Correlation between Angle of Internal Friction and Angle of Shearing Resistance in Consolidated quick Triaxial Compression Tests on Residual Clays

Relation entre l'angle de frottement interne et l'angle de résistance au cisaillement dans les essais de compression triaxiaux rapides des argiles résiduelles préconsolidées

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Summary

Considering that residual clays are generally in the form of a skeleton of coarse grains, each in his clay envelope, which gives the soil a typical non-dilatant clay behaviour, the author generalised *Skempton*'s conclusions, in his study of the immediate triaxial compression tests, to interpret results of slow and consolidated quick tests on residual clays. A formula was deduced which correlates the angle of internal friction with the angle of shearing resistance in consolidated quick tests on such clays. The results of the tests confirm the calculated values.

Sommaire

Considérant que les argiles résiduelles sont souvent constituées par un squelette de gros grains enveloppés par de l'argile, ce qui donne au sol un comportement typique d'argile non-dilatante, l'auteur applique l'étude de *Skempton* sur les essais de compression triaxiaux rapides à l'interprétation des résultats des essais lents et rapides des argiles résiduelles préconsolidées. L'auteur propose une formule qui établit une relation entre l'angle de frottement interne et l'angle de résistance au cisaillement dans les essais rapides des argiles résiduelles préconsolidées. Les résultats des essais confirment les valeurs calculées.

Residual clays are often formed in such a way that their structure consists of a skeleton of the coarse grains which touch each other without being cemented together, enveloped in a clay matrix. The coarse-grain skeleton contributes a shearing resistance similar to that of a loose sand and the clay matrix gives the soil a non-dilatant clay behaviour during any shearing process.

When a load is applied to such a soil there is an immediate deformation of the coarse structure with a partial transference of load to the pore-water in the clay. Under such circumstances it seems possible for the theory developed by *Skempton*, to determine the value of the neutral pressures in clays during immediate triaxial tests, to be generalised and applied with success to this type of soil.

Skempton (1948) has shown that the volume change ΔV of a cylindrical specimen with an initial volume V_0 when subjected to a change in axial effective pressure Δp_1, and a lateral effective pressure Δp_3, is:

$$\frac{\Delta V}{V_0} = \frac{C_c}{3}\left[\Delta p_1 + 2\,\Delta p_3 \frac{C_s}{C_c}\right] \tag{1}$$

where C_c is the compressibility (volume decrease per unit, taking into account triaxial effective pressure increase) and C_s is the expansibility (volume increase per unit, taking into account triaxial effective pressure decrease) of the clay structure.

Experience has shown that the decrease of void-ratio during a triaxial compression is similar to that observed during a normal consolidation test. The compressibility and expansibility coefficients C_c and C_s may be determined by such a test, ε_0 being the void-ratio of the sample:

$$C_c = \frac{1}{V}\frac{dV}{dp} = \frac{1}{1+\varepsilon_0}\frac{d\varepsilon}{dp} \tag{2}$$

$$C_s = -\frac{1}{V}\frac{dV}{dp} = -\frac{1}{1+\varepsilon_0}\frac{d\varepsilon'}{dp} \tag{3}$$

Above a certain load, we have during consolidation:

$$\varepsilon = \varepsilon_0 - K_c \; lg \; p/p_0 \tag{4}$$

and during expansion:

$$\varepsilon' = \varepsilon_0 + K_s \; lg \; p/p_0 \; .. \tag{5}$$

From (4) and (5) we get:

$$\frac{d\varepsilon}{dp} = -\frac{K_c}{2.3p} \text{ and } \frac{d\varepsilon'}{dp} = \frac{K_s}{2.3p}$$

Whence:

$$-\frac{C_s}{C_c} = \frac{d\varepsilon'}{d\varepsilon} = -\frac{K_s}{K_c} = \lambda \tag{6}$$

Which constitutes an easy way to determine the relation between C_s and C_c by a simple normal consolidation test.

