A Field Determination of Permeability

Détermination sur place de la perméabilité

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Summary

In order to determine the permeability of undisturbed soil in the field, a new method using a simple pool is proposed instead of the traditional assumption which is unreasonable or the troublesome method employed hitherto. In this Paper theoretical formulae on which this method depends are derived, and the result obtained with this method and its verification are discussed.

Sommaire

Nous proposons, pour remplacer l'ancienne méthode de détermination de la perméabilité d'un sol vierge, laquelle est irrationnelle, une méthode nouvelle qui ne demande qu'un bassin simple pour l'expérience. Nous avons établi les formules théoriques dont découle cette méthode et nous discutons ici les résultats de nos observations expérimentales et les preuves à l'appui.

Introduction

The coefficient of soil permeability, as defined in soil engineering, is founded on *Darcy*'s law, and the methods of measuring it are by means of a permeameter in a laboratory or the field method carried out without disturbing the layer condition of the soil.

When the laboratory method is carried out with a small portion of the sample, the collected sample being disturbed, the condition of the strata which affects the permeability usually differs from that in the field, making it very difficult to determine quantitatively their corresponding relation even with the penetration test and others. Furthermore, it is difficult to avoid the undesirable effect caused by the space between the sample and the wall of the measuring apparatus.

As the above defects are eliminated in the field method which has the advantage that it does not disturb the layer condition of the ground, various methods have been studied by many authors (*Krynine*, 1941). For instance, the method of measuring the travelling time of the high electrolyte (*Slichter*, 1902) and that of measuring the hydraulic gradient of water flowing from a channel by boring several holes near the measuring channel (*Terzaghi*, 1930) have been proposed. In either case, however, as considerable preparation is necessary in order to take the measurements and since the water as it is poured in forms a three-dimensional flow, theoretical analysis is difficult and, in most cases, the reliability of the measured results is considered limited, as they are based on unreasonable assumptions.

In our field method, the three-dimensional flow is changed into a two-dimensional one and the theoretical treatment is simplified in a suitable way. The theoretical formulae for this case are deduced so that the coefficient of permeability of the field ground can easily be calculated. These are the main characteristics of this method.

Fundamental Theoretical Equation

Gravity flow: Generally the condition of the water seeping into the soil can be expressed by the following complex conjugate function:

$$\omega = \Phi + i\Psi = f(z) = f(x + iy) \qquad (1)$$

where Φ and Ψ, which form a flow-net perpendicular to each other, represent the function of the velocity potential and the flow function of the two-dimensional flow, respectively.

The most important factor governing the condition of the water seeping into the dam and water leaking from a channel is obviously gravity. If such gravity flow is considered as a two-dimensional flow, potential function Φ and pressure function p both satisfy *Laplace*'s equation and Φ can be expressed in the following form:

$$\nabla^2\Phi = \nabla^2 p = 0, \quad \Phi = \frac{k_0}{\mu}(p \pm \gamma g y) \qquad (2)$$

where μ coefficient of viscosity of fluid, γ weight of unit volume of fluid, g gravity acceleration, k_0 coefficient of permeability, and the double sign corresponding to the upper and lower directions of the y-axis, respectively. If the atmosphere pressure is 0, the coefficient of effective permeability is defined as $k = \frac{k_0 \gamma g}{\mu}$ taken instead of k_0 and the downward direction of y-axis is assumed positive, then Φ in Equation (2) becomes:

$$\Phi = -ky \quad \ldots \quad (2')$$

Derivation of the solution of the flow from a channel: The representation of the complex function is given by the following equation which is applicable to the case when the water seeping from a channel with the section shown in Fig. 1 has a free water surface in the case of the two-dimensional flow:

$$z = \frac{1}{k}\left(-Hke^{\frac{\pi\omega}{Q}} - i\omega + \frac{Q}{2}\right) \quad \ldots \quad (3)$$

$$\text{i.e.}\quad \left.\begin{aligned} x &= \frac{1}{k}\left(-Hke^{\frac{\pi\Phi}{Q}}\cos\frac{\pi\Psi}{Q} + \Psi + \frac{Q}{2}\right) \\ y &= \frac{1}{k}\left(-Hke^{\frac{\pi\Phi}{Q}}\sin\frac{\pi\Psi}{Q} - \Phi\right) \end{aligned}\right\} \quad \ldots \quad (3')$$

