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Consolidation of a Cylindrical Clay Sample with External Radial Flow of Water

Consolidation d'un échantillon d'argile cylindrique à mouvement d'eau radial divergent

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Summary

The phenomenon of clay consolidation is a consequence of water flow through the soil pores by a pressure applied to the material, and it can be analysed as a function of two variables, volumetric change and time, either considered together or not.

While volume change is dependant on the compressibility of the specimen and the load applied, the rate of consolidation is also dependant on the drainage conditions of the sample.

This paper considers the process of consolidation of cylindrical samples with drainage made possible through the external cylindrical surface and its use as a soil test.

The consolidation of a compressed clay cylinder by drainage through the external cylindrical surface is somehow comparable with the process of drying a cylinder of infinite length in a confined atmosphere, where the loss of water decreases with the time elapsed. The soil volume decreases also at the same rate.

A test performed in such a manner is interesting because:—

- (1) The coefficient of permeability by radial flow, is obtained with no internal disturbance of the sample.
- (2) This process is quicker than any other for long sample using base drainage.
- (3) The sample may be of moderate diameter and the test performed by light loadings.
- (4) For long samples, the precision of measurement is increased to permit the use of low sensibility instruments.
- (5) The only practical detail to be observed when performing the test is to make the external drain between the sample and the consolidation ring, without fixing them by friction or causing disturbance in the sample. This difficulty has actually been overcome (Barros, J., 1951).

Sommaire

Le phénomène de la consolidation des argiles est une conséquence du mouvement de l'eau entre les interstices du sol sous l'effet d'une charge et il peut être analysé comme fonction de deux variables – variation volumétrique et temps, considérées séparément ou non.

Tandis que la variation de volume dépend de la compressibilité de l'échantillon et de la charge appliquée, la vitesse de consolidation dépend des conditions de drainage de l'échantillon.

Ce rapport décrit le mécanisme de la consolidation par drainage à travers la surface cylindrique externe de l'échantillon et son emploi pour les essais de sols.

Theoretical developments: The theory (Silveira, J., 1950) is based on the integration of the consolidation equation using cylindrical coordinates:

$$\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} = \frac{1}{C} \frac{\partial w}{\partial t} \quad (1)$$

where w neutral pressure (excess of hydrostatic pressure)

r Radial coordinate

a Diameter of cylinder

t Time

C Consolidation coefficient

This equation is solved by a function

$$w = RT$$

where $R = f(r)$ and $T = f(t)$ only

By substitution in (1)

$$C \left(\frac{R''}{R} + \frac{1}{r} \frac{R'}{R} \right) = \frac{T'}{T} = -p^2 \dots \quad (2)$$

The first is a *Bessel* equation, easily solved by operational calculus with *Laplace* transformation (Churchill, R., 1944).

$$L(rR'') = -s^2 y'(s) - 2s y(s) - R(0)$$

$$L(R') = s y(s) - R(0)$$

$$L\left(\frac{p^2 r R}{C}\right) = -\frac{p^2}{C} y'(s)$$

By substitution in (2)

$$\frac{y'(s)}{y(s)} = \frac{s^2}{s^2 + \frac{p^2}{C}} = \frac{s^2}{s^2 + c^2} \quad (3)$$

Standard transformations (Churchill, 1944) or the *Bromwich-Wagner's* inversion integral (McLachlan, 1942) give the solution:

$$R = A' J_0(cr) \quad (4)$$

The second is of immediate solution:

$$T = A'' \exp(-p^2 t) \quad (5)$$

This

$$w = A J_0(cr) \exp(-p^2 t) \quad (6)$$

constants (*p*) and (*A*) are determined by the boundary conditions of the problem, considering both bases of the cylinder to be impervious.

$$(a) \quad t = 0 \quad a \geq r \geq 0 \quad w = w_0$$

$$(b) \quad t \geq 0 \quad r = a \quad w = 0$$

$$(c) \quad t \geq 0 \quad r = 0 \quad \partial w / \partial r = 0$$

By the first — $w_0 = A J_0(cr)$;

By the second — $J_0(ca) = 0$ and (*ca*) must be any root (β_n) of this *Bessel* function, which number is infinite.

The determination of (*A*) is now made by *Fourier-Bessel serie's* (Byerly, B. E., 1893) development

$$An = \frac{2}{a^2 |J_1(\beta_n)|^2} \int_0^a r J_0\left(\frac{\beta_n r}{a}\right) \exp\left(-\frac{\beta_n^2 C t}{a^2}\right) w_0 dr$$

and, by substitution,

$$w = w_0 \sum_{n=1}^{\infty} 2J_0\left(\frac{\beta_n r}{a}\right) \exp\left(-\frac{\beta_n^2 C t}{a^2}\right) \frac{1}{nJ_1(\beta_n)} \quad (7)$$

The amount of consolidation is given by the integral (Terzaghi, 1943)

$$U_r = \frac{m_v \int_0^{2\pi a} \int_0^a (w_0 - w) r \cdot dr d\theta}{m_v w_0 \pi a^2} \quad (8)$$

¹⁾ $\exp(x) = e^x$

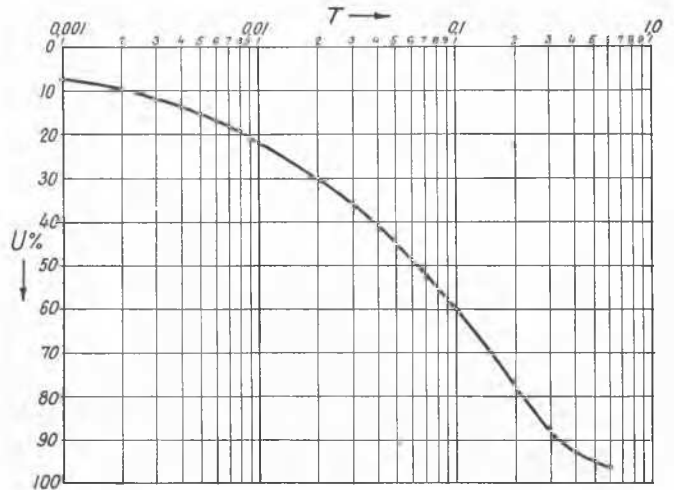


Fig. 1 Consolidation Chart for a Cylindrical Sample with External Radial Flow of Water
Diagramme de consolidation pour un échantillon d'argile avec drainage radial externe

the solution of which is

$$U_r = 1 - 4 \sum_{n=1}^{\infty} \exp\left(-\frac{\beta_n^2 C t}{a^2}\right) \frac{1}{\beta_n^2 n} \quad (9)$$

This expression was computed by *Barros* (1951) and a graph made for practical purposes was plotted (see Fig. 1) with time factor

$$T = Ct/a^2 \quad (10)$$

In conclusion, it is hoped that this work could be employed as a good soil test for horizontal permeability and to control and prepare consolidated samples for different purposes [in Soil Mechanics.

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