Now, suppose a cylindrical sample of a residual clay consolidates under a triaxial pressure σ_3, and then an axial load increase $\Delta\sigma_1 = \sigma_1 - \sigma_3$ is quickly applied. During the application of this axial load there is a certain deformation of the clay structure but, as there is no water escape from the pores of the clay matrix there is no change in volume. A neutral pressure occurs and the changes in effective pressures are:

In the axial direction:

$$\Delta p_1 = (\sigma_1 - u) - \sigma_3 = \Delta\sigma_1 - u$$

In the lateral direction:

$$\Delta p_3 = (\sigma_3 - u) - \sigma_3 = -u$$

As there is no volume change, we have:

$$\frac{dV}{dV_0} = 0 = \frac{C_c}{3}\left[(\Delta\sigma_1 - u) - 2\lambda u\right]$$

$$\Delta\sigma_1 - u = 2\lambda u$$

and

$$u = \frac{\Delta\sigma_1}{1 + 2\lambda} \tag{7}$$

If we call $\Delta\sigma_1 = R_a$, the compression strength at the moment of rupture the neutral pressure developed in the sample is:

$$u = \frac{R_a}{1 + 2\lambda} \tag{8}$$

Experience has shown (*Rutledge*, 1947) that the value of R_a is an exclusive function of ε_r, the void-ratio of the sample at the moment of rupture, whatever the type of test may be. As the void-ratio ε_r at the moment of rupture is plotted on a semi-logarithmic paper parallel to the consolidation curve (see Fig. 1 *a*), in the same manner R_a plotted against ε_r would be plotted parallel to the consolidation curve.

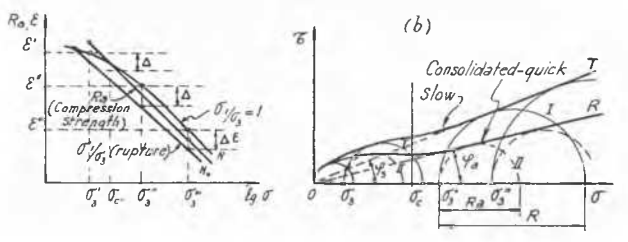

Fig. 1 (*a*) Pressure—Void-Ratio Curves for $\sigma_1/\sigma_3 = 1$, σ_1/σ_3 in Rupture and Compressive Strength—Void-Ratio Curve
Courbes pression – indice de vides, pour $\sigma_1/\sigma_3 = 1$, σ_1/σ_3 à la rupture, et résistance à la compression–indice de vides
(*b*) *Mohr*'s Envelopes for Slow and Consolidated-Quick Tests
Courbes enveloppes de *Mohr* pour les essais de compression lents et rapides, sur échantillons préconsolidés

The value of the effective pressures in the moment of rupture in a quick-consolidated test would be:

$$p_1 = \sigma_3 + R_a - u = \sigma_3 + R_a - \frac{R_a}{1 + 2\lambda}$$

$$p_3 = \sigma_3 - u = \sigma_3 - \frac{R_a}{1 + 2\lambda}$$

or:

$$\left.\begin{aligned} p_1 &= \sigma_3 + \frac{2\lambda R_a}{1 + 2\lambda} \\ p_3 &= \sigma_3 - \frac{R_a}{1 + 2\lambda} \end{aligned}\right| \tag{9}$$

We have also, above a certain value of p_3:

$$\frac{p_1}{p_3} = tg^2\left(45 + \frac{\varphi_s}{2}\right) = \frac{\sigma_3 + \dfrac{2\lambda R_a}{1 + 2\lambda}}{\sigma_3 - \dfrac{R_a}{1 + 2\lambda}} \tag{10}$$

where φ_s is the angle of internal friction as determined in a slow triaxial compression test (see Fig. 1 *b*). In addition, above a certain pressure, we have, in a consolidated-quick triaxial compression test:

$$\frac{\sigma_3 + R_a}{\sigma_3} = tg^2\left(45 + \frac{\varphi_a}{2}\right) \tag{11}$$

where φ_a is the angle of shearing resistance for consolidated-quick condition:

