The Effect of Stress History on the Relation between φ and Porosity in Sand

L'influence de l'historique des contraintes sur la relation entre φ et la porosité dans les sols sableux

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Summary

A series of triaxial tests has been carried out on sand including undrained and drained tests, both isotropically and anisotropically consolidated, together with a limited number of axial extension tests.

The results indicate that, within the range of stress used, the angle of internal friction measured in the compression tests is independent of the stress history and of the use of anisotropic consolidation. The results of the extension tests are less conclusive, and further data in this field is required.

The marked influence of anisotropic consolidation on the relation between undrained strength and consolidation pressure is noted. This appears to be an important factor in explaining the large discrepancy between the measured strength of normally consolidated natural strata and the results of consolidated undrained tests, in the case of soils of low plasticity index.

Sommaire

Une série d'essais triaxiaux ont été entrepris sur du sable avec des échantillons drainés et non drainés, consolidés d'une manière isotrope et anisotrope en même temps qu'un nombre limité d'essais axiaux d'extension.

Les résultats montrent que, dans la gamme des efforts employés, l'angle de frottement interne mesuré dans les essais de compression, est indépendant de l'historique des contraintes sollicitant l'échantillon et du mode de consolidation anisotrope. Les résultats obtenus par les essais d'extension sont moins concluants, et des données supplémentaires sur ce sujet sont nécessaires.

L'influence prononcée de la consolidation anisotrope sur le rapport entre la résistance non drainée et la pression de la consolidation est mise en évidence. Cela paraît être un facteur important pour expliquer la différence considérable entre les mesures de résistance des terrains naturels et les essais consolidés non drainés, pour des sols normalement consolidés et dont l'indice de plasticité est faible.

Introduction

The test results presented in this paper were obtained in the course of an investigation into the factors controlling the shear strength of sand under various conditions of loading and drainage. The test results relating to the conditions under which φ_u, the angle of undrained shearing resistance, is equal to zero in sands have been described in Géotechnique (*Bishop* and *Eldin*, 1950). Fuller details of the test results, including a discussion of the correlation between the pore pressure changes in undrained tests and the volume changes in drained tests are given by *Eldin* (1951).

The present paper is concerned mainly with the effect of stress history and mode of failure on the angle of internal friction, and with the effect of anisotropic consolidation on the strength in consolidated undrained tests.

Test Procedure

All tests were carried out on cylindrical samples $1\frac{1}{2}$ inches in diameter and $3\frac{1}{2}$ inches in length, enclosed in a rubber membrane approximately 0.01 inch in thickness. With the exception of one series of tests in which dry samples were used, all samples were formed by placing the sand under de-aired water, the sand itself having previously been boiled in water to remove all traces of air adhering to the particles. The lower range of porosities were obtained by vibration. This method has proved very successful in giving fully saturated samples, as is shown by the fact that a change in cell pressure is accurately reflected by an equal change in pore water pressure if no drainage is permitted (*Bishop* and *Eldin*, 1950). It also makes it possible to obtain normally consolidated samples of sand in the low pressure range without the danger of overconsolidation inherent in methods of saturation by drawing water through a dry sample under a vacuum. Further details of the apparatus and testing procedure will be found in the references given above.

The types of test performed were as follows:—

(*a*) Undrained compression tests with isotropic consolidation:—

This is the standard type of consolidated undrained test with pore pressure measurement. The samples were normally consolidated, and it was found important to allow adequate time for complete consolidation to occur, as a relaxation of effective stress otherwise takes place when the sample is sealed off before commencing the undrained stage of the test.

In strongly dilatant soils shear is accompanied by a drop in pore pressure, and hence, in order to avoid large negative pore pressures and the danger of cavitation, the cell pressure was raised after sealing off the sample. Under the conditions of full saturation for which $\varphi_u = 0$, this has no influence on the effective stresses in the sample.

