# The Influence of the Compressibility of Soil on Some Problems of Ground-Water Flow

## L'influence de la compressibilité du sol sur certains problèmes de l'écoulement des eaux souterraines

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### Summary

This paper deals with some complications in hydrological practice, due to the compressibility of the soil. First (chapter II) the phenomenon of the apparent resistance of a layer is discussed. In chapter III the author gives a method for the determination of a characteristic constant (the product of the permeability and the modulus of elasticity) from measurements of ground-water head in a shore with tidal movement. In chapter IV the influence of the compressibility on pumping tests (retardation-effect) is discussed. Chapter V. at last, deals with the piezometer test. The unexpected fact is demonstrated, that the compressibility of the soil has no influence on the results obtained by this test. Therefore the piezometer test proves to be a very simple and useful method for the determination of the permeability ($K$-value) of a soil layer.

### Sommaire

Cette communication traite de quelques phénomènes d'hydraulique souterraine dus à la compressibilité du sol. L'auteur donne d'abord (chapitre II) un exposé sur la résistance apparente d'une couche du sol. Dans le chapitre III il établit une méthode permettant l'évaluation d'une constante caractéristique du sol (le produit de la perméabilité et du module d'élasticité) se basant sur des mesures obtenues dans des terrains au bord de la mer où l'influence de la marée se fait sentir sur l'eau souterraine. Dans le chapitre IV l'auteur examine l'influence de la compressibilité du sol sur les essais de pompage (l'effet de retard). Le dernier chapitre est consacré à l'examen de l'influence de la compressibilité sur les mesures de perméabilité obtenues par l'observation des variations de niveau dans un tube piézométrique. L'auteur arrive au résultat inattendu que ces variations sont indépendantes de la compressibilité.

### Introduction

It is often taken for granted that in hydrological problems the compressibility of the soil is negligible. As a rule, this is right. But there are some cases, in which such an assumption does not seem to be valid. In this paper the influence of the compressibility will be more closely investigated.

It is a well known fact, that the total, vertical soil pressure $\sigma_g$ consists of two components: the water pressure $\sigma_w$ and the effective pressure $\sigma_k$.

$$\sigma_g = \sigma_w + \sigma_k$$

If $\sigma_g$ is a constant, $\frac{\partial \sigma_g}{\partial T}$ equals zero, and therefore

$$\frac{\partial \sigma_w}{\partial T} = -\frac{\partial \sigma_k}{\partial T}$$

Let $\varphi$ be the piezometric head of the ground-water, then:

$$\gamma_w \frac{\partial \varphi}{\partial T} = \frac{\partial \sigma_w}{\partial T}, \quad \text{or} \quad \frac{\partial \varphi}{\partial T} = -\frac{1}{\gamma_w}\frac{\partial \sigma_k}{\partial T}$$

($\gamma_w$ = spec. weight of water).

Therefore: an increasing head causes a decrease of the effective pressure, and this causes an increase of the voids ratio: the soil will absorb water. On the other hand: a decreasing head will decrease the voids ratio and the soil will expel its surplus of water.

Let $E$ be the modulus of elasticity of the soil and $V$ the considered volume, then

$$\frac{\partial \sigma_k}{\partial T} = -\frac{E}{V}\frac{\partial V}{\partial T} \quad \text{and therefore}$$

$$\frac{\partial V}{\partial T} = \gamma_w \frac{V}{E}\frac{\partial \varphi}{\partial T} \tag{1}$$

$$\text{or} \quad \frac{\partial \varphi}{\partial T} = \frac{E}{\gamma_w}\cdot\frac{1}{V}\cdot\frac{\partial V}{\partial T} \tag{1}$$

### The Apparent Resistance of a Layer

We consider a layer, with thickness $D$ and with a (vertical) permeability $K$, and resting on a layer of coarse gravel ($K = \sim$).

At the time $T = 0$ the head of water in the gravel may be decreased at once with $\varphi_0$, and this value may be maintained.