From (10) we can obtain:

$$R_a = \sigma_3 \frac{\left[tg^2\left(45 + \dfrac{\varphi_s}{2}\right) - 1\right](1 + 2\lambda)}{2\lambda + tg^2\left(45 + \dfrac{\varphi_s}{2}\right)} \tag{12}$$

and from (11)

$$R_a = \sigma_3\left[tg^2\left(45 + \frac{\varphi_a}{2}\right) - 1\right] \tag{13}$$

Comparing (12) and (13), where R_a is independent of the type of test on residual clay:

$$tg^2\left(45 + \frac{\varphi_a}{2}\right) = \frac{tg^2\left(45 - \dfrac{\varphi_s}{2}\right)(2 + 2\lambda) - 1}{2\lambda + tg^2\left(45 + \dfrac{\varphi_s}{2}\right)}$$

Experimental Results

Triaxial compression tests were extensively made on undisturbed samples of two residual clays. One originated from decomposition of gneiss and was a soft, greenish, saturated clay ($LL = 53$; $IP = 17$; moisture content: 39%, void-ratio: 1) the other clay originated from decomposition of granite and had experienced, after decomposition, an evolution of lateritic type. Its colour was brownish, consistence medium and not saturated ($LL = 68$; $IP = 26$; moisture content: 30%, void-ratio: 1).

Fig. 2 shows consolidation curves for both clays obtained in standard one-dimensional consolidation tests and triaxial compression. Fig. 3 and 4 show consolidation curves of triaxial compression, the relation between R_a, void-ratio and pressure-void-ratio during rupture for both clays. The residual clay from granite was not saturated in its natural state. There are two curves for the compression strength: one in the non-saturated natural state and the other for samples previously saturated in the laboratory. Fig. 5, 6 and 7 show *Mohr*'s envelopes for slow, consolidated-quick and quick tests on both clays. The first samples of granite clay tested had previously been saturated, and the later one had not been saturated.