(*b*) Undrained compression tests with anisotropic consolidation:—

The condition of isotropic consolidation used in the laboratory does not in general represent field conditions, under which normally consolidated strata are laid down under the condition of no lateral yield and, in consequence, are subjected to an anisotropic stress system. A simple procedure has been developed for simulating this condition in the triaxial apparatus (*Bishop*, 1950), and is illustrated in Fig. 1. If a small vertical compression dh is applied to a sample of initial cross-sectional area A_0, the volume of water which will be expelled if consolidation takes place with zero lateral strain is equal to $A_0 \cdot dh$; and the change in level in the burette will correspond to this value. The test procedure consists of applying the vertical

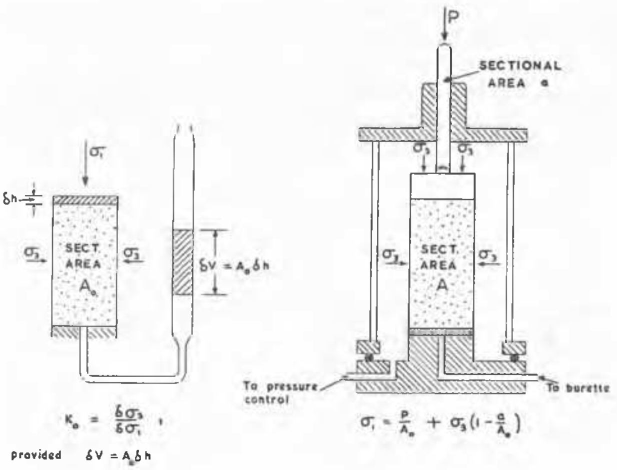

Fig. 1 Triaxial Consolidation with Zero Horizontal Strain
Consolidation triaxiale avec une déformation horizontale de zéro

deformation via the proving ring of the testing machine at a slow rate, and continuously adjusting the cell pressure by means of a screw-controlled piston so that the volume change indicated by the burette corresponds to the vertical deformation of the sample, indicated by a micrometer dial gauge.

The ratio of the increase in the horizontal stress to that in the vertical stress gives the value of the coefficient of earth pressure at rest. The undrained test with pore pressure measurement is then carried out as before.

(*c*) Drained compression tests with isotropic consolidation:—

This is the standard drained test, the samples being normally consolidated and then sheared by increasing the axial stress, the cell pressure remaining constant. Volume changes were recorded throughout.

(*d*) Drained compression tests with anisotropic consolidation:—

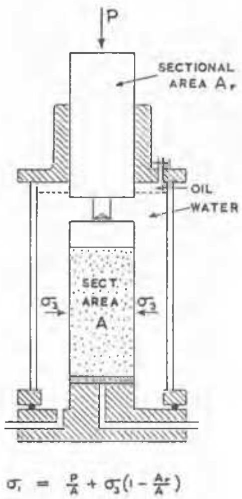

Fig. 2 Layout for Application of Constant Axial Stress
Arrangement pour l'application d'un effort axial constant

The samples were placed and consolidated as in (*b*). The drained test was then carried out under increasing axial stress and constant cell pressure.

(*e*) Drained compression tests with overconsolidation:—

The isotropic consolidation pressure was increased to 101.3 lbs. per sq. inch, and then reduced to 5.35 lbs. per sq. inch before testing under drained conditions. This gave an overconsolidation ratio of $\frac{101.3 - 5.35}{5.35} = 18$, which was the practical upper limit for the apparatus used.

(*f*) Drained compression tests, with failure caused by decreasing the minor principal stress:—

In these tests a constant axial load was maintained by the use of a dead load acting on $1\frac{1}{2}$ inch diameter ram in place of the usual ram, $\frac{1}{2}$ inch in diameter (Fig. 2). The sample was normally consolidated under an all round pressure of 101.3 lbs. per sq. inch. The ram was loaded to an equal stress and brought in contact with the sample, which was then caused to shear by reducing the cell pressure.