The continuity of water flow and the *Darcy* law leads then to the equation

$$\frac{\partial^2 \varphi}{\partial y^2} = \frac{1}{\varepsilon_s} \frac{\partial \varphi}{\partial T} \quad .. \tag{3}$$

wherein $\varepsilon_s = \dfrac{KE}{\gamma_s}$

The solution of the Equation (3) satisfying the following boundary conditions:

(1) When $T = \infty$, is $\varphi = \varphi_0 \dfrac{y}{D}$ (steady state of flow)

(2) When $y = 0$, is always $\varphi = 0$

(3) When $T = 0$, everywhere $\varphi = \varphi_0$ (except when $y = 0$) was given by *Terzaghi–Fröhlich* in the form:

$$\varphi = \varphi_0 \left\{ \frac{y}{D} + \frac{2}{\pi} \sum_{n=1,2...}^{\infty} \frac{1}{n} e^{-\varepsilon_s \frac{n^2 \pi^2}{D^2} T} \sin \left( n \pi \frac{y}{D} \right) \right\}$$

For brevity we put: $t = \varepsilon_s \dfrac{\pi^2}{D^2} T$ and we calculate $\varphi$ as a function of time, when $y = \frac{1}{2} D$. We obtain:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| when $t =$ | 0.01 | 0.1 | 0.5 | 1 | 2 | 3 | 4 | 5 |
| $\dfrac{\varphi_{\frac{1}{2}D}}{\varphi_0} =$ | 0.996 | 0.992 | 0.884 | 0.744 | 0.586 | 0.532 | 0.511 | 0.504 |

The time interval $T_p$, during which $\dfrac{\varphi_{\frac{1}{2}D}}{\varphi_0}$ has decreased from 1 to 0.550, may be named the "period" of the layer. After this "period" the influence of the compressibility has vanished, roughly taken, over 90%. The table above indicates, that

$$T_p = 2.5 \cdot \frac{D^2}{\pi^2 \varepsilon_s} \quad .. \tag{4}$$

*Example* (wherein $D = 2$ m):

(1) Sand may possess $\varepsilon_s = 10^6$ m²/day, thus $T_p = 0.086$ sec. Here $T_p$ is very small: within 1 sec the steady flow has established itself. In this case therefore the influence of the compressibility is negligible.

(2) Clay, which may have a permeability $10^{-4}\, K_{\text{sand}}$ and the compressibility of which may be ten times greater, has an $\varepsilon_s = 10$ m²/day. Now $T_p = 2.4$ hours and therefore in this case it will not be allowed to neglect the compressibility. The steady flow in this clay layer does not appear until some hours later.

The decrease of head at the surface of the layer ($y = D$) during the period $T_p$ is very small. We calculated ($D = 2$ m) in sand: $\Delta \varphi_D = 2.10^{-5}\, \varphi_0$ and in clay: $\Delta \varphi_D = 2.10^{-4}\, \varphi_0$.

From all this we may deduce the following conclusions:

If the resistance of the layer is derived from the decrease of head at the upper side of the layer, a nearly infinite resistance will result. If however the permeability is derived from the quantity of water, that flows out from the lower side of the layer, one may obtain a far too small value for the resistance, if measuring within the non steady flow period.

At the steady state $Q = \dfrac{K}{D} \varphi_0$, therefore the resistance (the real-one): $C = \dfrac{D}{K} = \dfrac{\varphi_0}{Q}$.

During the non steady flow, however,

$$Q_\varepsilon = K \left( \frac{\partial \varphi}{\partial y} \right)_0 = \frac{K}{D} \varphi_0 \{ 1 + 2 \sum_{n=1}^{\infty} e^{-n^2 t} \}$$

and therefore the apparent resistance

$$C_a = \frac{\varphi_0}{Q_\varepsilon} = \frac{D}{K} \cdot \frac{1}{1 + 2 \sum_{1}^{\infty} e^{-n^2 t}} = a C$$

We calculate from this:

| | | | | |
|---|---|---|---|---|
| when $T =$ | $\frac{1}{8} T_p$ | $\frac{1}{4} T_p$ | $\frac{1}{2} T_p$ | $T_p$ |
| then $a =$ | 0.316 | 0.447 | 0.680 | 0.860 |

This means: estimating the resistance of a clay-layer with $T_p = 2.4$ hours by measuring the outflow at the lower side after 18 minutes ($\frac{1}{8} T_p$), one obtains a resistance, which is less than $\frac{1}{3}$ of the real one. Measuring this after one minute, the apparent resistance will appear to be only a fraction of the real one. Therefore, estimating the resistance of an overlain layer in this way, one has to wait until the steady flow has established itself. This waiting period is proportional to the sqaure of the thickness $D$ (4). Dealing with clay we may expect a waiting-period of some hours, with very thick layers perhaps of some days.

