Three-dimensional Seepage-tests with viscous Fluids

Essais de perméabilité à trois dimensions avec des liquides visqueux

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Summary

The described procedure for the reproduction of two- and three-dimensional seepage flows is in compliance with the law of proportionality between the rate and hydraulic gradient of seepage (*Darcys*'s law). On account of the relatively small height of the boundary zone of capillarity and its only slight effect upon the process of seepage flow, it is possible also to reproduce complicated problems of seepage flow.

The tests made with horizontal filterwells to the present date, which have been dealt with in the following report, indicate the usefulness of the procedure. The same is suitable for the study of nonstationary flows with rapid changes of the fluid levels, because—beside capillarity—also other disturbing effects, such as e.g. the sealing off of the pores by air or the inhomogeneousness of the pervious material appear to be of minor importance. A colouring of the flow threads is possible, on the condition that the coulouring fluid has the same viscosity as glycerine.

Sommaire

Ce travail décrit un procédé qui permet d'étudier des problèmes hydrauliques concernant la question des nappes phréatiques à l'aide de modèles à échelle réduite et en utilisant le sable de quartz comme matériel filtrant et la glycérine comme liquide percolateur.

Ce procédé est applicable à la reproduction des courants d'infiltration à deux et à trois dimensions et est conforme à la loi de proportionnalité entre la vitesse de l'infiltration et la déclivité hydraulique (loi de *Darcy*). Vu la quasi insignifiante zone de capillarité, et son effet peu considérable sur le comportement du courant d'infiltration, il permet même la reproduction des problèmes d'infiltration les plus difficiles.

Les expériences sur les collecteurs d'infiltration horizontaux auxquelles nous avons procédé – traitées dans le rapport ci-dessous – sont un témoignage de l'utilité de ce procédé. Il est applicable à l'étude des courants non-permanents à rapides changements de niveau car les effets perturbateurs, tant le colmatage des vides interstitiels par l'air que le défaut d'homogénéité de la couche aquifère, semblent de peu d'importance. Il est possible de colorer les lignes de courant à condition que le liquide colorant ait la même viscosité que la glycérine.

Introduction

The general methods in use to date for the investigation of three-dimensional seepage flows by means of models with water and a relatively fine grained pervious material have all the disadvantage that the result of the measurements become badly distorted or are showing too big a spread on account of capillarity, selfsealing by air, structural inhomogenities of the pervious material, etc.

In connection with *Hele-Shaw* (1899) well-known method for the solution of two-dimensional seepage flow problems a method has been developed, by which also three-dimensional seepage flows can be more accurately investigated. Use was made of a coarse grained material consisting of well rounded quartz sand as pervious material and of a technical glycerine as viscous fluid.

After a general study of the method special tests were carried out upon the initiative of Dr. h.c. *H. Fehlmann*, Berne. Their aim was to investigate the hydraulic conditions of horizontal filterwells, a certain number of them having been built in Switzerland during the last years.

The technique of the tests has been perfected, so that, under observance of certain boundary conditions, it appears possible also to investigate complicated (three-dimensional) flow problems by means of model tests. The process is particularly suitable for the study of nonstationary seepage flows.

General

It has long been known that two-dimensional seepage flow problems can be investigated by means of a fluid which is flowing between two parallel glass plates. Compared with the usual methods using water flowing through sand, this method shows considerable advantages (*Hele Shaw*, 1899; *Günther*, 1940; *A. Casagrande*, 1951). It was therefore obvious to investigate the possibility of testing three-dimensional flow problems by means of a similar testing arrangement. For this purpose the natural pervious material was replaced by a fill of glass balls of a uniform diameter, and water by a viscous fluid (*Schweiz. Bauzeitung*, Jahrg. 68, 1950).

The verification of the fundamental relation of proportionality between the rate and the hydraulic gradient of seepage flow was carried out with the help of a glass cylinder filled with glass balls of 2 mm diameter, through which watery glycerine was flown at various seepage gradients ($J = 0$ to 3). Within the range of investigation, the tests showed the well-known proportionality between the rate and the gradient of seepage flow. As could forthwith be expected in accordance with the general wording of *Darcy*'s law, this law which is governing the groundwater currents, is thus also applicable in model tests, in which a coarse material with relatively large pores is flown through by a sufficiently viscous fluid, whereby a laminar flow in the pores is maintained. Furthermore it could be established that the criteria for the laminarity of flow conditions in permeable formations (*Fancher* etc., 1933; *Muskat*, 1937) are more or less fulfilled.