The major (axial) principal stress thus remains constant (except for a correction due to the change in cross sectional area of the sample), while the minor and intermediate principal stresses (equal to cell pressure) are decreased.

(*g*) Drained compression tests on dry sand:—

These are a repetition of the low pressure range series of tests, using sand which had been oven-dried and cooled in a desiccator. The method used for measuring changes in volume during consolidation and shear has been described in Géotechnique (*Skempton* and *Bishop*, 1950).

(*h*) Drained axial extension tests, with failure caused by increasing the major and intermediate principal stresses:—

These tests form part of a series of tests on saturated sand to investigate the influence of the intermediate principal stress, which in compression tests is equal to the minor principal stress and in extension tests is equal to the major principal stress. The minor (axial) principal stress was maintained at an approximately constant value by a dead load on the $1\frac{1}{2}$ inch diameter ram, which was connected to the sample cap by a slotted plate and key, engaged by a quarter turn of the ram. The cell pressure, which was equal to the major and intermediate principal stresses, was raised by a screw controlled piston.

(*i*) Drained axial extension tests, with failure caused by decreasing the minor principal stress:—

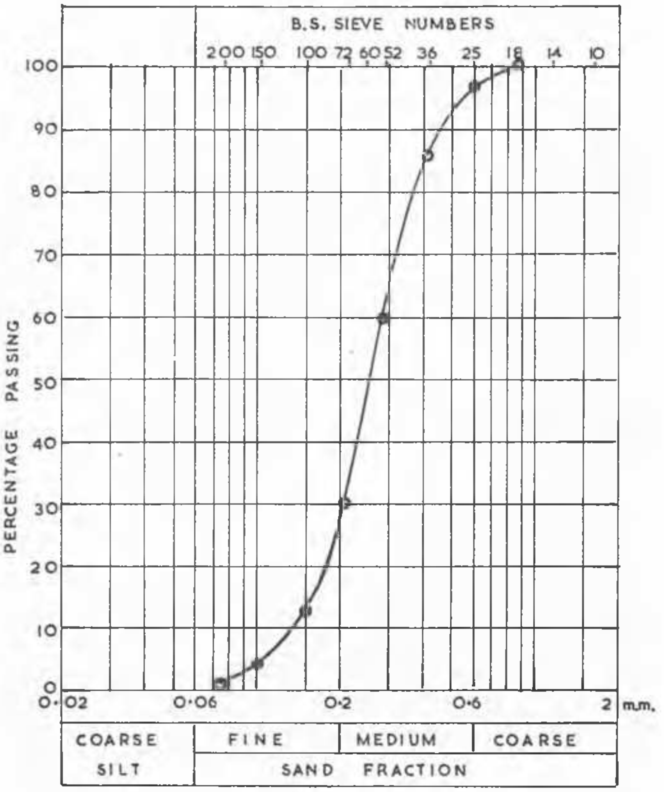

Fig. 3 Particle Size Distribution
Courbe granulométrique

In this series the cell pressure at the end of consolidation was maintained at a constant value. The axial load on the $1\frac{1}{2}$ inch diameter ram was reduced, and the sample was thus caused to shear under decreasing minor (axial) principal stress.

(*j*) Undrained axial extension tests, with isotropic consolidation:—

A few preliminary tests in this series were carried out, using the same assembly as in series (*i*), but measuring the pore pressure changes during undrained shear.

Corrections

Allowance was made for the change in cross sectional area of the sample, the deviator stress being calculated on the basis of a right cylinder with the same length and volume as the actual sample at all stages of the test. The validity of this method has been checked by detailed measurements with a travelling microscope of the changes in shape during typical tests. The errors are found to be very small, reaching 3 per cent at 20 per cent axial strain in compression and 2 per cent at 10 per cent axial strain in extension; though in the latter tests a modified correction is required when necking commences.