## Determination of $\varepsilon_s$ from Measurements

A layer, with a thickness $D$ and a (horizontal) permeability $K$, resting on an impermeable base may be overlain by a layer with thickness $D_1$, and permeability $K_1$ (vertical), above which is maintained a constant head $\varphi_p$. At $x = 0$ the open water (e.g. a river) may confine this aggregate of layers by a flat, vertical boundary. The level of the open water alternates according to: $\varphi_b = \varphi_0 + A_b \sin \dfrac{\pi}{2} \cdot \dfrac{T}{T_0}$.

In this case the flow of the ground-water is ruled by the partial differential equation:

$$\frac{\partial^2 \varphi}{\partial x^2} - \frac{\varphi - \varphi_p}{\lambda^2} = \frac{1}{\varepsilon_s} \cdot \frac{\partial \varphi}{\partial T}$$

where $\lambda = \sqrt{\dfrac{K D D_1}{K_1}}$ and $\varepsilon_s = \dfrac{KE}{\gamma_w}$

For tidal movement at $x = 0$ we know the solution:

$$\varphi = \varphi_p + A_b e^{-\beta x} \sin (n T - a x)$$

The following relations exist between $a$, $\beta$ and $n$:

$$\beta^2 - a^2 = \frac{1}{\lambda^2}, \quad \text{therefore } \lambda = \frac{1}{\sqrt{\beta^2 - a^2}}$$

$$a \beta = \frac{1}{2} \cdot \frac{n}{\varepsilon_s}, \quad \text{therefore } \varepsilon_s = \frac{n}{2 a \beta}$$

$$\text{and } n = \frac{\pi}{2} \cdot \frac{1}{T_0}$$

It is possible to estimate $a$ and $\beta$ from the graph of an observation well at a distance $x$ from the shore.

However, it is very difficult to estimate the distance $x$ between observation well and shore exactly. Therefore one has better arrange two wells, with distances $x_1$ and $x_2$ from the shore, and deal with the difference $x_2 - x_1$, which can be estimated exactly.

According to this method, $\varepsilon_s$ and $\lambda$ have been determined in Holland (on the river Rhine, province of South-Holland). The object was a coarse sand layer, with a thickness of about 20 m, overlain by a semi-pervious layer, with a thickness of about 3 m. We found: $\varepsilon_s = 1.65 \times 10^6$ m²/day and $\lambda = 440$ m.

*Remarks:* (1) The possibility, that the overlaying layer has a period $T_p$ of some importance (vide p. 220) has not been taken into account here. If $T_p$ has about the extent of the tidal period, the above derivation may not be valuable.

(2) When the level of the river fluctuates irregularly (no tidal-movement) it is impossible to determine $\varepsilon_s$ and $\lambda$ from measurements in observation wells on the shore, the formulae being far too intricate.

## The Non-Steady State during a Pumping Test

A layer, with thickness $D$ and horizontal permeability $K$ may rest on an impermeable base and may be overlain by a semi-permeable layer with thickness $D_1$ and vertical permeability $K_1$, above which is maintained a constant head $\varphi_p$. A well, screened in the layer $D$ (the length of the filter equals $D$) is pumped. If the level within the well is maintained at a constant depth, the induced flow will be a steady flow. This steady flow has been calculated by *de Glee* (1930), and according to his formulae it is possible to determine two hydrological constants $\left(a = \sqrt{\dfrac{K_1 D}{K D_1}} \text{ and } \varepsilon_p = KD\right)$ from observations made in some observation wells, the output of the pumped-well being known.

It is well-known however, that the steady state of flow, according to *de Glee*'s formulae, does not manifest itself at once. The compressibility of the soil causes a retardation. The non-steady state of flow during this retardation-period may be investigated in this chapter.