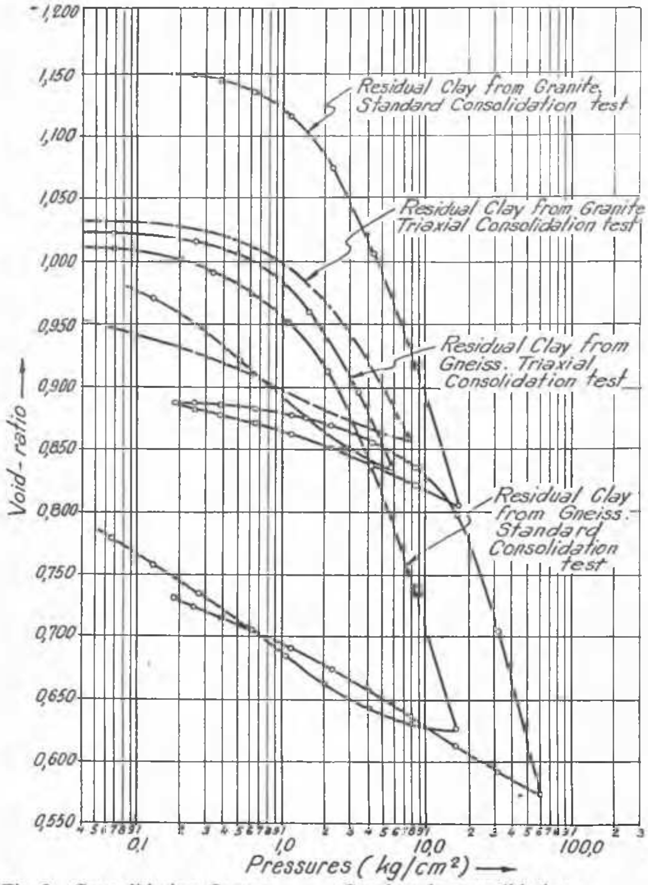

Fig. 2 Consolidation Curves Courbes de consolidation

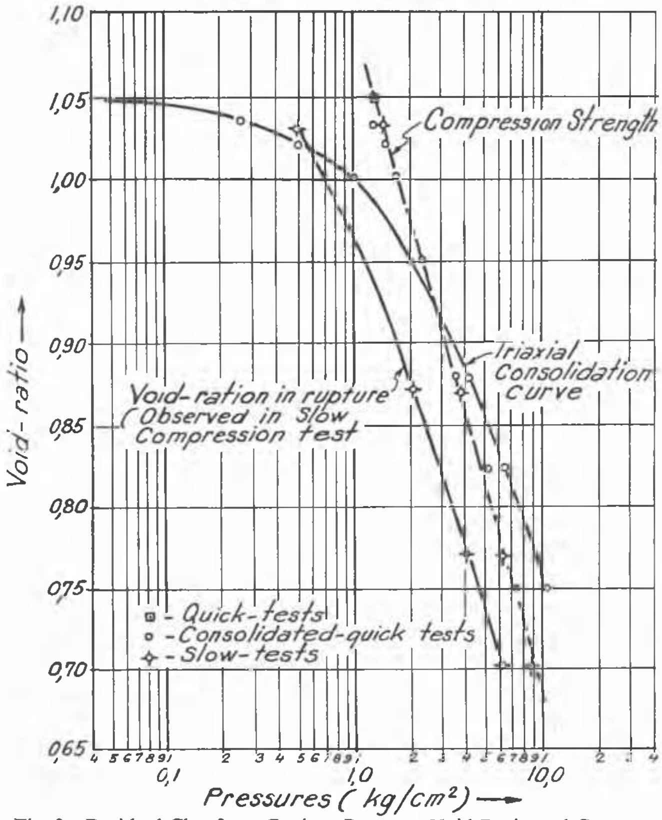

Fig. 3 Residual Clay from Gneiss—Pressure, Void-Ratio and Compressive Strength Curves
Argile résiduelle de gneiss – Courbes pression et résistance à la compression – Indice des vides

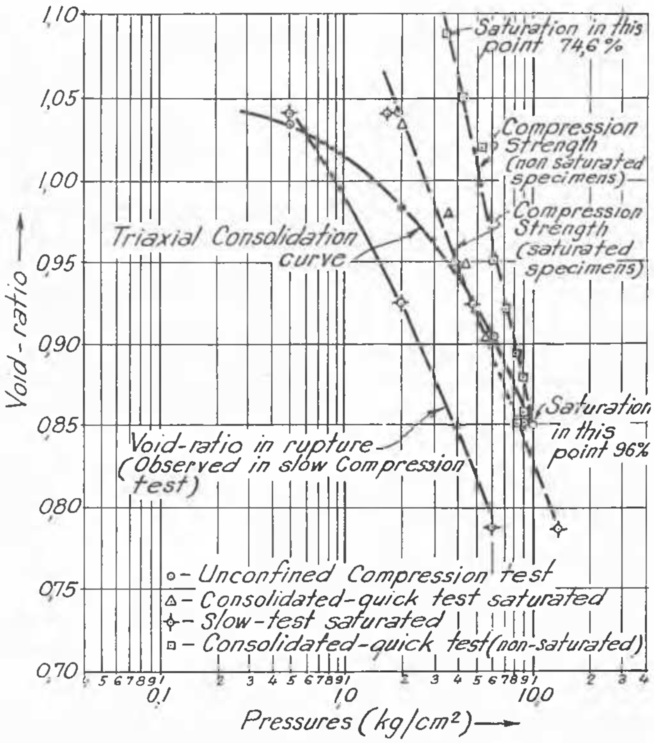

Fig. 4 Residual Clay from Granite—Pressure, Void-Ratio and Compressive Strength Curves
Argile résiduelle de granit – Courbes pression et résistance à la compression – Indice des vides