A correction is also made for the influence of the rubber membrane based on its measured strength, but is only important at low stresses (see *Eldin*, 1951, and *Henkel* and *Gilbert*, 1952, for a discussion of methods).

Description of Sand

The sand used in the tests was the medium-to-fine fraction from a well graded sand of the Folkestone Beds, and was obtained from a deposit being worked adjacent to the River Darent near Brasted, in Kent. The particle size distribution is shown in Fig. 3; the limiting porosities (dry) were found to be 46.2 per cent and 33.2 per cent (after *Kolbuszewski*, 1948).

Angle of Internal Friction

The magnitude of the angle of internal friction is dependent on the strain at which it is calculated, and also, in drained tests, on the rate of volume change at failure. In order to compare the angles of internal friction measured in the various types of tests, the following definitions are used.

(*a*) In undrained tests the maximum value of the principal stress ratio generally occurs at a small strain (of about 5 per cent), while the maximum value of the deviator stress occurs at a larger strain (15 to 30 per cent) and corresponds to a lower value of the principal stress ratio. The corresponding values of the angle of internal friction are referred to as φ_{rm} and φ_{rf} respectively, where

$$\sin \varphi = \frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'} \tag{1}$$

σ_1' denoting the major effective principal stress and σ_3' denoting the minor effective principal stress.

(*b*) In drained tests, in which one principal stress is kept constant, the maximum value of the principal stress ratio occurs when the deviator stress has its maximum value. The corresponding value of the angle of internal friction is denoted by φ_d. In dilating samples, which increase rapidly in volume at failure, an appreciable proportion of the apparent frictional strength is due to the work required to expand the sample against the confining pressure.

In a triaxial compression or extension test the stress system can be resolved into a ambient stress σ_a, and a deviator stress σ_d (Fig. 4). If Δ_v is the change in volume per unit volume corresponding to an axial strain of Δ_ε, and σ_d' is that part of

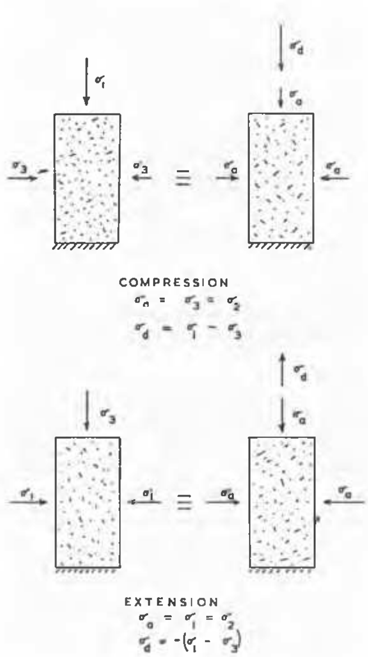

Fig. 4 Stress Systems in Compression and Extension
Arrangements des efforts pour compression et en extension

the deviator stress required to cause the sample to dilate, then it follows at once that

$$\sigma_d' = \frac{d\,\Delta_v}{d\,\Delta_e} \cdot \sigma_a \,.. \qquad (2)$$

The remaining part of the deviator stress $\sigma_d - \sigma_d'$ represents the frictional strength of the sample, and the corresponding angle of internal friction φ_{d_r} may be compared[1]) with φ_{r_m}.

The test results for all *compression tests on saturated samples* are presented in Fig. 5. The results[2]) indicate that, within practical limits of accuracy, and for the various stress ranges quoted, the following conclusions may be drawn for this sand:—

