The Determination of the Dynamical Characteristics of Soils, a Good Help in the Calculation of Dynamically Excited Foundations

Détermination des caractéristiques dynamiques des sols considérée comme un auxiliaire pour le calcul de fondations sollicitées par des efforts dynamiques

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Summary

The modern calculation methods applied for dynamically excited foundations require increased consideration of the dynamical behaviour of the soil. Though usually dynamical deformations of the soil are not proportional to its reaction forces, for characterizing the dynamical foundation properties of a soil system a dynamical stiffness or *bedding value* may be introduced; it is defined by the slope of the tangent to the dynamically determined stress strain-diagram. The following paper shows three different methods for calculating this dynamical bedding value, utilising investigations by means of a heavy vibrator. A numerical example illustrates the methods and gives an idea of their accuracy.

Sommaire

Le procédé moderne de calcul des fondations pour des systèmes émettant des forces dynamiques demandent de plus en plus que l'on considère les qualités dynamiques du sol. Bien que les déformations du sol ne soient généralement pas proportionnelles à ses forces de réaction, pour caractériser les propriétés dynamiques du sous-sol un *module instantané* de déformation pourrait être introduit, défini par la pente de la tangente dans le diagramme pression/tension déterminé par moyens dynamiques. Cette communication décrit trois méthodes de détermination du module instantané de déformation dynamique du sol par des essais avec un vibrateur lourd. Un exemple numérique illustre les phases de ces calculs et donne un aperçu de leur précision.

Dynamical soil investigations, performed during several years, have shown that the dynamical stress strain-diagram (called characteristic in dynamics) of all examined soils is a concave curve with linear tangent for high values (Fig. 1), i.e. the deformation of the soil due to increasing dynamical pressure increases more than proportionally; in other words, within certain limits the soil becomes weaker with increasing dynamical stress. Such characteristics are called sublinear. As in the stress-strain-diagram pressures are plotted against amplitudes, the pressures must be composed of static (weight of exciter per basement area) and dynamic pressures (exciting force per base-contact area).

Large scale test series have proved that the existence of a sublinear characteristic (Fig. 1) explains all so far inexplicable phenomena, such as: (1) decrease of self-frequency at increasing exciting force, (2) decrease of self-frequency at increasing static pressure, (3) increasing of self-frequency at increasing base area with constant pressure.

Thus, this characteristic seems to present a useful means for describing dynamical soil properties. From the characteristic we may derive its tangent, a value to be called "dynamical bedding value", because its dimension (kg/cm³) is the same as the dimension of the static bedding value. The dynamical bedding value does not aim to be a pure soil constant, because it depends, besides on properties of soil, on the total stress σ.

The following paper proposes simple methods for the determination of the characteristic derived from investigations by means of a vibrator. For this purpose it is necessary to state resonance-curves on each soil with at least three different exciting forces, i.e. values of excentricity.

A sublinear characteristic may be expressed by the formula

$$\sigma = ax + \frac{bx}{d+x} \qquad (1)$$

whereas the stresses σ derive from

$$\sigma = \sigma_{st}\,(1 + 0.04\,\varepsilon n^2); \qquad (2)$$

$\sigma_{st} = \dfrac{G}{F}$ denotes the statical stress, $\varepsilon = \dfrac{m_0 r g}{G}$ the excentricity factor and n the frequency of exciting. In such a manner from three tests three value-paires σ, x can be obtained where x signifies the peak-amplitude of the vibration of the soil surface.

To determine the constants a, b and d of equation (1), three equations are available; their solution gives

$$\left.\begin{aligned} a &= \frac{\sigma_1}{x_1} - \frac{b}{d+x_1} \\ b &= \frac{\dfrac{\sigma_2}{x_2} - \dfrac{\sigma_1}{x_1}}{\dfrac{1}{d+x_2} - \dfrac{1}{d+x_1}} \\ d &= \frac{A x_3 (x_1 - x_2) - x_2 (x_1 - x_3)}{x_1 - x_3 - A (x_1 - x_2)}, \text{ where} \\ A &= \frac{\dfrac{\sigma_3 x_1}{\sigma_1 x_3} - 1}{\dfrac{\sigma_2 x_1}{\sigma_1 x_2} - 1} \end{aligned}\right\} \quad (3)$$

The following example should explain this calculation method. From three tests, performed with a vibrator of $G = 2.7$ tons weight, ground surface $F = 1$ m² and the excentricity-factor $\varepsilon = \dfrac{\gamma}{133}$ cm, where γ ranges from $\gamma = 1$ for the first to $\gamma = 3$ for the third test, three resonance-curves, shown in Fig. 2, are measured, with the peak amplitude $x_1 = 0.05$, $x_2 = 0.04$ and $x_3 = 0.03$ cm. According equation (2) the stresses are

$\sigma_1 = 0.505$ kg/cm²
$\sigma_2 = 0.440$ kg/cm²
$\sigma_3 = 0.370$ kg/cm²

The evaluation of the formulas (3) gives
$A = 2{,}3$, further $d = 0.037$ cm, $b = 0.48$ kg/cm² and finally $a = 4.5$ kg/cm³.

