3(A_1x_1 + A_2x_2)\dfrac{A_1x_1 + A_2x_2}{A_1a_1 + A_2a_2}}{A_1a_1 + A_2a_2} =$$

$$= \frac{A_1a_1\left(b_1 + b_2\dfrac{x_1}{a_1} + b_3\dfrac{x_1x}{a_1a}\right) + A_2a_2\left(b_1 + b_2\dfrac{x_2}{a_2} + b_3\dfrac{x_2x}{a_2a}\right)}{A_1a_1 + A_2a_2}.$$

This would only be the same result as derived from (7) if we had $\frac{x_1^2}{a_1^2}$ and $\frac{x_2^2}{a_2^2}$ instead of $\frac{x_1x}{a_1a}$ and $\frac{x_2x}{a_2a}$. The difference:

$$\Delta = \frac{b_3}{a}\left[A_1x_1(t - t_1) + A_2x_2(t - t_2)\right] \;\;.. \qquad (8)$$

is generally negligible in the face of (7). For instance, with $a_1 = 0.57, x_1 = 0.12, t_1 = 0.21, a_2 = 0.73, x_2 = 0.94, t_2 = 0.78$, we obtain:

LL $\Delta = 0.03 = 3\%$

PI $\Delta = 0.02 = 2\%$

The expressions obtained allow the analytical construction of *Bonnenfant's* diagrams (1948), and provide an efficient tool for the study of sandy mixtures of soils and for the indirect determination of the Atterberg characteristics of non-plastic types of soils.

The Plasticity Chart

The classical work by *Casagrande* (1947) introduced the relation between *LL* and *PI* as an instument for the identification of cohesive soils. Fig. 5 presents the Plasticity Chart as used in *Casagrande*'s classification.

From equations (4) and (5), a quadratic relation of the form

$f(LL, PI) = 0$

may easily be derived.

If we plot this relationship in Fig. 5, we obtain a line that is but slightly different from line "*A*" of the Plasticity Chart.

In the research to be carried out, it will be examined whether the superposition of the effects of factors affecting *LL* and *PI* can account for the different types of soils listed in the chart.

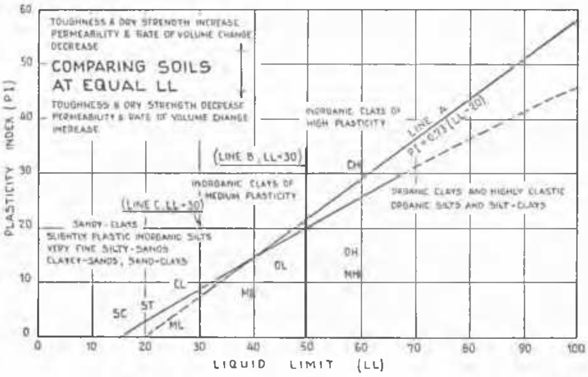

Fig. 5 Relation Between *LL* and *PI*, after *Casagrande* and as Deduced from Equations (4) and (5)
Relation entre *LL* et *IP* d'après *Casagrande* et déduite des équations (4) et (5)

The Classification of Soils

There are many systems of soil classification, none of them entirely sactisfactory. It was claimed by several investigators (see, for instance, *Turnbull*, 1948) that the grain-size distribution curve would provide a good basis for such a classification. A number of systems use together some grain-size distribution data and the values of *LL* and *PI*. Within this type falls the H.B.R. classification, one of the most widely used for road engineering purposes.

In order to illustrate the possibilities that arise from the cor-

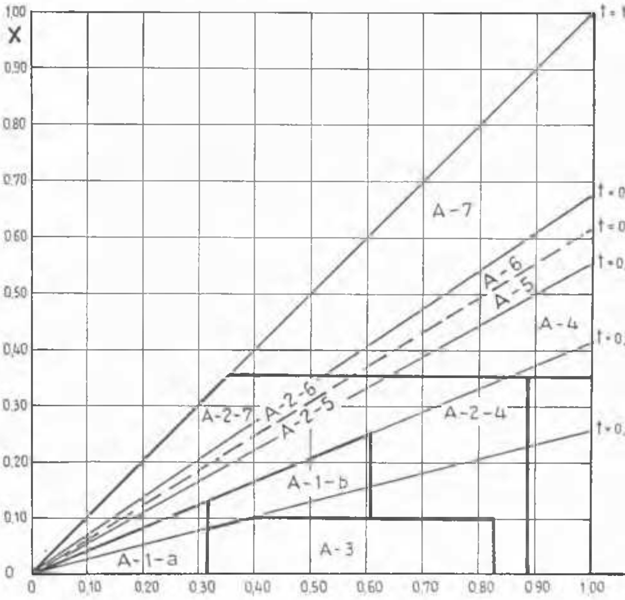

Fig. 6 Diagram for the Classification of Soils According the H.B.R. System
Diagramme pour la classification des sols selon la classification du Highway Research Board

relation between Atterberg characteristics and grain-size data the diagram shown in Fig. 6 has been drawn tentatively. Taking axes a and x and remembering that $t = x/a$, it is possible to place the boundaries of the various groups of soils. A more fluid condition takes place with the four groups A–2–5, A–2–6, A–5 and A–6, which show contradictory values for LL and PI as derived from (4) and (5): The line $t = 0.61$ is a purely arbitrary line drawn as a mean between $t = 0.55$ and $t = 0.67$.

