The Efficacy of Toe Drains in Controlling Seepage Uplift in Layered Pervious Foundations

L'efficacité des drains de pied pour le contrôle de la filtration au travers des fondations perméables et stratifiées

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Summary

Mathematical solutions are obtained for the problem of seepage through a two layered, pervious foundation, covered by an impervious top blanket, to a semi-circular toe drain. For cases where the lower pervious layer is somewhat more pervious than the upper pervious layer, the efficacy of the drain is seriously reduced.

Sommaire

L'auteur a obtenu des solutions mathématiques au problème de la filtration vers un drain semi-circulaire de pied au travers d'une double couche protégée par une couverture imperméable. Dans les cas où la couche inférieure est un peu plus perméable que la couche supérieure l'efficacité du drain est sensiblement réduite.

The relief of seepage uplift pressures in deep pervious foundations, covered by a natural, relatively impervious blanket downstream from a water retaining structure, has received increasing attention of soil engineers during the past few years (*Middlebrooks* and *Jervis*, 1947). The means of affecting such relief are drain wells (Technical Memorandum No. 3, 304) and trench or toe drains located at the downstream toe of a levee or dam (*Barron*, 1952). The efficacy of such toe drains is the subject of this paper which is based upon a master's thesis submitted by the writer to the Massachusetts Institute of Technology.

The flood plains of rivers draining areas, formerly occupied by glaciers commonly, have an upper deposit of silt or clay overlying a pervious deposit of sands and gravels which in turn rests upon a relatively impervious base. In general, the pervious layer becomes more pervious with increasing depth.

A mathematical solution of the general case has not been obtained but may be approximated by considering the pervious portion of the foundation to be composed of two layers of different perviousness. For the case where the pervious layers are of equal thickness the solution for a semi-circular drain penetrating into the upper part of the upper pervious layer has been obtained (Fig. 1) by means of conformal mapping.

The uplift potential, measured upward from the tailwater pool, of the upper pervious layer, I, is

$$\Phi_I = \Delta + \frac{q}{k_1}\left\{\ln\left[\frac{\cosh\frac{\pi}{2h}(y-L)-\cos\frac{\pi x}{2h}}{\cosh\frac{\pi}{2h}(y+L)-\cos\frac{\pi x}{2h}}\right] + m\ln\left[\frac{\cosh\frac{\pi}{2h}(y-L)+\cos\frac{\pi x}{2h}}{\cosh\frac{\pi}{2h}(y+L)+\cos\frac{\pi y}{2h}}\right]\right\} \qquad (1)$$

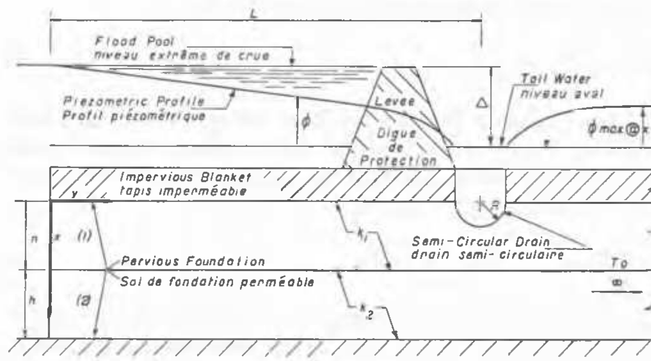

Fig. 1 Typical Section
Section typique

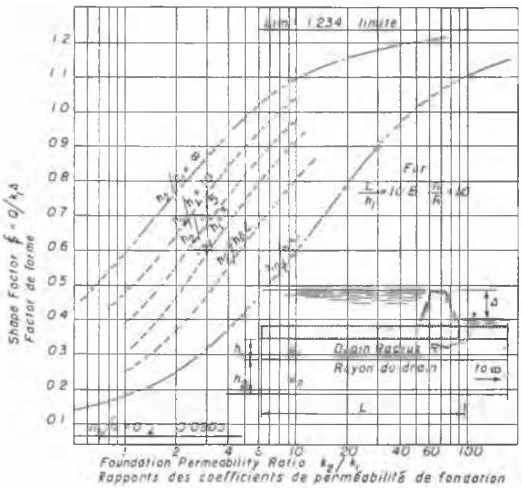

Fig. 2 Curves of Shape Factor Versus Foundation Permeability Ratio
Courbes donnant le facteur de forme en fonction du rapport des coefficients de perméabilité

and for the lower pervious layer, II, is

$$\Phi_{II} = \Delta + \frac{q}{k_2}\left\{\ln\left[\frac{\cosh\frac{\pi}{2h}(y-L)-\cos\frac{\pi x}{2h}}{\cosh\frac{\pi}{2h}(y+L)-\cos\frac{\pi x}{2h}}\right]\right\}(1-m) \quad (2)$$

where Δ is the net head between tailwater and flood pool
k_1 and k_2 are the foundation permeability coefficients of pervious layers I and II respectively
q is the flux strength of the toe drain
m is the potential reflection coefficient of the interface between layers I and II.