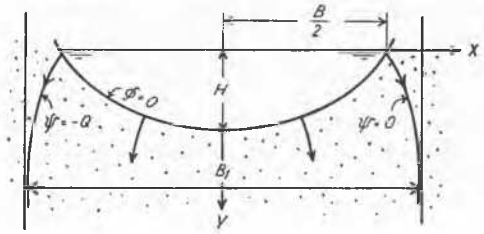

Fig. 1 Seepage out of a Ditch with free Surfaces Bounded by Vertical Asymptotes
Filtration hors du bassin à surfaces libres limitée par des asymptotes verticales (niveau souterrain bas)

where H is the depth of water in the channel at the centre of the cross-section, Q is the discharge of water from unit horizontal depth. The two streamlines, $\Psi = 0, -Q$ are free-surface streamlines. If Equation (2) is introduced into the above equations, the streamlines become the curves symmetrical along the y-axis represented by the following equations:

$$x = \frac{1}{k}\left(-Hke^{-\frac{\pi ky}{Q}} + \frac{Q}{2}\right), \quad x = \frac{1}{k}\left(Hke^{-\frac{\pi ky}{Q}} - \frac{Q}{2}\right) \quad (4)$$

If $\Phi = 0$ is substituted in Equation (3) to obtain the shape of channel to which Equation (3) is applicable, the following equation is derived:

$$\left\{x + \frac{1}{k}\left(\frac{Q}{\pi}\sin^{-1}\frac{y}{H} - \frac{Q}{2}\right)\right\}^2 + y^2 = H^2 \quad \ldots \quad (5)$$

If B is the width of the channel-section at the water surface and $x = \frac{B}{2}$ is substituted in this equation at $y = 0$,

$$B = \frac{Q}{k} - 2H, \quad B_1 = 2|x|_{y=\infty} = \frac{Q}{k}B + 2H \quad (6)$$

Thus B_1, the maximum width of the sheet of water seeping down at infinite depth becomes known.

The above simple results will, of course, apply only to the case when the shape of the channel is close to that given by Equation (5) and when a uniform soil layer is of a great depth, so that the water can maintain indefinitely its vertical downward seepage. However, among these boundary conditions, it has been stated that the former assumption is not so important compared with the latter (*Muskat*, 1937). Therefore, it can be said that the above formulae are applicable only to the case when an underground water-table exists deep in the earth. In many practical situation however, the water seeping down from the channel will reach the normal groundwater level at a relatively shallow depth, thus forcing the streamlines to assume a horizontal rather than a vertical trend. Fig. 2 represents such a case and the complex function is given by the following equation:

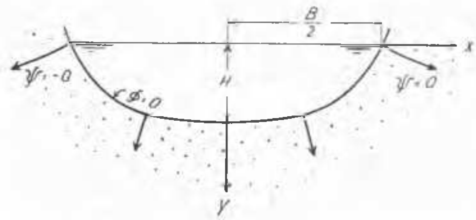

Fig. 2 Seepage out of a Ditch with Radially Spreading free Surfaces
Filtration hors du bassin à surfaces libres s'étalant radialement (niveau souterrain bas)

$$z = \frac{1}{k}\left(Hke^{-\frac{\pi\omega}{Q}} - i\omega + \frac{Q}{2}\right) \quad (7)$$

$$\text{i.e.}\quad \left.\begin{aligned} x &= \frac{1}{k}\left(Hke^{-\frac{\pi\Phi}{Q}}\cos\frac{\pi\Psi}{Q} + \Psi + \frac{Q}{2}\right) \\ y &= \frac{1}{k}\left(-Hke^{-\frac{\pi\Phi}{Q}}\sin\frac{\pi\Psi}{Q} - \Phi\right) \end{aligned}\right\} \quad (7')$$

The two symmetrical free-surface streamlines, $\Psi = 0, -Q$, are given as before in the following equations:

$$x = \frac{1}{k}\left(Hke^{\frac{\pi ky}{Q}} + \frac{Q}{2}\right), \quad x = \frac{1}{k}\left(-Hke^{\frac{\pi ky}{Q}} - \frac{Q}{2}\right) \quad (8)$$

A comparison of Equation (4) with Equation (8) reveals that the former has vertical asymptotes at the distance $\frac{Q}{2k}$ from the center of the channel which the free water-surface approaches deep in the earth, while with the latter the depth of the streamlines increases logarithmically as the distance from the channel increases.