The Tests

The study of known two-dimensional flow problems was undertaken. Among others the discharge into a drainage ditch, into a horizontal drainpipe and out of a canal, was investigated.

Fig. 1 represents the model for the investigation of a drainage ditch. It shows the surface of the seepage in the wall of the ditch, which becomes visible after the drawn down exceeds the critical value, as well as the only markedly weak zone of capillarity (grid = 5 cm).

Particular attention was given to tests with three-dimensional seepage flows. Firstly, the yield and the free surface as a function of the elevation of the undisturbed groundwater table were tested on a model vertical filter well. The results obtained coincide with the tests carried out by *Cambefort* (1948). Furthermore, the theories about the surface of seepage in filter pipes could be compared with actual measurements (*Cambefort*, 1948; *Ehrensberger*, 1928; *Kozeny*, 1947; *Némeny*, 1933; *Breitenöder*, 1942; *Vibert*, 1943; *Jaeger*, 1946, 1947).

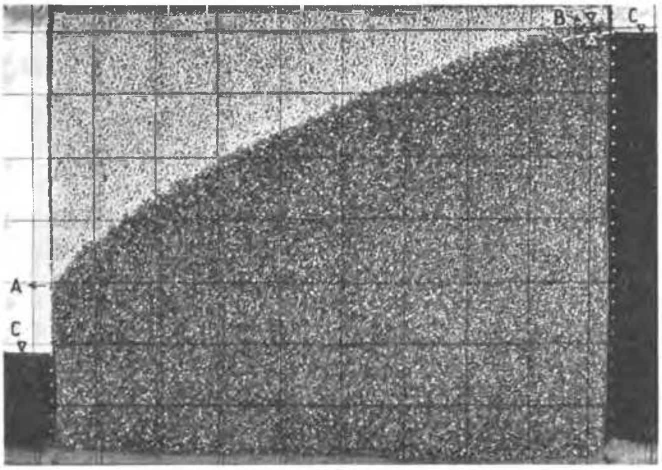

Fig. 1 Flow-Pattern of the Discharge into a Drainage-Ditch.
A = Discharge Point B = Capillary Fringe
C = Level of the Liquid
Courbe du débit d'écoulement dans un fossé de drainage.
A = source B = zone capillaire C = niveau du fluide

Fig. 2 Model of the Horizontal Filterwell. Filling in of the Pervious Material
Modèle de collecteur horizontal au moment de l'addition de l'aquifer

A further series of tests was concerned with horizontal filter wells, i.e. a well consisting of a vertical main shaft, from which radial horizontal filter pipes have been driven. This example is dealt with in detail on page 5.

Pervious Material

From a purely scientific point of view, a packing of glass balls with a grain diameter of 2 mm, is particularly suitable as a porous model material. With the suitable bedding obtained by filling in the dry glass balls with a funnel under free fall (see Fig. 2) differences in the porosity of 2.6% appeared which is negligeable in model tests. The disadvantage of this material is its high unit price.

For the purpose of reducing the costs, the glass balls for the large models were replaced by well rounded quartz sand with a grain diameter of 2.5 to 3 mm. This material also met all requirements.

Test Fluid

The coarse grained pervious material requires for the maintenance of a laminar flow the use of a viscous fluid. After long trials (since 1949) a mixture of glycerine and water was found with a viscosity higher than 120 cP. This was the only viscous liquid which was easily available ready made and which met the following requirements:

(1) viscosity independent from the state of movement (*Newton*'s liquid),
(2) the used mixture of glycerine and water is not subject to physical-chemical and bacteriological alterations.

A peculiar property of the glycerine offered considerable difficulty in the fulfilment of the first requirement. It was first necessary to establish the azeotropic equilibrium between glycerine and water in order to obtain a constant viscosity. Unfortunately this equilibrium depends on the temperature. Tests over a longer period of time have, therefore, to be carried out in a thermostatic room.

Measuring-Arrangements and Experiments

The measurement of the yield i.e. the quantity of discharge was always made with gauged containers.