Now $m = \frac{(k_1-k_2)}{(k_1+k_2)}$

and at the toe drain, where $(y-L) = R$ and $(y+L) \cong 2L$, the uplift Φ_1 is zero and so

$$q = \frac{-\Delta k_1}{2\left\{\ln\left[\frac{\sinh\frac{\pi R}{4h}}{\sinh\frac{\pi L}{2h}}\right] + m\ln\left[\frac{\cosh\frac{\pi R}{4h}}{\cosh\frac{\pi L}{2h}}\right]\right\}} \quad (3)$$

The seepage into the toe drain per unit length of drain is

$$Q = 2\pi q = k_1 \Delta \mathcal{S} \quad (4)$$

where $\mathcal{S}$ is a shape factor which is dependent upon the geometry of the problem and the ratio of k_2/k_1.

$$\mathcal{S} = \frac{Q}{k_1\Delta} = \frac{2\pi q}{k_1\Delta} \quad (5)$$

The maximum landside uplift occurs at $y = \infty$. However, within a short distance landside of the toe drain the uplift pressure becomes nearly equal to the maximum value. The maximum landside uplift ratio is

$$\frac{\Phi_{\max}}{\Delta} = 1 - \frac{L(1+m)\pi q}{h\, k_1 \Delta} \quad (6)$$

A solution has been obtained for the case where the thickness, h_1, of layer I is not equal to the thickness, h_2, of layer II, which is similar to that given by *Hummel* (1932), except that the reciprocal terms of *Hummel*'s solution are replaced by natural logarithmic terms. However, because of the slow rate of convergence of the logarithmic terms, the solution is not practical for computational purposes.

A solution has also been obtained for the limiting case where the lower pervious layer is infinitely thick. The potential reflection coefficient for the interface between the top impervious blanket and the upper pervious layer is 1.0; thus this interface is perfectly reflecting. This fact in combination with the partially reflecting interface between pervious layers I and II creates an infinite number of images for a line source or sink insofar as Φ_I, the potential in pervious layer I, is concerned. The n-th images of the strength $q' m^n$ are located at $x = \pm 2nh$. The images for Φ_{II}, the potential in the lower pervious layer II, are all located above the line source or sink. The n-th image of strength $q'(1-m)m^n$ is located at $x = -2nh$. The potential Φ_I is

$$\Phi_I = \Delta + \frac{q'}{2k_1}\left\{\ln\left[\frac{x^2+(y-L)^2}{x^2+(y+L)^2}\right] + \sum_{n=1}^{n=\infty} m^n \ln\left[\frac{(2nh-x)^2+(y-L)^2}{(2nh-x)^2+(y+L)^2}\right] + \sum_{n=1}^{n=\infty} m^n \ln\left[\frac{(2nh+x)^2+(y-L)^2}{(2nh+x)^2+(y+L)^2}\right]\right\} \quad (7)$$

The potential Φ_{II} is

$$\Phi_{II} = \Delta + \frac{q'(1-m)}{2k_2}\sum_{n=1}^{n=\infty} m^n \ln\left[\frac{(2nh-x)^2+(y-L)^2}{(2nh-x)^2+(y+L)^2}\right] \quad (8)$$

The seepage, Q, per unit length of semi-circular drain is

$$Q = \pi q' = \Delta k_1 \mathcal{S} \quad (9)$$

where

$$q' = \frac{-k_1\Delta}{\ln\frac{R}{2L} - \sum_{n=1}^{n=\infty} m^n \ln\left(1+\frac{L^2}{n^2h^2}\right)} \quad (10)$$

so

$$\mathcal{S} = \frac{\pi q'}{\Delta k_1} = \frac{Q}{\Delta k_1} \quad (11)$$

The influence of the variations of the ratios h_2/h_1 and k_2/k_1 on the maximum landside uplift and seepage are shown on

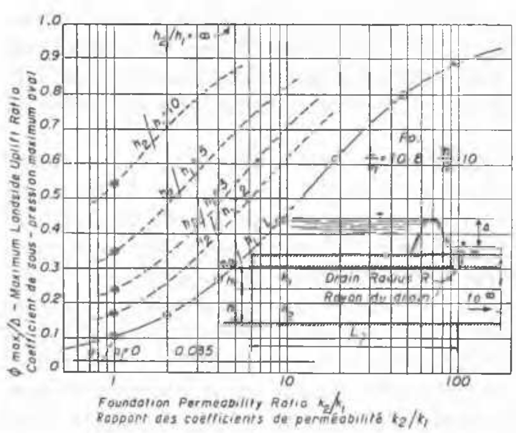

Fig. 3 Curves of Maximum Landside Uplift Ratio Versus Foundation Permeability Ratio
Courbes donnant les coefficients de sous-pression maximum en fonction du rapport des perméabilités

Figs. 2 and 3 for the case where $L/h = h/R = 10.0$. The dashed lines are the interpolated curves for the cases where h_1/h_2. However, the values for these curves, at $k_1 = k_2$ were computed using equations (5) and (6) after setting $h_1 + h_2 = 2h$.

It is readily apparent, for cases where the lower part of a pervious deposit is only somewhat more pervious than the upper part, that the seepage uplift pressure reduction on the base of a top impervious blanket produced by a landside toe drain is rather small and that for such cases a more efficacious means (such as properly spaced fully penetration relief wells) must be used.

References

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