Substituting $\Phi = 0$ so that the shape of the channel in this case satisfies Equation (7), gives

$$\left\{x + \frac{1}{k}\left(\frac{Q}{\pi}\sin^{-1}\frac{y}{H} - \frac{Q}{2}\right)\right\}^2 + y^2 = H^2 \quad (9)$$

If $x = \frac{B}{2}$ is substituted when $y = 0$,

$$B = \frac{Q}{k} + 2H \quad (10)$$

is obtained.

It should be noted that while Equation (9) is in form identical with Equation (5), one must take the positive radical $\sqrt{H^2 - y^2}$ in solving Equation (9) for x, whereas in Equation (5) the negative values of this radical must be used in solving x.

Solution for the case when the shape of the channel consists of straight lines: Fig. 3 shows the seepage flow from a channel formed with straight lines when an underground water-table exists at great depth. Here, too, a consistent set of potential and streamline distributions and the shapes of the free-surfaces can be determined exactly by applying a succession of complex variable transformations, but the exact profile of the channel to which they correspond is found only at the end of the solution. The solution for this case was given by *Wedernikow* (1934), and the result is represented by the following equation corresponding to Equation (6):

$$B = \frac{Q}{k} - 2H\frac{K}{K'}$$

$$B_1 = 2|x|_{y=\infty} = \frac{Q}{k} = B + 2H\frac{K}{K'} \tag{11}$$

where K, K' are the complete elliptic integrals of the first kind with moduli k^*, $\sqrt{1-k^{*2}}$. When $K = K'$, viz, $k^* = \pm\sqrt{\frac{1}{2}}$, Equation (11) is identical to Equation (6).

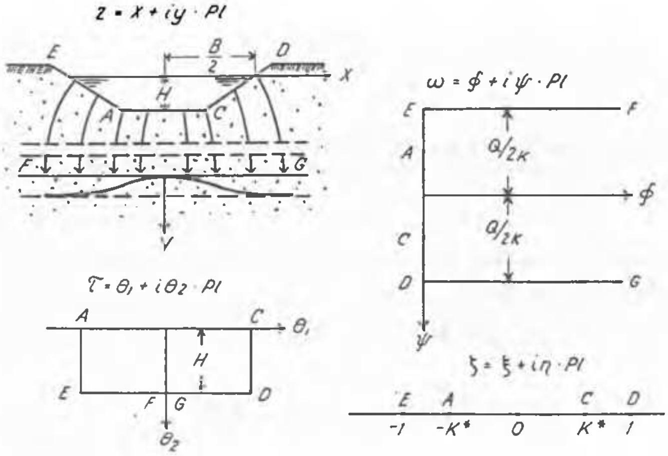

Fig. 3 Seepage out of a Ditch into the Ground with Deep lying Water Table and its Transformation
Filtration hors du bassin dans le sous-sol avec nappe phréatique à niveau bas et sa transformation

Test Measurement and Result

Method of measurement: Two test pools A, B with cross-sections as shown in Fig. 4 were made in the Kantō loam at the site of the Asamizo depositing reservoir, Kanagawa Prefecture. Water was continuously poured in to maintain the water level of the pools always at 50 cm. The quantity of water seeping from the pools was measured in the following way: A bucket with a capacity of 10 litres was used for the measurement and the time required to fill it was taken. The seepage flow in this case is a three-dimensional flow and diffuses to all sides from the pools. Then the pools were enlarged in the direction perpendicular to the cross-section, and by subtracting the volume of water seeping from the original test-pool from that of the enlarged one, the effect of the flow near both ends of the section was eliminated. In this manner, Q, the volume of water flowing out from the section and the unit length of horizontal depth was obtained. The flow from which the effect of the ends is eliminated in this way can be considered as a two-dimensional flow, resulting in the formulae derived from the fundamental theoretical equations stated above being applicable. The length to which the test pool was extended and the number of times it was enlarged were twice 2 m for A and once 4 m for B, and the differences of each cases were taken.

