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The Bearing Capacity of Footings on a two-layer cohesive Subsoil

La capacité portante des empattements dans un sous-sol cohésif à deux couches

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Summary

Fellenius' method has been used to determine the ultimate bearing capacity of long strip footings, and the bearing capacity coefficients for $\varphi = 0$ soils have been obtained:—

(1) For a footing on the surface of a layer of cohesion c_1 resting on a thick layer of cohesion c_2.

(2) For a footing on the surface of a layer in which the cohesion varies linearly with depth from c_1 at the surface to c_2 at a depth d, and resting on a thick layer of cohesion c_2.

The thickness d of the upper layer is expressed in terms of the width of the footing $2b$.

Sommaire

Pour déterminer la capacité portante des semelles de longueur infinie on s'est servi de la méthode d'analyse de *Fellenius*. On a déterminé les coefficients de capacité portante pour les sols dont $\varphi = 0$.

1° Quand le niveau de la fondation se trouve à la surface d'une couche de cohésion c_1 et d'une épaisseur d qui repose sur une épaisse couche de cohésion c_2.

2° Quand le niveau de la fondation se trouve à la surface d'une couche dont la cohésion varie linéairement en fonction de la profondeur de c_1 à la surface supérieure à c_2 à une profondeur d, et qui repose sur une épaisse couche de cohésion c_2.

L'épaisseur d est calculée en fonction de la largeur de l'empattement 2_b.

Introduction

The ultimate bearing capacity of a long strip footing on the surface of a cohesive soil ($\varphi = 0$) is a constant factor multiplied by the cohesion, i.e.

$$q = c \cdot N_c$$

The values of the factor N_c vary within limits according to the method of analysis used, and may be said to be between 5.1 (*Prandtl*, 1920, *Skempton*, 1951) and 5.7 (*Terzaghi*, 1951). *Fellenius*' method (*Fellenius*, 1929) gives a value of 5.5 and this method has been used in the following analysis as it gives a value about midway in the range of bearing capacity factors for homogeneous clays, and is also easier to handle in the mathematical analysis.

Fellenius' method assumes that the slip surface is cylindrical, and hence in considering a section of unit thickness, the problem becomes a two dimensional one with a circular arc.

Case 1—A Layer of Cohesion c_1, and Thickness d Resting on a Thick Layer of Cohesion c_2

Method of Analysis

For a circle with centre 0 (Fig. 1) the condition of limiting equilibrium is obtained by calculating the moments of the forces about 0, and we have:

$$2qb\,(r \cdot \sin\theta - b) = 2r^2\theta c + 2r^2 \cdot nc \cdot \cos^{-1}(\cos\theta + d/r) \quad (1)$$

where q is the load per unit area on the footing. If the lengths are written in terms of b, so that $r' = r/b$ and $d' = d/b$, then:

$$N_c = q/c = r'^2 \left\{ \frac{\theta + n \cdot \cos^{-1}(\cos\theta + d'/r')}{r' \cdot \sin\theta - 1} \right\} \quad \ldots \quad (2)$$

In this equation, r' and θ are variables and n and d' are parameters whose values are fixed by the conditions, and N_c is the bearing capacity factor required.

Hence, for a minimum value of N_c corresponding to the circle giving the minimum bearing capacity:

$$\frac{\partial N_c}{\partial \theta} = \frac{\partial N_c}{\partial r'} = 0 \quad .. \tag{3}$$

For the homogeneous case ($n = 0$) this equation may be solved analytically and gives a value of $N_c = 5.51$.

The parameter n may have any value from -1 to ∞, and when it is not zero, the analytical solution of equation (3) is difficult, and the solutions for different values of n and d' were obtained by calculating N_c by means of equation (2) for different values of r', θ for fixed values of n, d'. Curves of