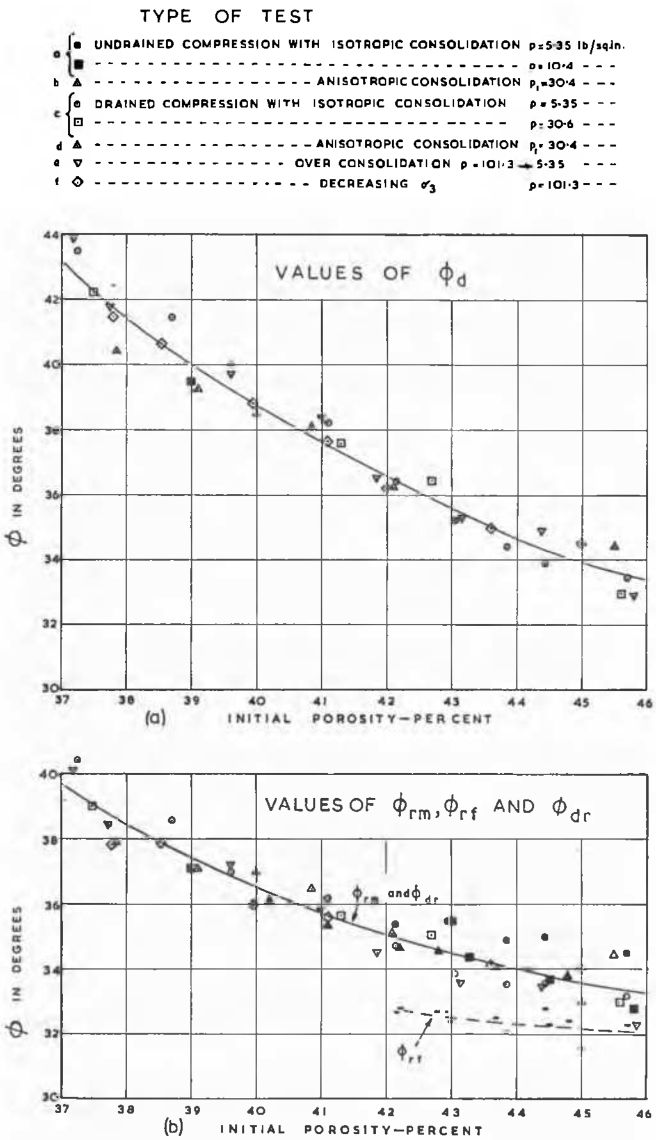

Fig. 5 Results of Compression Tests on Saturated Samples
Résultats des essais de compression sur des échantillons saturés

[1]) The relation of shear strain to axial strain differs in the two tests owing to dilatancy, and exact agreement cannot be expected. This principle is discussed in relation to shear box tests by *Taylor* (1948) and *Bishop* (1950).

[2]) The results are compared on the basis of the initial porosity of the samples, measured at the end of preparation under a suction pressure of about 1 lb. per sq. inch.

(*a*) The various values of the angles of internal friction are independent of the magnitude of the stresses.

(*b*) The values of φ_d and φ_{r_m} are independent of stress history, and hence structural anisotropy induced during consolidation or shearing is not significant, within this stress range, in its effect on the angle of internal friction.

(*c*) The angle of internal friction measured under an increasing major principal stress is the same as that measured under a decreasing minor principal stress (though in the latter case a much smaller strain is required to mobilise it). This fact, together with the values of coefficient of earth pressure at rest given by the anisotropically consolidated tests, provides an explanation of the small strains required to mobilies active pressure on a retaining wall and the large strains required for passive pressure, which are difficult to correlate with conventional shear tests.

(*d*) The frictional strength in a drained test, as represented by φ_{d_r} (in which allowance has been made for the work done in changing the volume), is in good agreement with the maximum value of the frictional strength mobilised in an undrained (i.e. constant volume) test, represented by φ_{r_m}. The value φ_{r_f}, corresponding to maximum deviator stress in an undrained test, is about 2 degrees lower than φ_{r_m} and up to 4 degrees lower than φ_d. A direct comparison of the undrained and drained failure envelopes in dilatant soils cannot therefore in general be expected to give agreement.