Result of measurement:

(*a*) A test-pool ($B = 2.5$ m, $H = 0.5$ m, $b = 1.5$ m, $m = \frac{B-b}{2H} = 1$)

Average seepage discharge of original pool 19.5 cm³/sec

When enlarged once 38.9 cm³/sec difference 19.4 cm³/sec

When enlarged twice 57.1 cm³/sec difference 18.2 cm³/sec

average 18.8 cm³/sec

Average discharge from section A–A per unit horizontal depth in cm, $Q_A = 18.8 \div 200 = 0.094$ cm³/sec

(*b*) B test-pool ($B = 3.0$ m, $H = 0.5$ m, $b = 1.0$ m, $m = \frac{B-b}{2H} = 2$)

Average seepage discharge of original pool 112.2 cm³/sec

When enlarged 149.2 cm³/sec difference 37.0 cm³/sec

Average discharge from section B–B per unit horizontal depth in cm, $Q_B = 37.0 \div 400 = 0.092$ cm³/sec.

Application of theoretical equation: As the test pools A and B used for this measurement, for the convenience of construction, have straught the rigorous method explained already must be applied, but in accordance with the thesis that a slight change in the shape of the pool does not invalidate the application of the formulae from which the simple result given by Equations (3) ~ (10) is obtained, the coefficient of permeability k will be calculated adversely from Equation (6) or (10). In this case, the choice between the two equations depends upon the direction of diffusion of the seepage flow governed by the depth of the underground water-table. In this test measurement, the depth of the loam layer in which the test pools were dug is about 15 ~ 16 metres judging from the result of a boring test, and it is known that the underground water-level exists in the gravel below the loam. As is clear when observed in a side trench built under the pool bottom, it is confirmed that the seepage water flows perpendicularly downwards and not horizontally, but, to make sure, k will be computed using both Equations (6) and (10) which gives the width B of the test pools. Thus from Equation (6):

$$k_1 = \frac{Q}{B + 2H} \tag{6'}$$

and from Equation (10):

$$k_2 = \frac{Q}{B - 2H} \tag{10'}$$

Applying the measured result to the above equations, gives, from Equation (6′),

$$k_{1A} = \frac{Q_A}{B + 2H} = \frac{0.094}{350} = 2.68 \times 10^{-4} \text{ cm/sec,}$$

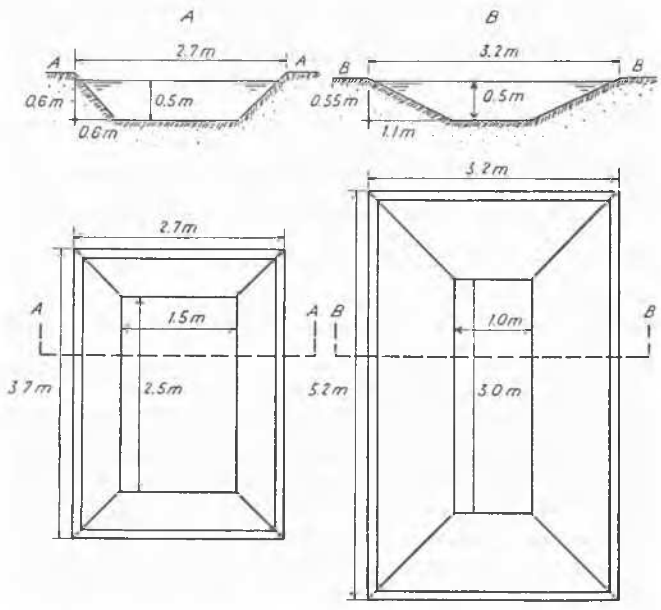

Fig. 4 Dimensions of Test Pools
Dimensions des bassins d'expérience

$$k_{1B}=\frac{Q_B}{B+2H}=\frac{0.092}{400}=2.30\times10^{-4}\ \text{cm/sec},$$

average 2.49×10^{-4} cm/sec,

from Equation (10′),

$$k_{2A}=\frac{Q_A}{B-2H}=\frac{0.094}{150}=6.26\times10^{-4}\ \text{cm/sec},$$

$$k_{2B}=\frac{Q_B}{B-2H}=\frac{0.092}{200}=4.60\times10^{-4}\ \text{cm/sec},$$

average 5.43×10^{-4} cm/sec.

Therefore, it is found that the coefficient of permeability k of the Kantō loam layer in which the measurement was taken is within the following values:

2.49×10^{-4} cm/sec $\leqq k \leqq 5.43\times10^{-4}$ cm/sec.