Two methods were used for the establishment of the levels of the liquids. The first was based on the use of piezometric tubes (interconnected standpipes); the fluid levels were lifted up by vacuum while the second method was developed as an electrical contact gauge whereby upon the contact of a probe with the liquid (free liquid level in the standpipe) electrical contact was made. Both methods showed about the same degrees of accuracy (spread $\pm\frac{1}{2}$ mm). As the magnitude of the pervious material in the model amounted to an average of 200 mm, this error in measuring can be qualified as insignificant.

The physical properties of the glycerine permits the use of a relatively coarse grained medium as pervious material. This assures, on the other hand, a small height of capillary rise (1.0 to 1.5 cm), which causes no noticeable effect on the yield on account of transportation in the boundary zone of capillarity.

Glycerine absorbs very little air. The pores of the pervious material, therefore, are hardly in danger of becoming clogged up by successively collected air. A test of a duration of two months in continuous operation and considerably fluctuating "groundwater" level showed for the coefficient of permeability $k_{10°}$ only a decrease from $3.45 \cdot 10^{-2}$ cm/sec to $3.25 \cdot 10^{-2}$ cm/sec. On the other hand it was noticed that at the first filling permeability decreased considerably in some hours. A sufficiently long initial time is therefore required until the state of equilibrium is established.

A technical glycerine of high viscosity (waste product from the manufacturing of soap) was used in the tests. This caused considerable damage by corrosion on nearly all metallic parts of the models as well in the pipelines as in the containers. A covering of the models with an acid resistant coating was therefore indispensable.

The Horizontal Filterwell as an Example of a Test-Series

Insight into the nature and the working of the testing methods dealt with in this report is obtained from the tests on horizontal filterwells. The tests were made with the aid of Dr. *H. Fehlmann* (AG für Grundwasserbauten, Bern). The task consisted primarily in establishing the effect of the number of drains, the length and the thickness of the pervious material upon the yield of the well.

For this purpose a model was built in the shape of a quarter circle, with a radius of 200 cm (Fig. 3). The vertical collecting-well (with a diameter of 11 cm and impermeable walls) is located in the centre of the circle. Five horizontal drainpipes (relative to the quarter circle) which are sited just on top of the impermeable bottom layer lead directly into this centre well.

For the investigation of the effect of a smaller number of drains, some drains were eliminated, while the testing on the effect of the drain length was realized by a partial elimination of the respective drainpipes. The test fluid is supplied to the model from a supply tank along the outside of the circular wall. The pervious material was, as already mentioned, rounded quartz sand (grain diameter 2.5 to 3.0 mm) and the layer had a thickness of 40 cm (Fig. 3). The test was carried out in such a way that, as well in the supply tank as in the central collecting well, a fixed elevation was maintained. After equilibrium had been established, which was the case after an operating time of 4 to 5 hours, the yield in the central shaft was measured. At the same time the pressure heads were observed at 52 measuring points (measuring points approx. 2.5 cm above the impermeable bottom layer) and the physical properties of the glycerine established.

Fig. 3 Model of the Horizontal Filterwell with Drains Directly on Top of the Impermeable Bottom Layer Before the Filling in of the Previous Material. A Centre Well, B Supply Tank
Modèle de collecteur horizontal, dont les drains sont en contact direct avec la surface imperméable, avant addition de l'aquifer. A: le puits, B: le réservoir

The tests gave a clear picture of the working of this type of well. They were carried out under the following assumptions:

(1) Horizontal, impermeable layer as low limit of the pervious material.
(2) Horizontal, stationary groundwater table.
(3) Homogeneous pervious material of fixed grain size.
(4) Constant reach (limited to $R = 200$ cm by the supply tank).
(5) Drainpipes of the same length and the same diameter (outside diameter 1.1 cm) for a particular test.
(6) Drains arranged symmetrically (with the exception of 5 drainpipes relative to the quarter circle).
(7) Intake- and energy-losses in the drains are negligeably small (i.e. unlimited draw-off capacity). For this purpose the cross-sections of the drains were constructed in such a way that the inflow of the liquid passes through filterpipes with an outside diameter of 1.1 cm; the outflow which was placed in the impermeable layer passes through a slot into a pipe of a larger diameter (Fig. 4).

In the following a few typical test results are described for the better characterization of this type of well.