The results of the *drained compression tests on dry samples* are given in Fig. 6, and a comparison is made with the drained compression test results on saturated sand under the same test conditions. It will be noted that both φ_d and φ_{d_r} are higher in the case of dry sand by about 2 degrees for loose sand and 6 degrees for dense sand.

This difference is larger than that usually assumed, but it should be noted that the dry sand was oven dried and coded in a desiccator to free it from moisture, and that the saturated sand was laid down under water. A test in which the sand was placed dry and then flooded before testing gave a result intermediate between the other two types of test. The results suggest that the presence of water at the intergranular contacts may alter the coefficient of friction mobilised during sliding, but also that the structure of the sand depends on the method of deposition.

The results of the *drained extension tests on saturated samples* are given in Fig. 7. It will be seen that the values of φ_d are independent of stress history and are in agreement with those obtained from the compression tests. The values of φ_{d_r} are, however, widely different from those of the compression tests, tending to decrease rapidly (instead of increasing) with decreasing porosity. There was, however, some indication from the undrained tests that this effect may be associated with a more rapid fall in the angle of internal friction with strain than in the case of the compression tests. The data is not sufficiently conclusive to present at this stage.

Tests carried out on sand by *Habib* (1951) indicated considerably lower values of φ_d in extension than in compression. Tests by *Taylor* (1941) had shown a similar difference, but it appeared to be influenced by the size and shape of the samples. In undrained tests on clay (*Taylor*, 1951) obtained a higher angle of internal friction in extension than in compression. Further data is therefore required to clarify the effect of the intermediate principal stress on the angle of internal friction, but it appears that the magnitude and uniformity of the strains may be an important factor in controlling the values measured.

Angle of Shearing Resistance

(*a*) Angle of Undrained Shearing Resistance φ_u. If several identical samples of sand are consolidated under the same cell pressure, and are then subjected to undrained compression tests under different cell pressures, it is found that there is no gain in strength with increase in cell pressure. A typical set of test results is illustrated in Fig. 8. This result is dependent on the conditions (1) that the samples are fully saturated, and (2) that negative pore water pressures large enough to cause cavitation are not set up. A detailed discussion is given elsewhere (*Bishop* and *Eldin*, 1950).

(*b*) Increase in undrained strength with consolidation pressure:—

The undrained strength of a sample consolidated under a given isotropic stress p is independent of further changes in cell pressure, and may therefore be conveniently expressed as an apparent cohesion c_u, equal to half the compression strength. For normally consolidated samples c_u is almost directly proportioned to consolidation pressure, and the ratio $\left(\frac{c_u}{p}\right)_n$ is a useful parameter to express the gain in strength with consolidation pressure. For anisotropically consolidated samples the major effective principal stress under which the sample is consolidated is taken as the consolidation pressure.

The results of the three series of undrained tests are presented in terms of this parameter in Fig. 9. It will be seen

