INTERNATIONAL SOCIETY FOR SOIL MECHANICS AND GEOTECHNICAL ENGINEERING

Stresses in a Saturated Soil Mass During Electro-Osmosis

Contraintes exercées dans une masse de sol saturé au cours de l'électro-osmose

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Summary

Based on the theory of electro-osmosis and the principle of fluid flow, the forces created in a saturated soil mass by the combined hydraulic and electro-osmotic action are analyzed. The force which will affect the intergranular stresses is the resultant of the electric force and hydraulic drag upon the soil particles. Under steady state flow conditions, this force is shown to be equal to the seepage force which is determined only by the hydraulic gradient in the pore water. Equations are developed to correlate the pore water heads (hydraulic potentials) and the applied electric potentials. These equations were found to be in good agreement with experiment.

The nature of the mechanical effects of electro-osmosis in producing consolidation is explained by reference to the above relationships.

Sommaire

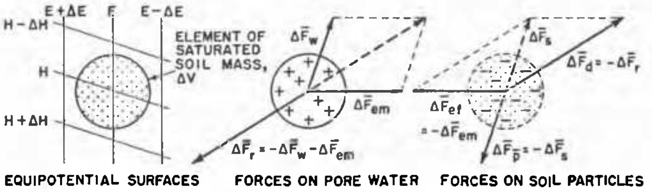
Se basant sur la théorie de l'électro-osmose et sur les principes de l'écoulement des fluides, les auteurs analysent les forces produites dans une masse de sol saturé par la combinaison des actions hydraulique et électro-osmotique. La force qui agit sur les contraintes intergranulaires est la résultante d'une force électrique et de la trainée hydraulique sur les particules du sol. En cas d'écoulement stationnaire, cette force se trouve être égale à la force de filtration qui est déterminée uniquement par le gradient de la pression de l'eau interstitielle, Les auteurs exposent des equations établissants la corrélation entre la pression de l'eau interstitielle (potentiel hydraulique) et le potentiel électrique appliqué et trouvent un bon accord avec l'expérience.

La nature des actions mécaniques de l'électro-osmose sur la consolidation est exposée sur la base des relations citées plus haut.

Forces Acting upon Soil Particles in Electro-Osmosis

When an external electric potential is applied to a saturated soil mass, which possesses a network of electrical double layers at the water-soil interface, two equal and opposite systems of electric forces are created simultaneously. One system of the electric forces acting upon the negatively charged adsorbed layer tends to move the soil particles toward the anode, and the other system acting upon the positively charged movable layer moves the pore water toward the cathode. For a closely packed soil mass, the motion of the soil particles is restrained, but the pore water is free to move. The flow of pore water thus produced is known as electro-osmosis. Due to the relative motion between the water and soil a pair of resistant forces are developed which are proportional to the velocity of flow. After the resistant forces grow large enough to counterbalance the electric forces, the pore water flow becomes steady and the resultant force upon the soil particles due to electro-osmotic action reduces to zero.

If the hydraulic potential or pore water head, H, is not a constant throughout the soil mass (i. e. grad $H \neq 0$), the seepage flow caused by the hydraulic gradient must be superposed on the electro-osmotic flow. In this general case, it is not difficult to see that under the steady state flow conditions the resultant force, $\Delta \bar{F}_s$, acting upon the soil particles by the combined hydraulic and electro-osmotic action is equal to the hydraulic seepage force, $\Delta \bar{F}_w$, alone. This is demonstrated in Fig. 1.



