On the Bearing Power of Soil under a Uniformly Distributed Circular Load

De la force portante du sol sous une charge circulaire uniforme

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Summary

The author reports of a process to determine the bearing value and the sliding surface of soil under an axial symmetric uniform circular load, under the same considerations as in a two-dimensional problem reported in a previous paper (*Mizuno*, 1948).

The transition region between the wedge of active earth pressure below the loading area and the region of passive earth pressure in the outermost part is divided into a number of small solids. For these solids the equilibrium condition of forces are applied successively and at the same time the sliding surface is drawn by means of the sliding condition, stress components being found by *Mohr*'s circle. And in such a way at last the bearing capacity can be obtained.

At the end of this paper, a diagram of the bearing power is given.

Sommaire

L'auteur rend compte d'un procédé pour déterminer la résistance du sol et sa surface de glissement, sous une charge de forme circulaire symétrique par rapport à l'axe vertical, et distribuée uniformément. Ce résultat s'obtient par la même considération que dans le cas du problème à deux dimensions rapporté précédemment (*Mizuno*, 1948).

La région de transition, entre le coin de pression active sous la base d'appui et la région de pression passive le plus extérieure, est divisée en beaucoup de petits solides. On applique successivement aux dits solides les conditions d'équilibre et de glissement, toutes les composantes des efforts étant données par le cercle de *Mohr*. Ainsi, on détermine finalement la surface de glissement et la capacité portante.

L'auteur donne à la fin de ce traité un diagramme de charge portante.

General Assumptions

In Fig. 1, let p be the load intensity from a structure having the diameter $2b$ and q be the surcharge intensity outside the structure base.

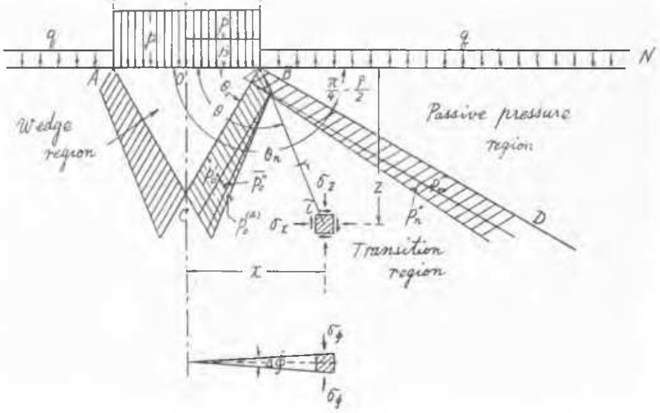

Fig. 1 General Assumptions on Loads and Stresses
Suppositions générales concernant charges et efforts

The author assumes that, below the bearing area, the wedge of active earth pressure would take place, although some authors suggest other shapes of wedge (*K. Terzaghi*, 1948; *J. Ohde*, 1952). The passive earth pressure region is assumed to arise in the outermost part. Then, if ρ is the angle of internal friction,

$$\angle ABC = \theta_0 = \frac{\pi}{4} + \frac{\rho}{2} \tag{1}$$

and

$$\angle NBD = \pi - \theta_n = \frac{\pi}{4} - \frac{\rho}{2} \tag{2}$$

At the instant of sliding, the uniform load p is considered to be divided into the following two parts: p' which is resisted by the uniformly distributed reaction p_0' on the conical surface BC and AC, and p'' which is resisted by the triangular reaction $\bar{p}_0''$ on the same surface. We have

$$p = p' + p'' \tag{3}$$

On the wedge surface, the active earth pressure $p_0^{(a)}$ due to the weight of soil is to be added to the above reactions.

Stresses along BC and BD

As, at the instant of sliding, p_0' and $p_0'' = \bar{p}_0'' + p_0^{(a)}$ along BC should make an angle ϱ with the normal to this line, we obtain the following values by the equilibrium conditions of the cone ABC.

$$p_0' = p' \cot \theta_0 \tag{4}$$

$$p_0'' = \frac{3p''r}{b} \cot \theta_0 \cos \theta_0 \tag{5}$$

$$p_0^{(a)} = \gamma r \cos \theta_0 \;\; \cdot\cdot \tag{6}$$

Where, γ is the unit weight of soil.