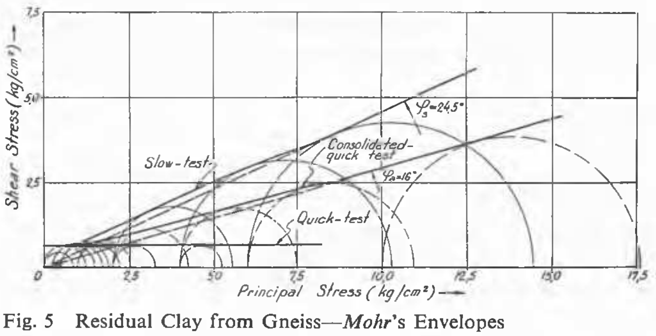

Fig. 5 Residual Clay from Gneiss—*Mohr*'s Envelopes
Argile résiduelle de gneiss – Courbes enveloppes de *Mohr*

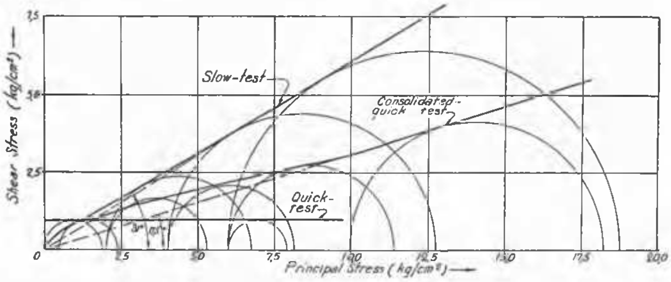

Fig. 6 Residual Clay from Granite (Previously Saturated Samples)—*Mohr*'s Envelopes
Argile résiduelle de granit (échantillons préalablement saturés) – Courbes enveloppes de *Mohr*

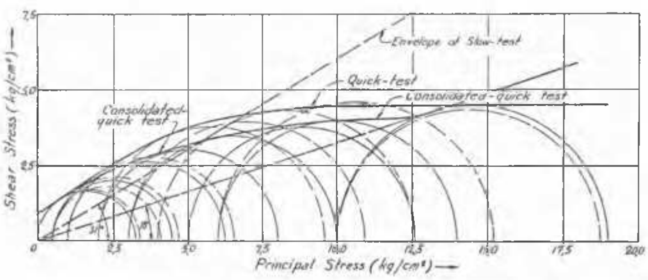

Fig. 7 Residual Clay from Granite (Non Saturated Samples)—*Mohr*'s Envelopes
Argile résiduelle de granit (échantillons non saturés) – Courbes enveloppes de *Mohr*

For the residual clay from gneiss we obtain from the consolidation curves: $K_s = 0.115$; $K_c = 0.39$; $\lambda = 0.295$ and from the slow triaxial test $\varphi_s = 24°30'$. With these values formula (14) gives $\varphi_a = 16°$, which was the value of φ_a obtained in the test (see Fig. 5).

For the residual clay from granite we obtain from consolidation tests $K_s = 0.025$; $K_c = 0.400$; $\lambda = 0.062$ and from triaxial tests with initial saturation $\varphi_s = 31°$. Applying formula (14) we can calculate $\varphi_a = 17°$; the consolidated-quick tests on previous saturated samples gives $\varphi_a = 17°30'$. Without initial saturation the consolidated-quick test gives $\varphi_a = 18°$.

References

Rutledge, P. C. (1947): Review of the Cooperative Triaxial Program of the War Department, Corps of Engineers. Waterways Exp. St., April.

Skempton, A. W. (1948): A Study of the Immediate Triaxial Test on Cohesive Soils. Proceedings 2nd Int. Conf. Soil Mec. and Found. Eng., Rotterdam.