In Fig. 5 the yield $Q_{10°}$ is shown as a function of the lowering of the fluid level in the central collecting well (relative) with drainpipes of various lengths. The effect of the lowering of the fluid level upon the yield is similar to the same upon the yield of the vertical filter-well.

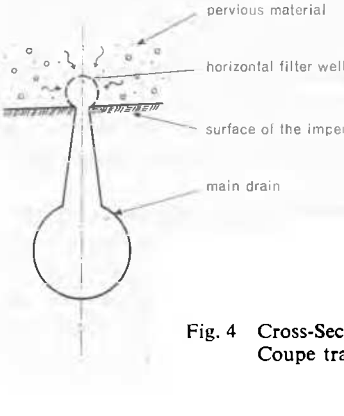

Fig. 4 Cross-Section through an Individual Drain
Coupe transversale d'un tuyau de drainage

Fig. 6 shows the dependence of the yield on the number of drains at constant length of the drainpipes, as they were directly recorded from the tests.

In Fig. 7 the effect of the number of drains is clearly visible. As was to be expected the yield cannot be raised ad libitum by increasing the number of drains (with constant length of the drainpipes).

Simultaneously with the measurements of the yield, the pressure heads at approx. 2.5 cm above the impermeable layer were also observed as already mentioned above.

Fig. 8 shows a representation of the lines of equal pressure heads plotted for the case of four drains, resp. 20 drains with relation to the full circumference. The general draw down is clearly visible and shows additional secondary depressions which depend on the position of the drains. Remarkable is e.g. the draw down in the vicinity of the centre well for the case of 4 drains. The whole is presenting a somehow simplified picture, which is nevertheless based upon nearly 100 measuring points related to the quarter circle.

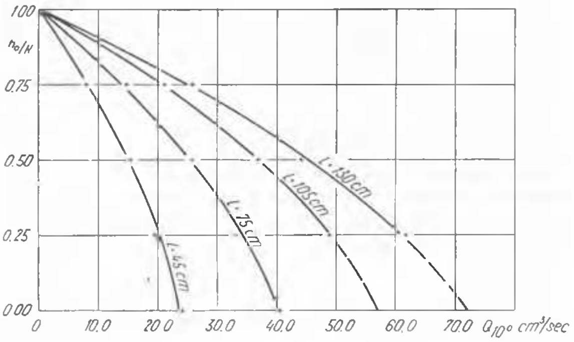

Fig. 5 Yield $Q_{10°}$ in Function of the Relative Craw Down at Various Lengths of the Drainpipes
Fluid Level in the Supply Tank $H = 20.9$ cm
Fluid Level in the Centre Well $h_0 = 0.0$ to 20.9 cm
Number of Drains = 4 Relative to the Full Circle
Outside Diameter of Drainpipes = 1.1 cm
Lengths of Drains from Axis of Well $L = 45/75/105/130$ cm
$R = 200$ cm
Yield Reduced to 10 °C, Relative to the Full Circle $Q_{10°}$ in cm³/sec
Coefficient of Permeability at 10 °C $k_{10°} = 3 \cdot 35.10^{-2}$ cm/sec
Débit $Q_{10°}$ en fonction de l'abaissement relatif des différents drains
Niveau du fluide dans le réservoir $H = 20{,}9$ cm
Niveau du fluide dans le puits central $h_0 = 0{,}0\text{–}20{,}9$ cm
Nombre de drains: 4, en proportion du cercle entier
Diamètre extérieur des tuyaux de drainage = 1,1 cm
Longueur des drains mesurée à partir de l'axe du puits $L = 45, 75, 105, 130$ cm
$R = 200$ cm
Débit, calculé pour le cercle entier sur la base de 10 °C $Q_{10°}$ en cm³/sec
Coefficient de perméabilité $k_{10°} = 3 \cdot 35.10^{-2}$ cm/sec

The tests, firstly limited to one level of the supply tank $H = 20.9$ cm, were also extended to the levels $H = 10.45$ cm, 31.35 cm and 41.8 cm. Essential differences could hereby not be established, but the thickness of the water bearing layer is not insignificant.

The differences in the working between the horizontal and the vertical wells are quite obvious. At the horizontal wells e.g. the shape of the fluid surface is decidedly influenced by the number of drains (compare Fig. 7). Furthermore the extent and the hydraulic gradients of the draw downs are entirely different from those in the vertical filterwells.