Similarly p_n' and p_n'' along BD, the former being the stress due to the surcharge q and the latter the stress due to the weight of soil, should make an angle ϱ with the normal to BD. Then they are given as follows:

$$p_n' = -q \cot \theta_n \tag{7}$$

$$p_n'' = -\gamma r \cos \theta_n \tag{8}$$

Equilibrium Conditions in the Axial Symmetric Problem

If we designate the stress components in a cylindrical coordinate system (x, z, φ) by σ_x, σ_Z, τ and σ_φ as shown in Fig. 1, we have the following equations of equilibrium, compression being taken as positive.

$$\left.\begin{aligned} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau}{\partial z} + \frac{\sigma_x - \sigma_\varphi}{x} &= 0 \\ \frac{\partial \tau}{\partial x} + \frac{\partial \sigma_Z}{\partial z} + \frac{\tau}{x} - \gamma &= 0 \end{aligned}\right| \tag{9}$$

For the active earth pressure, the stress components are

$$\left.\begin{aligned} \sigma_x &= k(p + \gamma z) \\ \sigma_Z &= p + \gamma z \\ \tau &= 0 \end{aligned}\right| \tag{10}$$

where

$$k = \frac{1 - \sin \varrho}{1 + \sin \varrho} \tag{11}$$

Therefore, from (9) we get

$$\sigma_\varphi = \sigma_x \tag{12}$$

For the passive earth pressure, similarly

$$\left.\begin{aligned} \sigma_x &= \frac{1}{k}(q + \gamma z) \\ \sigma_Z &= q + \gamma z \\ \tau &= 0 \end{aligned}\right| \tag{13}$$

$$\sigma_\varphi = \sigma_x \tag{14}$$

Accordingly also in the transition region, we might assume appropriately

$$\sigma_\varphi = \sigma_x \tag{15}$$

in order that the stresses be continuous, although a rigorous value of σ_φ is not known in this region.

Next, we assume approximately that the stresses along any radial line through B consist of two terms; the one, independent of the radial distance r from B, and the other, proportional to r. Strictly speaking, this is not correct in this case because these stresses do not satisfy equation (9).

However, the above two fundamental assumptions have been proved to be practically adequate by numerical calculations, in which the equilibrium conditions of moments are almost satisfied. An example will be shown later.

Process of Finding p'

In this case, the own weight γ of soil is not taken into account, and the stresses along any radial line are considered to distribute uniformly.

In Fig. 2, let CSD be a sliding surface. A solid $BRST$, whose base RST is tangent to the sliding surface at S and whose vertex angle is equal to a small angle $\Delta\theta$, is supposed to be bound with two vertical planes passing through the symmetrical axis OC and making a small angle $\Delta\varphi$ with each other. In these two vertical planes there are no shearing stresses. $<RBS$ and $<SBT$ are taken $\Delta\theta/2$ each. The transition region is divided into a number of such solids.

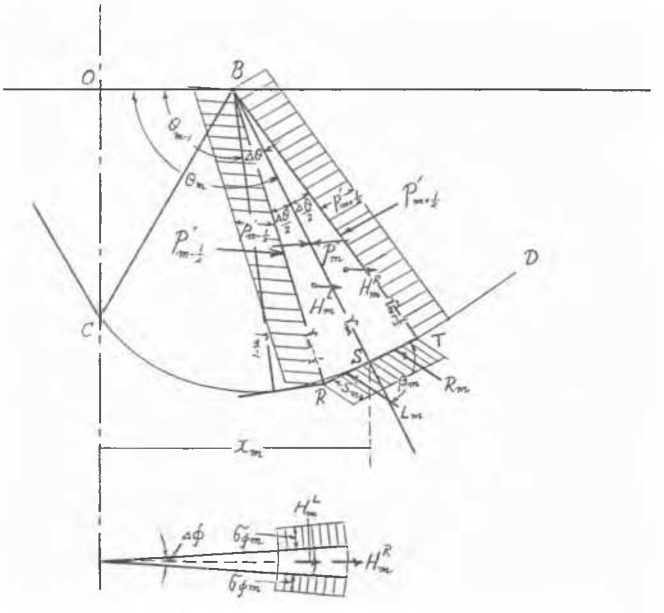

Fig. 2 Stresses on a Divided Solid
Efforts sur un solide divisé

The forces acting upon the solid BRS are $P'_{m-\frac{1}{2}}$ and P'_m which are respectively resultants of uniformly distributed stresses on the surfaces BR and BS, the reaction L_m along the sliding surface, and the horizontal force H_m^L resulted from the circumferential stress σ_φ. Likewise upon the solid BST will act P'_m, $P'_{m+\frac{1}{2}}$, R_m and H_m^R.