Considerations on the Measured Result

Verification of the order of the coefficient of permeability: Judging from the formation of the Kantō loam in this district, it had been expected that the coefficient of permeability lies between "permeable" and "impermeable", namely the order of 10^{-4} cm/sec. The result computed by the permeability computation method based upon the rate of water-level depression with the elapse of time gives $k = 4.94\times10^{-4}$ cm/sec, which is clearly within the range of the above mentioned k.

Observation of the width of the sheet of the seepage flow and its change with time at a great depth below the bottom of the pool: In order to examine how the water seeping from the test pool C in Fig. 5 diffused at a great depth underground and how it changed with time, the width of diffusion after the water had been poured into the pool (including that due to capillary action) was observed in a horizontal side-trench connected to a vertical shaft 8 m deep. The result was as stated above, namely, it was found that as the Kanto loam was thick and the underground water level was quite deep at this place, the free streamline of the water seeping from the pool had vertical asymptotes.

As is clear from Fig. 5, the change in the width of the sheet of the seepage flow with the elapse of time can be divided roughly into three phases. Namely, from the time water begins to enter the trench until about 100 hours later (I), the seepage water tends to diffuse horizontally due to the capillarity of the ground. During the next 150 hours (II), it continues to flow vertically downwards, becoming almost an imperceptible flow. From then (III), the flow concentrates to the centre of the pool-bottom due to the primary piping-action, that is, the width of the sheet diminishes with the elapse of time.

The theoretical width of the seepage calculated by Equation (6) which is applicable to phase (II) becomes the range shown by the chain line in Fig. 5. This differs slightly from the result of actual measurement. It is considered that this discrepancy is due to the capillarity of the soil and the difference in the shape of the cross-section of the test pool; if omitted, the theoretical and the experimental results become closer, and it may be admitted that the accuracy of this method of measurement is high.

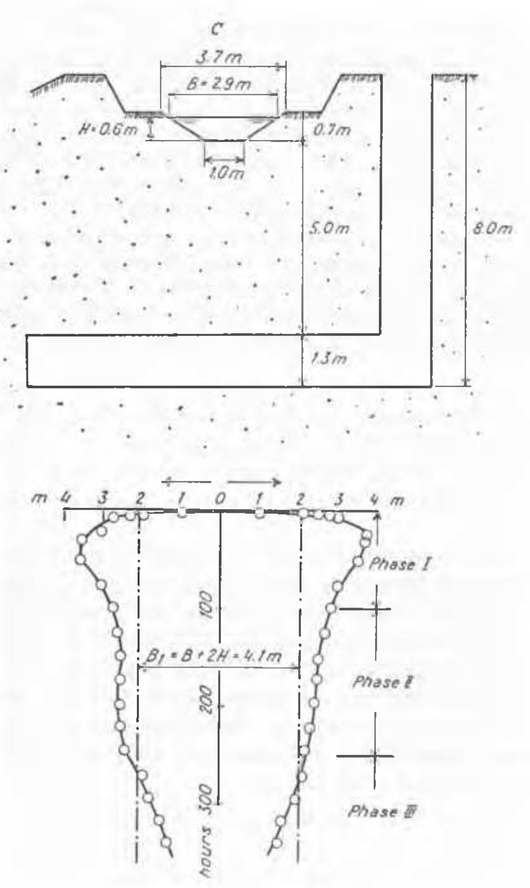

Fig. 5 Verification of the Maximum Width of the Sheet of the Seepage and its Change with Time
Vérification du courant d'eau filtrant et ses variations en fonction du temps

Conclusion

In this paper, in order to measure the coefficient of permeability of loam or other soil which has a comparatively medium permeability, a new method has been proposed: a very simple field pool may be made and enlarged at the site of an earth dam or other dam for the purpose of eliminating many great defects accompanying the conventional measuring method performed in the laboratory using a small quantity of the sample. Thus, a rational method of computing the coefficient of permeability of the foundation soil has been obtained through the hydraulic analysis formerly considered difficult. In order to verify the propriety of this method, the coefficient of permeability of the Kantō loam layer at the Asamizo depositing reservoir of the Yokohama Municipal Water Supply Bureau has been measured, and considering the verification of its order and the adaptability of the fundamental theoretical formulae, an accurate result has been obtained.

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