$\Delta\bar{F}_w$ = − w grad H · ΔV = HYDRAULIC SEEPAGE FORCE

$\Delta\bar{F}_{em}$ = − e grad E · ΔV = ELECTRIC FORCE ON MOVABLE LAYER IN PORE WATER

$\Delta\bar{F}_{ef}$ = e grad E · ΔV = ELECTRIC FORCE ON ADSORBED LAYER ON SOIL PARTICLES

$\Delta\bar{F}_r$ = RESISTANT FORCE AGAINST PORE WATER FLOW

$\Delta\bar{F}_d$ = DRAG FORCE UPON SOIL PARTICLES

$\Delta\bar{F}_s$ = RESULTANT FORCE ON SOIL PARTICLES DUE TO COMBINED HYDRAULIC AND ELECTRO-OSMOTIC ACTION = $\Delta\bar{F}_{ef} + \Delta\bar{F}_d = \Delta\bar{F}_w$

$\Delta\bar{F}_{\bar{p}}$ = RESULTANT OF INTERGRANULAR PRESSURES OVER BOUNDARY SURFACE OF THE ELEMENT WHICH IS REQUIRED TO BALANCE $\Delta\bar{F}_s$

Fig. 1 Forces Under Steady Flow Conditions
Forces dans le cas d'écoulement stationnaire

The unit weight of water, the electric charge of the adsorbed layer on the soil particles per unit volume of the soil mass, and the electric potential are denoted by w, $(-e)$, and E respectively. Then, upon the soil particles in the elementary volume, ΔV, the electric force is

$$\Delta \bar{F}_{ef} = e \operatorname{grad} E \cdot \Delta V \quad \ldots \tag{1}$$

and the resultant force is

$$\Delta \bar{F}_s = -w \operatorname{grad} H \cdot \Delta V \quad \ldots \quad \ldots \tag{2}$$

The force $\Delta \bar{F}_s$ will affect the intergranular pressures over the boundary of the element.

Equations of Combined Hydraulic and Electro-Osmotic Flow

Although it has already been pointed out that the force which affects the intergranular pressures between soil particles under the combined hydraulic and electro-osmotic action is the actual hydraulic seepage force alone, the distribution of pore water pressure that determines the magnitude and direction of the seepage force is nevertheless related to the applied electric potential. The relationship between the pore water head (hydraulic potential) and the electric potential can be shown by writing the equations of combined hydraulic and electro-osmotic flow. These equations are established on the following basic assumptions:

(1) The soil is perfectly saturated;

(2) The hydraulic flow obeys Darcy's law, i.e. a linear relationship exists between velocity and hydraulic gradient;

(3) The electro-osmotic flow obeys a similar law, i.e. a linear relationship exists between velocity and electric potential gradient;

(4) The electric current obeys Ohm's law, i.e. a linear relationship exists between current density and electric potential gradient;

(5) Both the pore water flow and the electric current obey the law of continuity.

Referred to Cartesian coordinates, the following notations are used:

x, y, z	coordinates of a position within the soil;
v_x, v_y, v_z	velocity (apparent) components of the combined flow at (x, y, z);
k_{hx}, k_{hy}, k_{hz}	hydraulic permeabilities at (x, y, z) in the directions of the three coordinate axes respectively;
k_{ex}, k_{ey}, k_{ez}	electro-osmotic permeabilites at (x, y, z) in the directions of the three coordinate axes respectively;
j_x, j_y, j_z	components of electric current density at (x, y, z);
$\varrho_x, \varrho_y, \varrho_z$	apparent electrical resistivities at (x, y, z) in the directions of the three coordinate axes respectively.

Within the flow system, at any position (x, y, z) the velocity of flow can be expressed by its components as

$$\left.\begin{aligned} v_x &= -k_{hx}\frac{\partial H}{\partial x} - k_{ex}\frac{\partial E}{\partial x} \\ v_y &= -k_{hy}\frac{\partial H}{\partial y} - k_{ey}\frac{\partial E}{\partial y} \\ v_z &= -k_{hz}\frac{\partial H}{\partial z} - k_{ez}\frac{\partial E}{\partial z} \end{aligned}\right\} \tag{3}$$