Acknowledgment

We particularly wish to acknowledge the co-operation of Mr. *W. Schaad* and Mr. *G. Amberg*. To Messrs. *H. B. Fehlmann* and *J. Fehlmann* we owe our thanks for their support and co-operation in the construction of the model horizontal filterwell and in the carrying out of the tests.

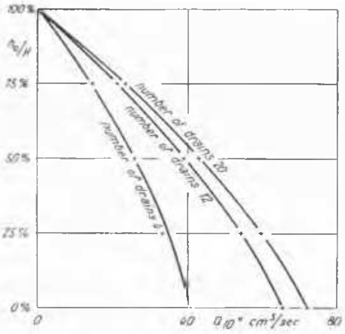

Fig. 6 Yield $Q_{10°}$ in Function of the Relative Draw Down for Various Numbers of Drains (relative to the full circle)
Fluid Level in the Supply Tank $H = 20.9$ cm
Fluid Level in the Centre Well $h_0 = 0.0$ to 20.9 cm
Length of Drains $L = 75$ cm; $R = 200$ cm
Outside Diameter of Drainpipes = 1.1 cm
Yield at 10 °C Relative to the Full Circle $Q_{10°}$ in cm³/sec
Coefficient of Permeability at 10 °C $k_{10°} = 3.35 \times 10^{-2}$ cm/sec
Débit $Q_{10°}$ pour différents nombres de drains en fonction de l'abaissement relatif (par rapport au cercle entier)
Niveau du liquide dans le réservoir $H = 20{,}9$ cm
Niveau du liquide dans le puits central $h_0 = 0{,}0\text{–}20{,}9$ cm
Longueur des drains $L = 75$ cm, $R = 200$ cm
Diamètre extérieur des tuyaux de drainage = 1,1 cm
Débit, calculé pour le cercle entier sur la base de 10 °C $Q_{10°}$ en cm³/sec
Coefficient de perméabilité à 10 °C $k_{10°} = 3.35 \times 10^{-2}$ cm/sec

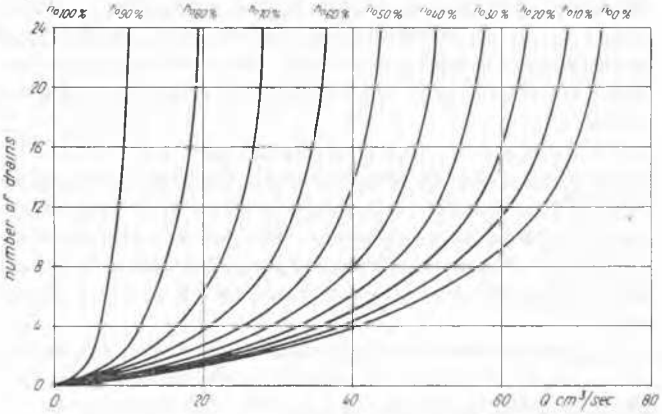

Fig. 7 Effect of the Number of Drains upon the Yield $Q_{10°}$
Fluid Level in the Supply Tank $H = 20.9$ cm
Fluid Level in the Centre Well h_0 in % of H
Length of Drains from Axis of Well $L = 75$ cm; $R = 200$ cm
Outside Diameter of Drainpipes = 1.1 cm
Yield at 10 °C Relative to the Full Circle $Q_{10°}$ in cm³/sec
Coefficient of Permeability at 10 °C $k_{10°} = 3.35 \times 10^{-2}$ cm/sec
Influence du nombre des drains sur le débit $Q_{10°}$
Niveau du liquide dans le réservoir $H = 20{,}9$ cm
Niveau du liquide dans le puits central h_0 en % de H
Longueur des drains mesurée à partir de l'axe du puits $L = 75$ cm; $R = 200$ cm
Diamètre extérieur des tuyaux de drainage = 1,1 cm
Débit, calculé pour le cercle entier sur la base de 10 °C $Q_{10°}$ en cm³/sec
Coefficient de perméabilité à 10 °C $k_{10°} = 3{,}35 \times 10^{-2}$ cm/sec