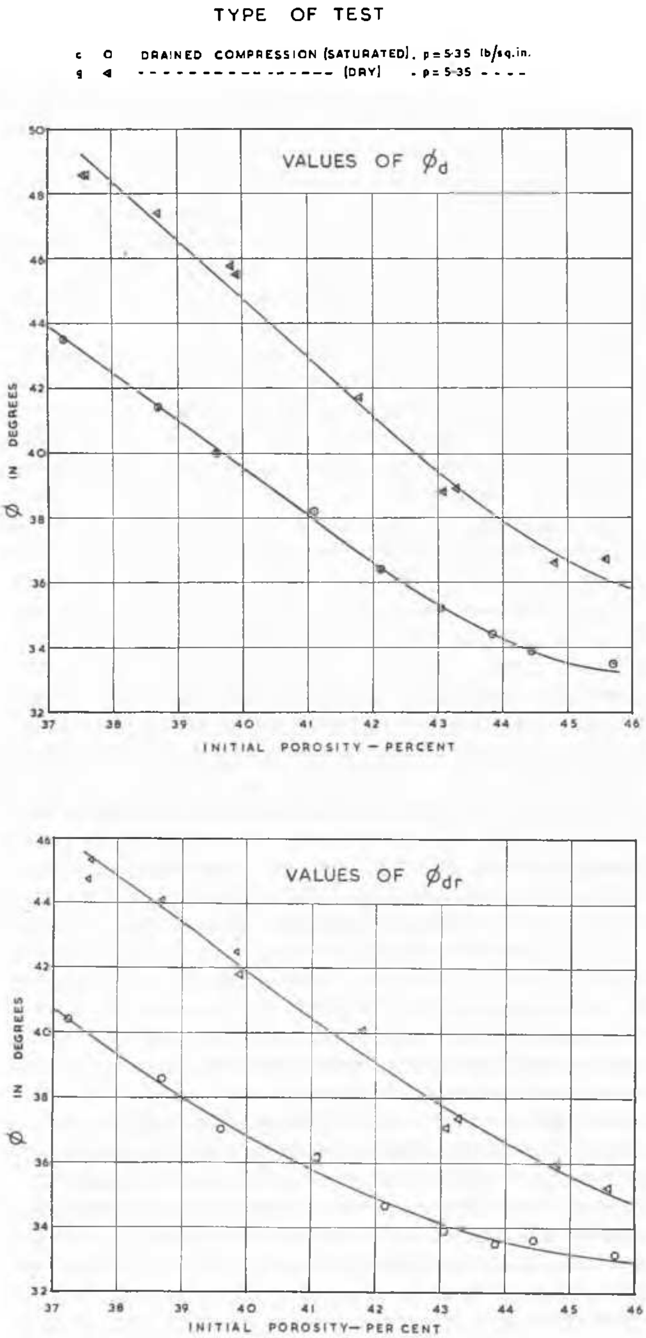

Fig. 6 Results of Compression Tests on Dry Samples
Résultats des essais de compression sur des échantillons secs

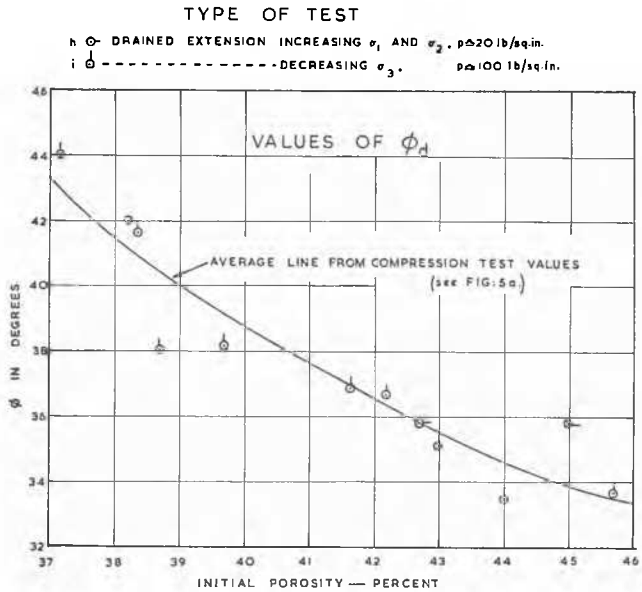

Fig. 7 Results of Extension Tests on Saturated Samples
Résultats des essais d'extension sur des échantillons saturés

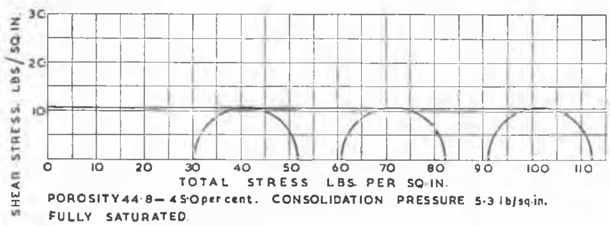

Fig. 8 Undrained Compression Tests on Saturated Samples
Essais non drainés de compression sur des échantillons saturés