The law of continuity of flow gives

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0 \tag{4}$$

For electric current Ohm's law can be written as

$$\left.\begin{aligned} j_x &= -\frac{1}{\varrho_x}\frac{\partial E}{\partial x} \\ j_y &= -\frac{1}{\varrho_y}\frac{\partial E}{\partial y} \\ j_z &= -\frac{1}{\varrho_z}\frac{\partial E}{\partial z} \end{aligned}\right\} \tag{5}$$

And the continuity equation of electric current is

$$\frac{\partial j_x}{\partial x} + \frac{\partial j_y}{\partial y} + \frac{\partial j_z}{\partial z} = 0 \quad \ldots \tag{6}$$

The above eight equations of combined flow contain eight unknowns: three velocity components (v_x, v_y, v_z), three components of electric current density (j_x, j_y, j_z), one hydraulic potential or pore water head (H), and one electric potential (E). The other nine physical quantities, permeabilities and resistivities, must be determined experimentally.

One Dimensional Electro-Osmosis

Fig. 2 shows the general one dimensional case of combined hydraulic and electro-osmotic flow through a soil prism of cross sectional area A. At any section (x), the hydraulic permeability, the electro-osmotic permeability, and the electrical resistivity of the soil are denoted by k_h, k_e, and ϱ respectively.

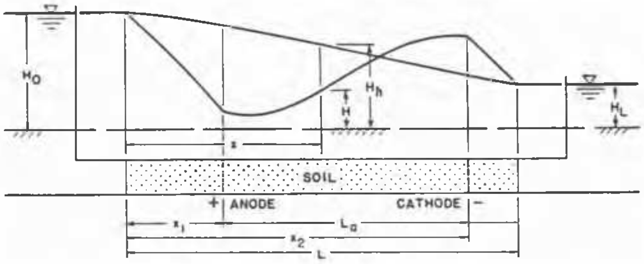


Fig. 2 General One Dimensional Combined Hydraulic and Electro-Osmotic Flow
Cas général d'écoulement à une dimension; écoulements hydraulique et électro-osmotique combinés

The pore water head, H, the velocity of flow, v, the electric potential, E, and the electric current, I, are then determined by equations (3) to (6) to be:

$$\left.\begin{aligned} H &= H_h + \frac{E_a}{\varrho_a L_a} B_a \left(-\frac{K_x}{K_L}\right) && (0 \leqslant x < x_1) \\ H &= H_h + \frac{E_a}{\varrho_a L_a} B_a \left(\frac{B_x}{B_a} - \frac{K_x}{K_L}\right) && (x_1 < x < x_2) \\ H &= H_h + \frac{E_a}{\varrho_a L_a} B_a \left(1 - \frac{K_x}{K_L}\right) && (x_2 < x \leqslant L) \end{aligned}\right\} \tag{7}$$

$$v = \frac{H_O - H_L}{K_L} + \frac{E_a}{\varrho_a L_a}\frac{B_a}{K_L} \tag{8}$$

$$E = E_a - \frac{E_a}{\varrho_a L_a}\varrho_x (x - x_1) \qquad (x_1 < x < x_2) \tag{9}$$

$$I = \frac{E_a}{\varrho_a L_a} A \tag{10}$$

wherein

$$H_h = H_O - \frac{K_x}{K_L}(H_O - H_L),$$

$$K_x = \int_0^x \frac{dx}{k_h},\ K_L = \int_0^L \frac{dx}{k_h},$$

$$B_x = \int_{x_1}^{x} \frac{k_e}{k_h}\varrho\, dx,\ B_a = \int_{x_1}^{x_2} \frac{k_e}{k_h}\varrho\, dx,$$

$$\varrho_x = \frac{1}{x - x_1}\int_{x_1}^{x} \varrho\, dx,\ \varrho_a = \frac{1}{L_a}\int_{x_1}^{x_2} \varrho\, dx.$$

In equation (7) it is indicated that the pore water head H in the combined hydraulic and electro-osmotic flow is generally different from the head H_h which would be the pore water head at the same section, if the external electric potential were not applied (i.e., $E_a = 0$). Since E_a, ϱ_a, L_a, B_a, B_x, K_L, and K_x are all positive quantities, and $B_x < B_a$ and $K_x < K_L$, the pore water head upstream from the anode is always reduced and that downstream from the cathode is always increased by the electro-osmotic effect; there may, however, be both an increase and a decrease between the electrodes.