Now, for the convenience, the reaction stress s_m and the circumferential stress $\sigma_{\varphi m}$ are assumed constant all over these two small solids. As the horizontal component of σ_φ is

$$t = 2\sigma_\varphi \sin \frac{\Delta\varphi}{2} \doteqdot \sigma_\varphi \Delta\varphi \tag{16}$$

H_m^L and H_m^R are respectively

$$\left.\begin{aligned} H_m^L &= \sigma_{\varphi m} \Delta\varphi(\Delta BRS) \\ H_m^R &= \sigma_{\varphi m} \Delta\varphi(\Delta BST) \end{aligned}\right\} \tag{17}$$

L_m and R_m are respectively

$$\left.\begin{aligned} L_m &= s_m\,(\text{area } RS) \\ R_m &= s_m\,(\text{area } ST) \end{aligned}\right\} \tag{18}$$

And $P'_{m-\frac{1}{2}}$, P'_m, $P'_{m+\frac{1}{2}}$ are

$$\left.\begin{aligned} P'_{m-\frac{1}{2}} &= p'_{m-\frac{1}{2}}\,(\text{area } BR) \\ P'_m &= p'_m \quad (\text{area } BS) \\ P'_{m+\frac{1}{2}} &= p'_{m+\frac{1}{2}}\,(\text{area } BT) \end{aligned}\right\} \tag{19}$$

Since s_m and $\sigma_{\varphi m} = \sigma_{xm}$ are the components of p'_m, these are obtained by *Mohr*'s circle from p'_m as shown in Fig. 3. At the same time the angle β_m at which the sliding surface intersects with the radial line BS, is given by the same stress circle.

$P'_{m-\frac{1}{2}}$ is known by the foregoing computation. If a trial value of the angle δ_m which p'_m makes with the normal to BS line is selected, $\sigma_{\varphi m}$, s_m and β_m are determined, taking p'_m as a unit, and areas BRS, RS, BS as well as forces H^L_m, L_m, P'_m are obtained. Then from the force polygon, by means of the equilibrium condition, p'_m and δ_m values are computed, and this result is compared with the first trial value. According to such a trial calculation, we get the final values of p'_m and δ_m. In the next course, H^R_m, R_m and $P'_{m+\frac{1}{2}}$ are calculated.

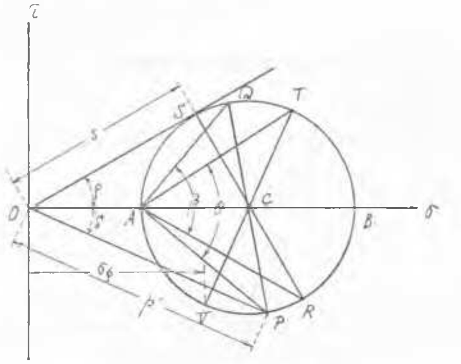

Fig. 3 Expression of Stress-Components by *Mohr*'s Circle
Expression de composantes des efforts par cercle de *Mohr*

Thus, starting from θ_0, we can obtain successively P' and the sliding surface until θ_n at last. At θ_n, of course, the directions of P'_n and the sliding surface must coincide with those of the passive pressure region. The computed P'_n being put equal to the passive pressure, the bearing value p' is obtained as a function of the surcharge q.

Besides, P'_m must act at the centroid of area BS, or the pressure line must pass through the centroid of any radial surface. This condition is necessary for the equilibrium of moments