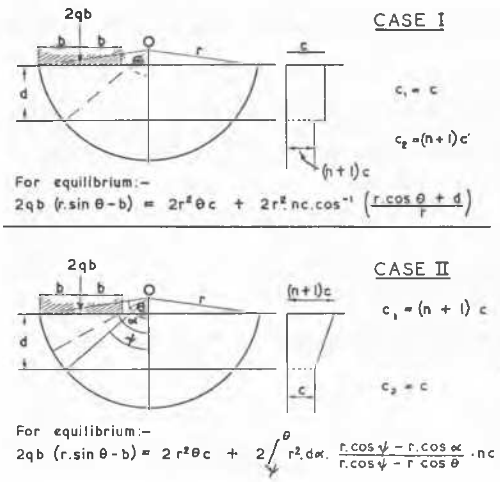

Fig. 1 Method of slip circle analysis
La méthode d'analyse par les cercles de glissement

N_c against θ for fixed values of r', and N_c against r' for fixed values of θ were drawn. From the series of curves of N_c against θ, the values of r' and θ when $\frac{\partial N_c}{\partial \theta} = 0$ were obtained and similarly the values of r' and θ when $\frac{\partial N_c}{\partial r'} = 0$.

Finally, the values of r', θ to give $\frac{\partial N_c}{\partial \theta} = 0$ and $\frac{\partial N_c}{\partial r'} = 0$ were plotted, and at the point of intersection, the values of r' and θ for the required condition $\frac{\partial N_c}{\partial r'} = \frac{\partial N_c}{\partial \theta} = 0$ were obtained and the minimum value of N_c calculated by substituting these values of r', θ in equation (2).

Results of Analysis

The values of the bearing capacity factor N_c obtained as described above are given in Fig. 2. When the cohesions c_1 and c_2 and the thickness d of the upper layer have been determined, it is a simple matter to obtain the value of N_c for the value of c_2/c_1 which has been determined and the value of d/b which is required. The ultimate bearing capacity is $c_1 \cdot N_c$ and the bearing capacity of footings of different widths can be determined quickly.

The following points in connection with these curves are worth noting:—

(i) When d/b is large, the condition represents a homogeneous material, with cohesion c_1 and the value of N_c is 5.51.

(ii) When d/b is zero the condition represents a homogeneous material with cohesion c_2 and the straight line through the origin passes through $N_c = 5.51$ when $c_2/c_1 = 1$, and in effect represents the homogeneous condition when $q = 5.51\,c_2$.

(iii) When c_2/c_1 is less than 1, the lower layer is weaker than the upper one. The value of N_c for any given value of d/b increases as c_2/c_1 increases until it reaches the limiting value of 5.51, and it then remains at this value. At this limiting point, the slip circles lie wholly within the upper layer, and any further relative increase in the strength of the lower layer does not increase the bearing capacity.

(iv) When c_2/c_1 is greater than 1, the lower layer has a greater strength than the upper one. For a particular value of d/b, the bearing capacity rises as the relative strength of the lower layer rises, but at the same time a smaller proportion of the total length of the slip surface passes into the lower, stronger layer. At a limiting point, the slip circle becomes tangential to the upper surface of the lower layer, and after this, any further increase in the strength of the lower layer