The constant B_a, which involves the ratio of k_e to k_h and appears in both equations (7) and (8), is the predominating factor influencing the amount of change in pore water head and velocity of flow for a given applied electric potential E_a. It is clear therefore that the mechanical effectiveness of electro-osmosis is primarily dependent upon the relative magnitude of the hydraulic and electro-osmotic permeabilities.

Experimental Results

The preliminary results of a quick critical hydraulic gradient test on a soil sample of sandy loam with and without electro-osmotic influence, given in Fig. 3, indicates that the critical hydraulic gradient of the soil is solely dependent on the actual apparent hydraulic gradient and is independent of the electro-osmotic action. This confirms the theory stated previously.

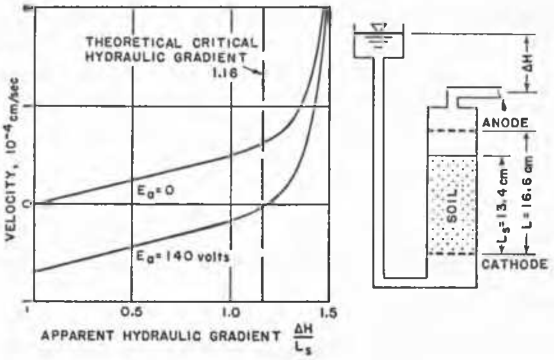


Fig. 3 Results of a Quick Critical Hydraulic Gradient Test
Résultats d'un essai rapide du gradient hydraulique critique

However, the above test alone is not sufficient to demonstrate whether or not there would be any change in the pore water pressures within the soil prism. Therefore, direct measurements of pore water heads and electric potentials along soil prisms of silty loam during prolonged periods of electro-osmosis were made. The remolded soil was compacted into a cylindrical transparent lucite testing chamber with an inside diameter of 7.6 centimeters, and was arranged between two perforated electrodes, steel anode and copper cathode, about 50 centimeters apart. The water heads at both ends of the soil prism were kept on the same level. The pore water heads were measured by means of hypodermic needles, which were connected to a pressure balancing device for instantaneous pressure measurements. The results of a complete test are given in Fig. 4.

It was discovered that the electrical resistivity along the soil prism changed continuously with time and was far from uniform after prolonged electro-osmotic treatment. The pore water heads were also observed to vary correspondingly. These observations suggested that a relationship existed between the electrical and mechanical properties of the soil, and this led to the development of the general equations of combined hydraulic and electro-osmotic flow. The close agreement between the measured pore water heads and the values computed from equation (7) based on the independently determined physical quantities, k_h, k_e, and ϱ, of a soil prism after a 3-day electro-osmotic treatment is shown in Fig. 5.

Besides the relationship between the electrical resistivity and pore water pressures, two other phenomena were noted: (1) the development of fissures and cracks, and (2) the accumulation of gas in the soil. They are shown in Fig. 6. The fissures and cracks, when not closed up by compaction, increased the hydraulic permeability of the soil as indicated in the figure. The effect of gas accumulation in the soil upon both the pore pressure and the electrical resistance, became noticeable in the later stage of electro-osmosis (after the third day, in Fig. 4). This, of course, would alter the original theory that was derived for the saturated soil only. This discrepancy between the theoretical and experimental results appearing in Fig. 5 is likely due to this effect.

Soil Consolidation under Electro-Osmosis

Three typical types of soil consolidation under electro-osmosis are discussed in the following paragraphs.