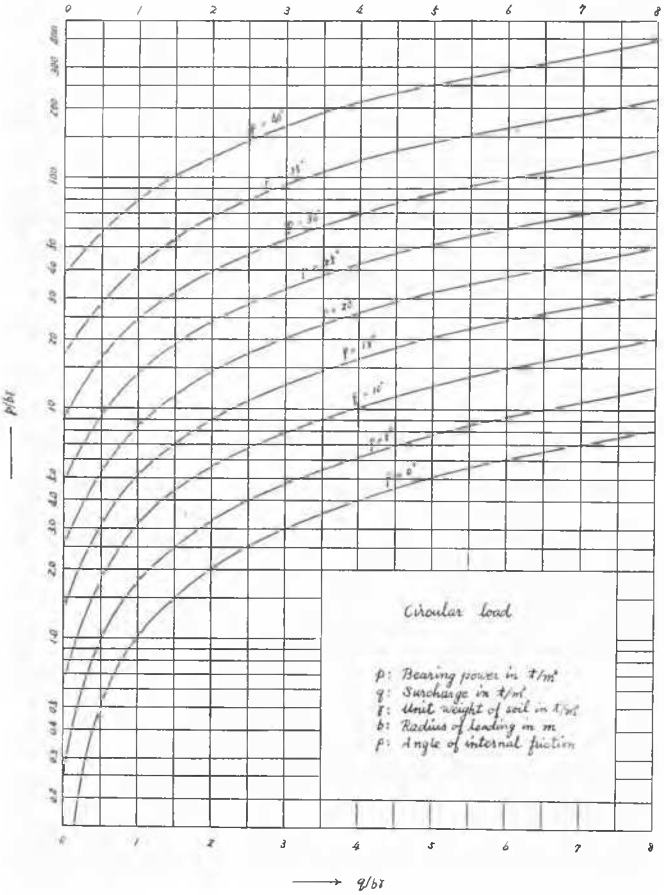

Fig. 5 Diagram of Bearing Power
Diagramme de la force portante

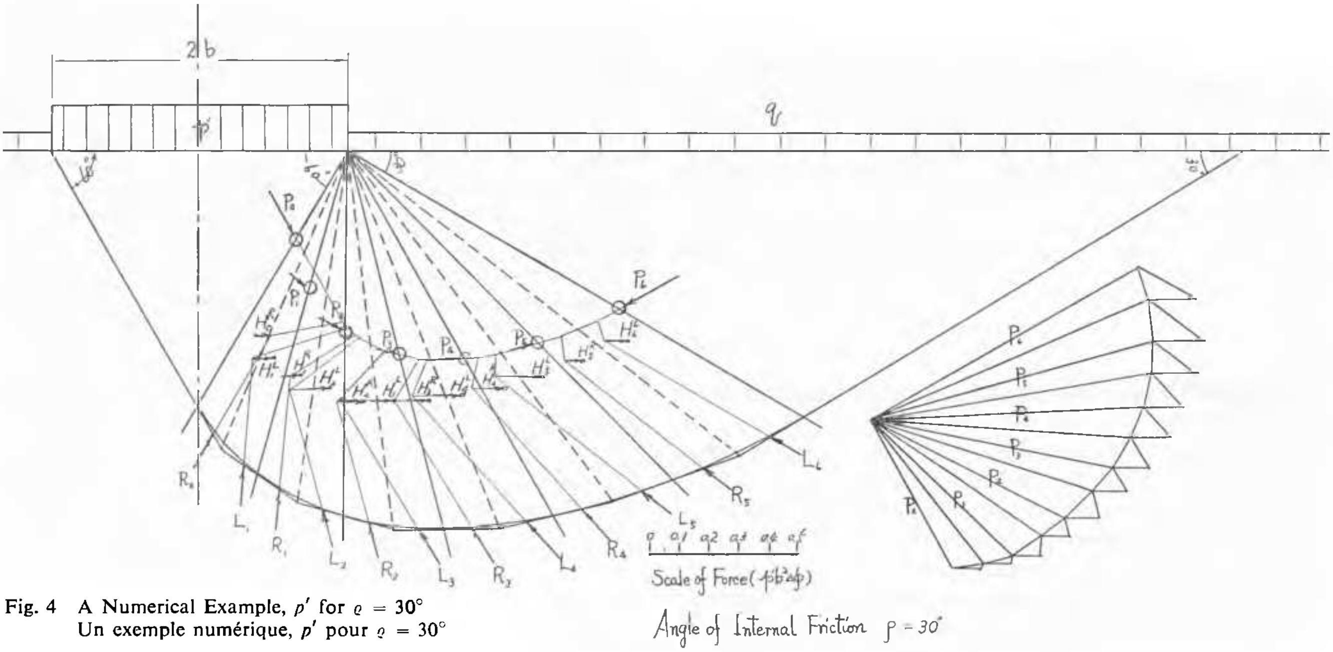

Fig. 4 A Numerical Example, p' for $\varrho = 30°$
Un exemple numérique, p' pour $\varrho = 30°$

and is seen to be almost satisfied in the numerical example in Fig. 4.

This figure is a graphic representation of p'-calculation for $\varrho = 30°$.

Process of Finding p''

Here, the surcharge q ist out of consideration and only the resistance due to soil weight γ is required. In this, all the stresses are proportional to the radial distance from B, and so the calculation is more troublesome than in the preceding case. Moreover the weight of every divided solid must be taken into the equilibrium condition.

At the beginning of calculation, p'' needs to be assumed as a multiple of $b\gamma$. A good trial value of p'' is about half of the corresponding p'' value in the two-dimensional case and it is improved by repeated calculations.

Diagram of Bearing Capacity

An exact value of the total bearing capacity would not agree with the simple sum of p' and p'' found respectively in the two foregoing paragraphs, as known from the author's previous paper for two-dimensional case (*T. Mizuno*, 1948). But since these sums will give somewhat safer values, they are plotted as bearing power values in Fig. 5 for various ϱ values.

Likewise as in the previous paper, for cohesive soil we shall be able to obtain the bearing value from this diagram by introducing the *Caquot* method.

References

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