From this the equation of the characteristic is found to

$$\sigma = 4.5\,x + \frac{0.48\,x}{x + 0.037} \qquad (4)$$

Now it may be shown, how using equation (4) the dynamic bedding value, i.e. the derivation of the characteristic, can be

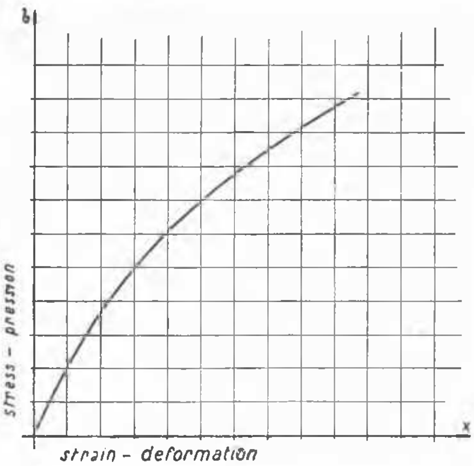

Fig. 1 Sublinear Characteristic of Soil
Caractéristique sublinéaire du sol

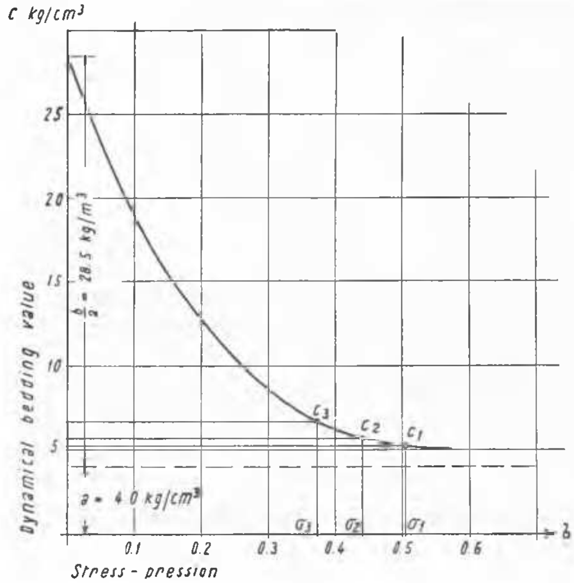

Fig. 2 Dynamical Bedding Value Plotted Against Total Stress
Module instantané de déformation dépendant de la pression totale

found for the purpose of calculating a machine foundation. Its statical pressure may be $\sigma_{st} = 1.1$ kg/cm², the excentricity factor $\varepsilon = 0.1$ cm and the normal frequency $n = 5$ cycles/sec. Hence it follows from equation (2)

$\sigma = 1.1\,(1 + 0.04\,\varepsilon n^2) = 1.21$ kg/cm².

From equation (1) the amplitude x can be calculated according to the stress σ

$$x = x(\sigma) = \frac{\sigma - b}{2a} - \frac{d}{2} + \sqrt{\left(\frac{\sigma - b}{2a} - \frac{d}{2}\right)^2 + \frac{\sigma d}{a}} \qquad (5)$$

With the above calculated values a, b, d of the characteristic and $\sigma = 1.21$, equation (5) gives $x = 0.18$ cm. The tangent (derivation) of the characteristic in the point $x = 0.18$ is found by

$$\frac{d\sigma}{dx} = C = a + \frac{bd}{(x+d)^2} = 4.88 \text{ kg/cm}^3;$$

that is the dynamical bedding value, applicable to the further calculation of a machine foundation.

More quickly, but less exactly, the constants of the characteristic and the bedding value C, according to the actual stress σ can be found on the supposition, that the characteristic consists of short straight-lines, that means, that for constant frequency and therefore constant exciting force an harmonic vibration is supposed.

Then, the self-frequency $n_e = \dfrac{1}{2\pi}\sqrt{\dfrac{c}{m}}$ and from $c = CF$, $m = \dfrac{G}{g}$ and $\sigma_{st} = \dfrac{G}{F}$ it derives

$$C = \frac{4\pi^2}{g}\,\sigma_{st}\,n_e^2 = 0.04\,\sigma_{st}\,n_e^2 \;.. \qquad (6)$$

Plotting the values C according to equation (6) against the total stress σ a curve of exponential type is obtained (Fig. 3); its equation may be written

$$C(\sigma) = a + \frac{b}{d}\,e^{-a\sigma} \qquad (7)$$

The constants of this equation signify

a: value of the asymptote for $\sigma = \infty$

$a + \frac{b}{d}$: value of C for $\sigma = 0$

α: parameter of the curve.

a can be easily found graphically from the drawing.