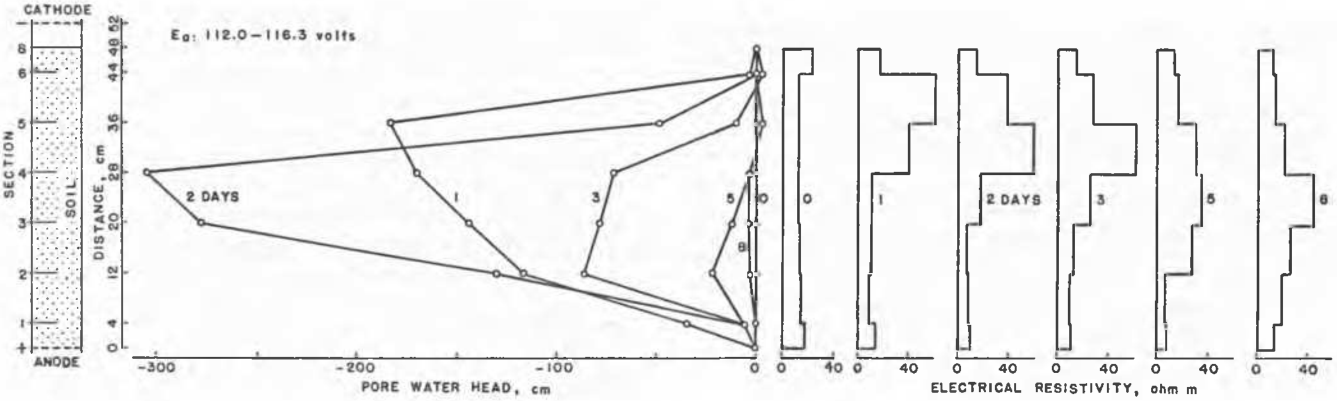
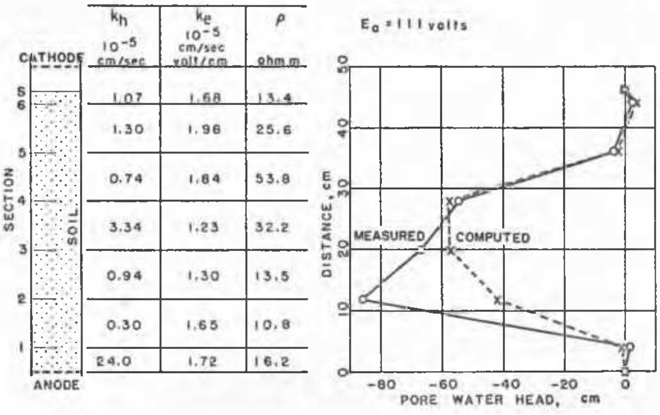
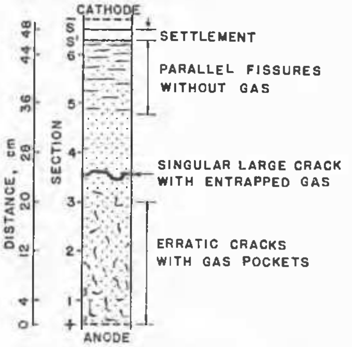


Fig. 4 Pore Water Heads and Electrical Resistivities During Electro-Osmosis
Pression de l'eau interstitielle et résistivité électrique durant l'électro-osmose



k_h 10^{-5} cm/sec	k_e 10^{-5} cm/sec volt/cm	ρ ohm m
1.07	1.68	13.4
1.30	1.96	25.6
0.74	1.84	53.8
3.34	1.23	32.2
0.94	1.30	13.5
0.30	1.65	10.8
24.0	1.72	16.2

Fig. 5 Comparison of Measured and Computed Pore Water Heads
Comparaison des valeurs mesurées et calculées de la pression de l'eau interstitielle



SECTION	k_h, 10^{-5} cm/sec	
	BEFORE	AFTER
6–S(S')	0.0996	48.4
5–6	0.373	1.98
4–5	2.09	1.80
3–4	0.406	5.29
2–3	0.248	9.20
1–2	0.172	6.35
+–1	0.0790	2.50
APPARENT	0.214	3.12