that for the range of stresses used $\left(\frac{c_u}{p}\right)_n$ is approximately a constant for any given intitial porosity for isotropically consolidated samples. The marked influence of anisotropic consolidation can be seen, samples consolidated under conditions of no lateral yield having less than one third of the undrained strength of samples isotropically consolidated under the same pressure. For natural strata whose initial porosity corresponds to the loosest possible packing for saturated sand (between 46 and 47 per cent in this case) a value of $\left(\frac{c_u}{p}\right)_n$ of between 0.3 and 0.2 would thus be expected. It is interesting to compare this value with the values of $\left(\frac{c_u}{p}\right)_n$ for natural deposits given by *Skempton* (1948) in relation to their Atterberg Limits, which tend towards 0.2–0.15 for soils of low plasticity. The use of anisotropic consolidation appears to bring the laboratory and field values into much closer agreement. In the case of clays and silts the effect is less marked, but still of great practical importance. Other data is given by *Victor de Mello* (1951), *Bishop* and *Henkel* (1953), and a theoretical discussion by *Hansen* and *Gibson* (1949).

For extension tests the values of $\left(\frac{c_u}{p}\right)_n$ for isotropic consolidation tend to agree with those of the compression tests for high porosities, but increase less rapidly as the porosity decreases.

Values of the Coefficient of Earth Pressure at Rest, K_0

In the anisotropically consolidated tests the initial isotropic stress under which the sample was set up ($\simeq$ 5 lbs. per sq. inch) was appreciable compared with the changes in stress during anisotropic consolidation. The resulting overall stress ratio at the end of consolidation was about 0.5. A more accurate estimate of K_0 as a soil property was made by taking the ratio of the increase in lateral stress to the increase in vertical stress under conditions of no lateral yield, and using a wide range of stresses. The values of K_0 obtained in this way are plotted against porosity in Fig. 10, and vary from about 0.45 for loose sand to about 0.35 for dense sand under conditions of normal consolidation.

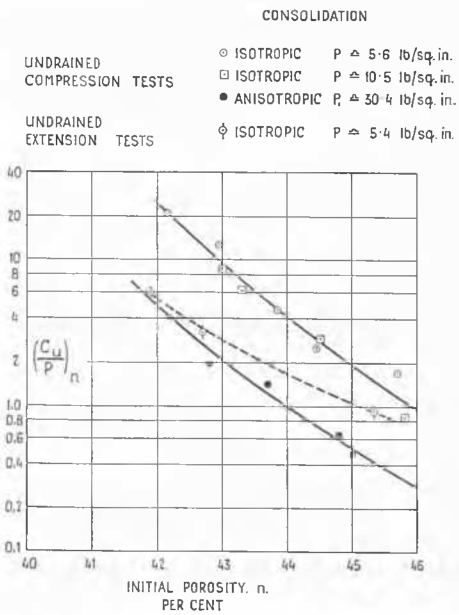

Fig. 9 Influence of Anisotropic Consolidation on Undrained Strength
Influence de la consolidation anisotrope sur la résistance d'un échantillon non-drainé

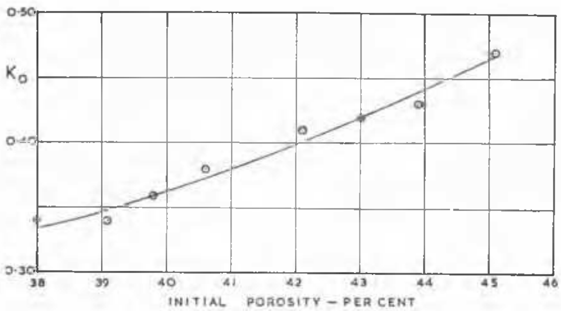

Fig. 10 Variation of Coefficient of Earth Pressure at Rest with Porosity
Variation du coefficient de la poussée des terres au repos, avec porosité

Acknowledgment

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