From equation (7) $\alpha = \dfrac{ln \dfrac{C_1 - a}{C_2 - a}}{\sigma_2 - \sigma_1}$ and $\dfrac{b}{d} = (C_3 - a) e^{\alpha \sigma_3}$

According to the simplifications used for this method, the result is not very exact. The three peak amplitudes in Fig. 2 appear at the frequencies $n_1 = 22$ Hz, $n_2 = 23$ Hz, $n_3 = 25$ Hz.

Hence from equation (6)

$C_1 = 0.04 \cdot 0.27 \cdot 484 = 5.22$ kg/cm³ and $\sigma_1 = 0.505$ kg/cm²
$C_2 = 0.04 \cdot 0.27 \cdot 529 = 5.70$ kg/cm³ $\sigma_2 = 0.440$ kg/cm²
$C_3 = 0.04 \cdot 0.27 \cdot 625 = 6.75$ kg/cm³ $\sigma_3 = 0.370$ kg/cm²

Drawing these values in Fig. 3, the asymptote is found to $a = 4.0$ kg/cm³.

Further $\alpha = \dfrac{ln \dfrac{C_3 - a}{C_1 - a}}{\sigma_1 - \sigma_3} = \dfrac{ln \dfrac{2.75}{1.22}}{0.134} = 6.1$

and $\dfrac{b}{d} = (C_2 - a) e^{\alpha \sigma_3} = 24.5$ kg/cm³.

The suitable bedding value C ist obtained from equation (7) $C = 4.0 + 24.5\, e^{-6.1 \cdot 1.21} = 4.02$ kg/cm³ (the exact value was found above to be 4.88).

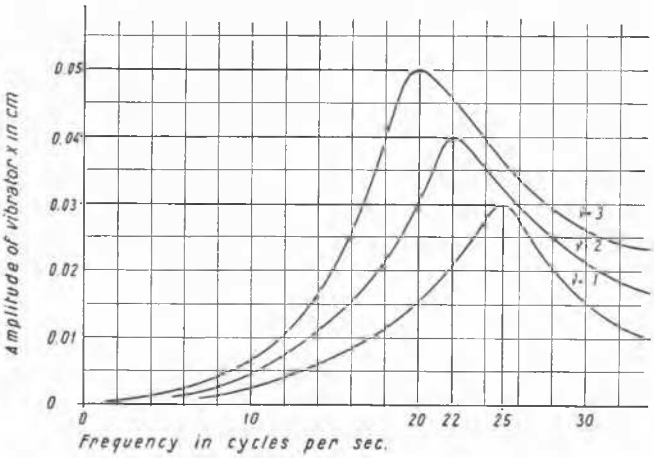

Fig. 3 Tension-Frequency-Curves with Varied Exciting Forces
Tension-fréquence-courbes avec différentes forces dynamiques

There exists a third, very rough method to determine C by the following table. The table-values C_n are found with a vibrator of $G = 2.7$ tons, $F = 1$ m², therefore $\sigma_{st} = 0.27$ kg/cm² and an exciting moment $m_0 r = 0.02$ kg/s^2, i.e. $\varepsilon = \frac{1}{133}$ cm.

Exciting frequencies n were ranging from 5 to 50 cycles/sec. Hence total stress was

$\sigma = 0.27\,(1 + 3.10^{-4} n^2)$.

Approximately C may be found using the Table 1 if the values C_n according to the indicated soil quality are reduced

$C = C_n - \beta \Delta \sigma$,

where $\Delta \sigma = \sigma - 0.27$ means the difference between the total stress of the foundation and the total stress of the vibrator. β is a factor adapted to the different soils as follows:

high cohesive soils $\beta \approx 2{,}5$ cm⁻¹
weak cohesive soils $\beta \approx 5{,}0$ cm⁻¹
cohesionless soils $\beta \approx 10{,}0$ cm⁻¹

Table 1 Values of C_n

Soil	*Self-frequency of Vibrator on the Soil; Cycles/sec*	C_n *kg/cm³*
Sand, fine, very clayey	20.0–25.5	4.0–6.5
Fine to medium sand, very clayey	23.5–27.5	5.5–7.5
Medium sand	26.5–28.5	7.0–8.0
Medium- to coarse sand, loose	28.5	8.0
Medium- to coarse sand, compacted	36.0	13.0
Gravel	31.0	9.5
Gravel, dry, argillaceous	31.0–39.0¹)	9.0–20.0¹)
Clay, wet	23.5	5.5
Clay, dry	32.5	10.5
Marl, wet	27.5	7.5
Marl, dry	33.0	11.0

¹) exceptionally high compacted

For the above mentioned example, C can be found by using Table 1 in the following way:

Soil kind under investigation: compacted medium to coarse sand

$C_n = 13$ kg/cm³; $\beta = 10$ cm⁻¹; $\sigma = 1.1$ kg/cm²

$\sigma = 1.1 - 0.27 = 0.83$ kg/cm²; $\beta \Delta \sigma = 8.3$ kg/cm³

$C = C_n - \beta \Delta \sigma = 4.7$ kg/cm³ in fair agreement with the exact result.