Fig. 6 Soil After Electro-Osmotic Treatment
Sol après traitement électro-osmotique

(1) *Consolidation due to electro-osmotic rise:* Fig. 7 (*a*) shows a prism of a homogeneous soil of cross sectional area A and length L between two electrodes, with an applied electric potential E_a. If the water level at the cathode is raised relative to the water level at the anode to such a height P, so that the forward electro-osmotic flow is just counter-balanced by the backward hydraulic flow, the apparent (but not actual) velocity of flow will be zero. The electro-osmotic rise P can be computed from equation (8)

$$P = \frac{k_e}{k_h} E_a \tag{11}$$

From equation (7)

$$H = H_0 + \frac{x}{L} P \tag{12}$$

And the force acting upon the soil particles in any elementary volume $A dx$, by equation (2), is

$$\Delta \bar{F}_s = -w \frac{\partial H}{\partial x} A\, dx = -w \frac{P}{L} A\, dx \tag{13}$$

$\Delta \bar{F}_s$ must be balanced by the difference of intergranular pressures at two ends of the element, that is:

$$\left.\begin{aligned} &\frac{\partial \Delta \bar{p}}{\partial x} A\, dx = \Delta \bar{F}_s \\ &\text{or} \\ &\frac{\partial \Delta \bar{p}}{\partial x} = -w \frac{P}{L} \end{aligned}\right\} \tag{14}$$

from which the increment of intergranular pressure $\Delta \bar{p}$ created by electro-osmotic rise is found to be

$$\Delta \bar{p} = wP\left(1 - \frac{x}{L}\right) = w \frac{k_e}{k_h} E_a \left(1 - \frac{x}{L}\right) \tag{15}$$

This increase of intergranular pressure will cause consolidation of the soil prism as under a body force of $w\frac{P}{L}$ per unit volume of the soil mass.

If instead of the water level at the cathode being raised, the anode is made of a solid impervious plate so that no water can flow into the soil through the anode, the pore water head in the soil immediately downstream from the anode will decrease to an amount equal to (H_L–P). The pore water head and the induced intergranular pressure along the soil prism are shown in Fig. 7 (*b*). This gives the same consolidation effect upon the soil as that given by the electro-osmotic rise.

(2) *Consolidation due to pore water tension induced by the effect of non-uniform distribution of electrical resistivity in soil:* During the early stages of electro-osmotic treatment, pore water tensions (decrease of pore water head) develop within the soil prism. These tensions are related to the change of electrical resistivity of the soil (Fig. 4), and the forces created by the inward hydraulic gradients from both ends of the soil prism are sufficient to bring about a consolidation of the soil prism.

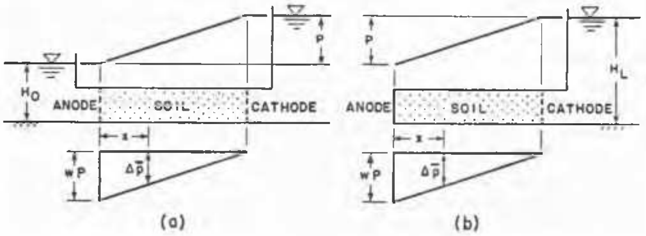


Fig. 7 Effect of Electro-Osmotic Rise
Effet de l'ascension électro-osmotique

(3) *Sudden settlement of silt due to electric shock:* It has already been pointed out that the two equal and opposite systems of electric forces produced upon the pore water and soil particles by an externally applied electric potential will balance each other because of the resistance induced between them, after the pore water flow becomes steady. However, before this steady condition is reached, those two forces systems are not in static equilibrium. The unbalanced electric force on the moveable layer accelerates the pore water flow, and that acting on the soil particles may cause consolidation of the soil. However, for the compacted soil or cohesive soil with low hydraulic permeability this initial consolidation is very small. But for a loose silt the initial unbalanced electric force may be sufficient to produce a rapid settlement.

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