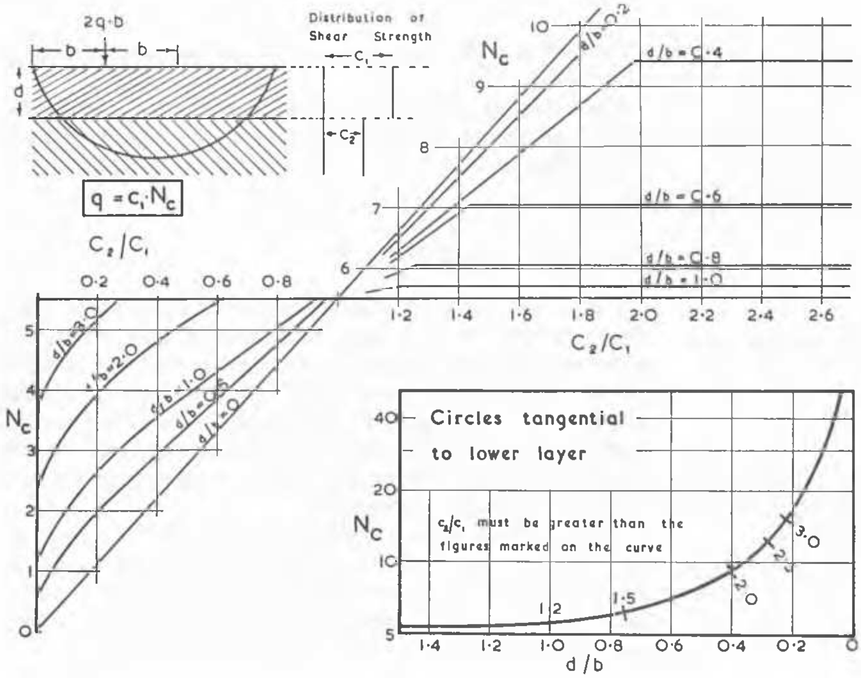

Fig. 2 Bearing capacity factors
Les facteurs de la force portante

will not influence the bearing capacity, as the slip surface will continue to be tangential to it. This is represented by the sudden change in the slope of the lines for values of c_2/c_1 greater than 1. After this sudden change in the slope of the line, the value of N_c for any particular value of d/b is unaltered as c_2/c_1 is increased, and the line is then parallel to the c_2/c_1 axis.

(v) For the homogeneous case, the depth of penetration of the slip surface is $1.3b$, so that if the lower layer is stronger than the upper one, it will not influence the bearing capacity if the thickness of the upper layer is $1.3b$ or more.

(vi) A curve is plotted in Fig. 2 giving values of the factor N_c for circles which are limited by hard layers. In order to cause the slip circles to be tangential to the lower layer for different values of the ratio d/b, the value of c_2/c_1 must be greater than certain limiting figures which are marked on the curve.

Case II—A Layer in which the Cohesion Varies Linearly with Depth from c_1 at the Surface to c_2 at a Depth d and Resting on a Thick Layer of Cohesion c_2

Method of Analysis

The method of analysis in this case is similar. The moments of the forces about 0 (Fig. 1—Case II) are:
Resisting moment of shear forces =

$$2r^2\theta c + 2\int_{\psi}^{\theta} r^2 \cdot d\alpha \cdot nc \cdot \frac{r \cdot \cos\psi - r\cdot\cos\alpha}{r\cdot\cos\psi - r\cdot\cos\theta} =$$

$$= 2r^2 c\left[\theta + \frac{n}{\cos\psi - \cos\theta}\{(\theta-\psi)\cos\psi - (\sin\theta - \sin\psi)\}\right]$$

Disturbing Moment =

$$2qb\,(r\cdot\sin\theta - b)$$

Writing all lengths in terms of b, so that $r' = r/b$ and $d' = d/b$ and equating the moments:

$$2q\,(r'\cdot\sin\theta - 1) =$$

$$= 2r'^2 c\left[\theta + \frac{n}{\cos\psi - \cos\theta}\{(\theta-\psi)\cos\psi - (\sin\theta - \sin\psi)\}\right] \quad (4)$$

Now $d' = r'(\cos\psi - \cos\theta)$
so that $\cos\psi - \cos\theta = d'/r'$
and $\psi = \cos^{-1}(d'/r' + \cos\theta)$

Equation 4 thus becomes

$$N_c = q/c =$$

$$\frac{r'^2}{(r'\cdot\sin\theta - 1)}\left[\theta + \frac{nr'}{d'}\{(\theta-\psi)\cos\psi - (\sin\theta - \sin\psi)\}\right] \quad (5)$$

In the same way as before, the variables are r', θ and the parameters are n, d' and ψ is a function of θ, r' and d'.

The minimum value of N_c for given values of n and d' is given by:

$$\frac{\partial N_c}{\partial\theta} = \frac{\partial N_c}{\partial r'} = 0$$

as before and the solutions were obtained in a similar way.

Results of Analysis

The values of the bearing capacity factor N_c for these conditions is given in Fig. 3. The form of the curves is similar to that in Case I, but there are important differences.

As before, large values of d/b represent the homogeneous case with cohesion c_1 and $N_c = 5.51$ and if d/b is zero, this represents a homogeneous case with cohesion c_2 and $q = 5.51\,c_2$.