In spite of the preliminary character of the correlations (4) and (5), it must be emphasised that the performance of this diagram for some hundreds of soils tested is stimulating.

The Stabilisation of Soils

It must be borne in mind that the main purpose of the programme of research that led to the above results is its possible application to soil stabilisation problems. In the section dealing with soil mixtures, some of there applications were outlined. Moreover, it is hoped that better understanding of empirical rules, like those used in American practice (namely, the A.S.T.M. specifications (A.S.T.M., 1949) (Wartime Road Problems)) may be achieved through the link formed between Atterberg limits and grain-size data. The condition of low organic content, generally imposed for granular stabilisation, simplifies the problem so far as this perturbing factor is concerned.

Figs. 7 and 8 show, in the diagram $x - a$, the boundaries of utilisation of soils according to the A.S.T.M. specifications for surface-courses (types A and B, quoted as SC–A and SC–B, respectively), and for base courses (types A and B, quoted as BC–A and BC–B, respectively).

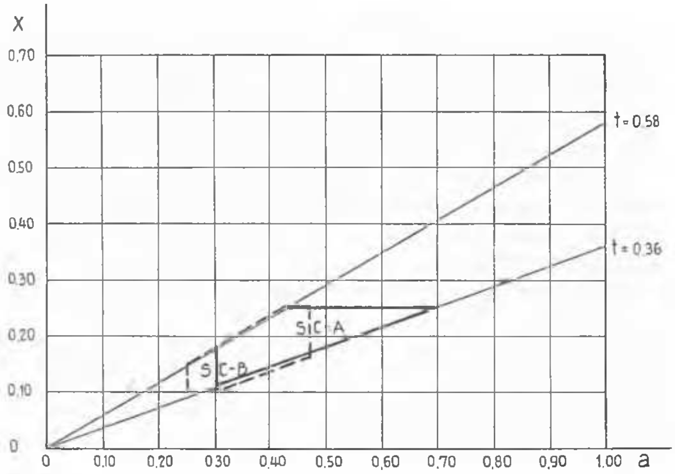

Fig. 7 Boundaries of Utilization of Soils for Stabilized Surface-Courses
Limites d'utilisation de sols pour couches d'usure stabilisées

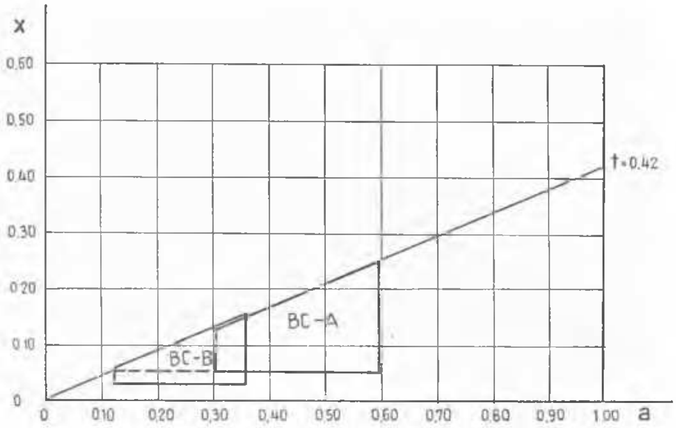

Fig. 8 Boundaries of Utilization of Soils for Stabilized Base-Courses
Limites d'utilisation de sols pour couches de fondation stabilisées

If we superimpose the diagrams on the diagram shown in Fig. 6, it will be seen that soils suitable for granular stabilisation fall within the limits of the A–1–a, A–1–b, A–3, A–2–4 and A–2–5 groups.

There is close agreement between the boundary a-values, as deduced both from Atterberg limits and grain-size distribution curves, in the BC–B type only. The agreement is good enough for the higher a-value in the SC–A and BC–A types, for which the lower a-value has been taken from the limiting grain-size distribution curves, in order to assure better compactibility. For the SC–B type, both a-limits must be deduced from the grain-size distribution curves. So far as the t-values deduced both from the LL and PI specifications are concerned, they are coincident for base-course values, and close enough for surface-course values. It appears, therefore, that is unnecessary to specify both of them.

The subject will be submitted to a deeper analysis in more advanced stages of research, for the above results seem worthy of careful consideration.

Acknowledgments

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