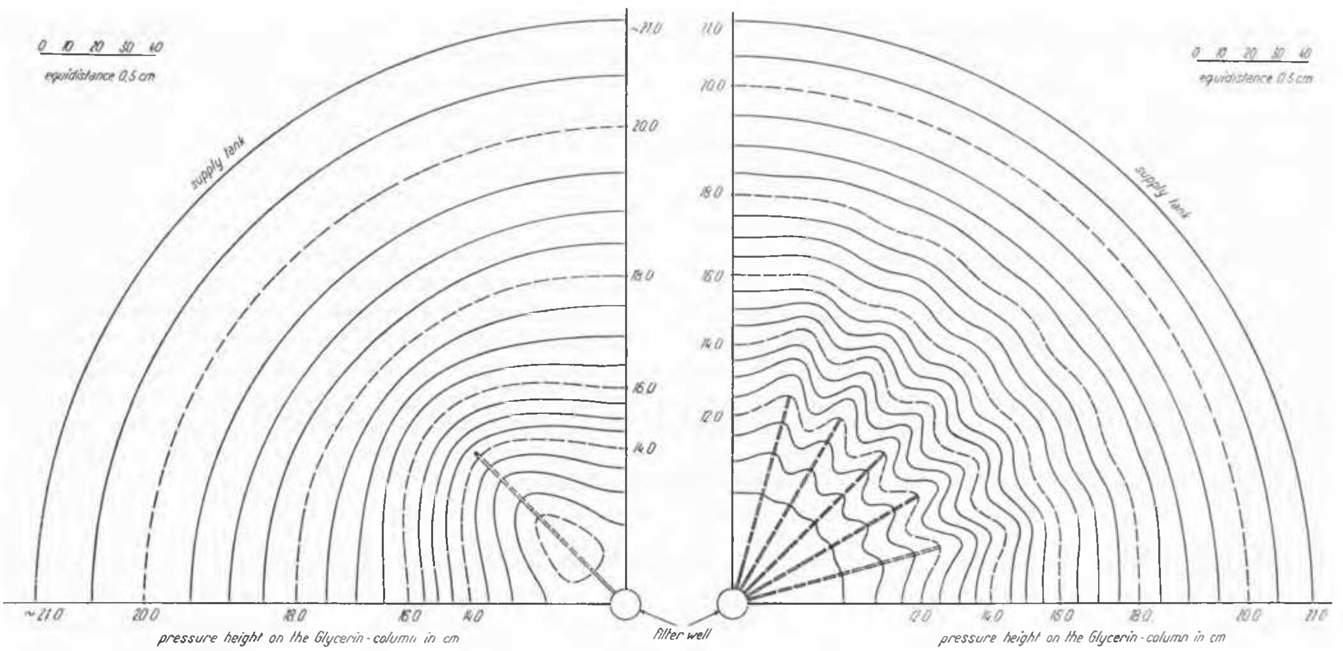

Fig. 8 Lines of Equal Pressure with 4, resp. 20 Drains Relative to the Full Circle
Pressure Heads Shown in cm Glycerine Column (Line at 21 cm has been Extrapolated)
Glycerine = 1.26 g/cm³ at Measuring Temperat. (approx. 12 °C)
H = Fluid Level in the Supply Tank = 20.9 cm
h_0 = Fluid Level in the Centre Well = 10.45 cm
Length of Drains (from Axis of Well) = 75 cm, Outside Diameter of Drains = 1.1 cm
Reach R = 200 cm
Coefficient of Permeability k_{10° = 3.35×10^{-2} cm/sec
Yield Q_{10° with 4 Drains = 26.0 cm³/sec } Relative to Full
with 20 Drains = 43.2 cm³/sec } Circle

Lignes d'égale pression avec 1, respectivement 5 drains, c'est-à-dire 4 ou 20 drains pour le cercle entier
Hauteur de pression en cm sur la colonne-glycérine (la ligne à 21 cm a été extrapolée)
Poids spécifique de la glycérine = 1,26 g/cm³ à la température de 12 °C environ
H = niveau du fluide dans le réservoir = 20,9 cm
h_0 = niveau du fluide dans le puits central = 10,45 cm
Longueur des drains mesurée de l'axe du puits = 75 cm, diamètre extérieur des drains = 1,1 cm
Rayon d'influence R = 200 cm
Perméabilité k_{10° = $3,35 \times 10^{-2}$ cm/sec
Débit Q_{10° pour 4 drains = 26,0 cm³/sec } en proportion du
pour 20 drains = 43,2 cm³/sec } cercle entier

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