When c_2/c_1 is less than 1, which is when the lower layer is weaker than the upper one, the value of N_c for any value of

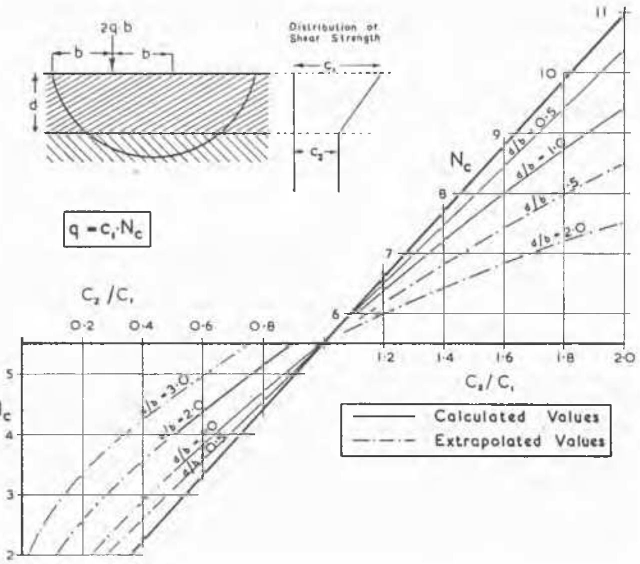

Fig. 3 Bearing capacity factors
Les facteurs de la force portante

d/b comes to a limiting value of 5.51 for the same reason as with Case I.

Since there is no abrupt change of strength at any depth, there is no abrupt change in the slope of the curve for values of c_2/c_1 greater than 1.

Practical Applications

The $\varphi = 0$ method of analysis is extensively used in Great Britain, where there are many instances of a close correlation between the figures obtained by this method and observations of full scale effects in practice (5). By this extension of the analysis for homogeneous soil conditions, the ultimate bearing capacity for many conditions encountered naturally can be estimated quickly without recourse to a slip circle analysis which takes a considerable time.

The figures at present obtained apply only to long strip footings on the surface. For footings below the surface, it is probably a close approximation to assume that the increase in the bearing capacity factor a depth-width ratio D/b follows that given by *Skempton*, 1951, which is as follows:—

Increase in bearing capacity factor with depth

D/b	N_{cD}/N_c
0	1.00
1	1.15
2	1.24
4	1.36
6	1.43
8 or more	1.46

Where D is the depth of the footings below the surface, $2b$ is the width of the footing, N_{cD} is the bearing capacity factor at the depth D, and N_c that at the surface.

For rectangular or square footings of width $2b$ and length L, the bearing capacity factor could also be increased, as suggested by *Skempton*, 1951, as follows:

$$\frac{N_c \text{ rectangle}}{N_c \text{ strip}} = 1 + 0.2\left(\frac{2b}{L}\right)$$

Having obtained the final value of N_c in this way, the ultimate bearing capacity is:

$$q = c \cdot N_c + \gamma \cdot D$$

where γ is the density of the soil.

Example of Use of Method

Suppose there is a layer of clay of cohesion 0.8 tons/sq.ft. of thickness 10 ft. lying upon a thick bed of clay of cohesion 1.2 tons/sq.ft. and it is required to estimate the ultimate bearing capacity of a footing 20 ft. long and 10 ft. wide founded 6 ft. below the surface. The density of the clay is 0.05 ton/cu.ft.

In this case:—

$c_1 = 0.8$ $\quad c_2 = 1.2$ $\quad c_2/c_1 = 1.5$
$b = 5$ ft. $\quad d = 4$ ft. $\quad d/b = 0.8$

From the curves in Fig. 2, $N_c = 6.1$.
The ratio D/b is 6/5, and so N_c is increased in the ratio of 1.18 giving

$$N_c = 1.18 \times 6.1 = 7.2$$

The ratio of width to length,

$$\frac{2b}{L} = \frac{1}{2}$$

and N_c is therefore again increased in the ratio:

$$1 + 0.2 \times \tfrac{1}{2} = 1.1$$

giving the final value of N_c as $1.1 \times 7.2 = 7.9$.
The estimated value of the ultimate bearing capacity of this footing is therefore:

$$\begin{aligned} c_1 \cdot N_c + \gamma \cdot D &= 0.8 \times 7.9 + 0.05 \times 6 \\ &= 6.3 + 0.3 \\ &= 6.6 \text{ tons/sq.ft.} \end{aligned}$$

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