The flow is ruled by the partial differential equation:

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{1}{x}\frac{\partial \varphi}{\partial x} - \frac{\varphi - \varphi_p}{\lambda^2} = \frac{1}{\varepsilon_s}\frac{\partial \varphi}{\partial T} \tag{5}$$

wherein $\lambda = \sqrt{\dfrac{K D D_1}{K_1}}$ and $\varepsilon_s = K\dfrac{E}{\gamma_w}$

Let be $\varrho = \dfrac{x}{\lambda}$ and $t = \dfrac{\varepsilon_s}{\lambda^2} T$, we obtain:

$$\frac{\partial^2 \varphi}{\partial \varrho^2} + \frac{1}{\varrho}\frac{\partial \varphi}{\partial \varrho} - (\varphi - \varphi_p) = \frac{\partial \varphi}{\partial t} \tag{6}$$

We want a solution in the shape:

$$\varphi - \varphi_p = f(t) \cdot F(u), \quad \text{wherein: } u = \frac{\varrho}{2\sqrt{t}} = \frac{x}{2\sqrt{\varepsilon_s T}}$$

Putting this solution into (5), we obtain:

$$\frac{1}{F}\frac{d^2F}{du^2} + \left(2u + \frac{1}{u}\right)\cdot\frac{1}{F}\cdot\frac{dF}{du} = 4t\left(\frac{1}{f}\cdot\frac{df}{dt} + 1\right) \tag{7}$$

This may be true only when both sides of the equation are equal to a constant. Let this constant be: 4 m². We obtain two equations:

$$\frac{d^2F}{du^2} + \left(2u + \frac{1}{u}\right)\frac{dF}{du} = 4\,\mathrm{m}^2 F \quad .. \tag{8}$$

and

$$t\left(\frac{1}{f}\frac{df}{dt} + 1\right) = \mathrm{m}^2 \quad .. \tag{9}$$

The Equation (9) has the general solution: $f = C_1 t^{m^2} e^{-t}$.

The general solution of (8) depends on m, and we are not able to write it down at once. However, if we choose m = 0, the thus obtained solution proves to be apposite to the considered case.

When m = 0, we obtain the solution

$$F = \tfrac{1}{2} C_2 \int_{\infty}^{x} \frac{1}{u^2} e^{-u^2} d(u^2) = \tfrac{1}{2} C_2 E_i(-u^2)$$

and therefore:

$$\varphi - \varphi_p = \tfrac{1}{2} C e^{-t} E_i(-u^2)$$

(when $T = 0$ is $\varphi = \varphi_p$; when $x = \infty$ is $\varphi = \varphi_p$ too).

The flow through a cylindric surface with radius $x$ is:

$$Q = KD \cdot 2\pi x\left(\frac{\partial \varphi}{\partial x}\right) = 2\pi K D C e^{-\left(\frac{T}{T_0} + \frac{x^2}{4\varepsilon_s T}\right)},$$

wherein $T_0 = \dfrac{\lambda^2}{\varepsilon_s}$.

The output $Q_p$ from the pumped well therefore will be ($x = r$):

$$Q_p = 2\pi K D C e^{-\left(\frac{T}{T_0} + \frac{r^2}{4\varepsilon_s T}\right)}$$

This $Q_p$ reaches its maximum value when $\dfrac{\partial Q_p}{\partial T} = 0$

$Q_m = 2\pi K D C e^{-\frac{r}{\lambda}}$, and this value is reached at the time $T = \tfrac{1}{2}\dfrac{r}{\lambda} T_0$, therefore:

$$Q_p = Q_m \cdot e^{\frac{r}{\lambda}} \cdot e^{-\left(\frac{T}{T_0} + \frac{r^2}{4\varepsilon_s T}\right)}$$

The value $r$ being very small compared with $\lambda$, the value $\dfrac{r}{\lambda}$ will be a small one, and therefore $e^{\frac{r}{\lambda}} \approx 1$. We put $e^{\frac{r}{\lambda}} = 1$. The value $\dfrac{r}{\lambda}$ being small, the maximum $Q_m$ will appear after a very short time-interval: $T = \tfrac{1}{2}\dfrac{r}{\lambda} T_0$. Applying the formulae only: when $T \gg \tfrac{1}{2}\dfrac{r}{\lambda} T_0$, it is allowed to neglect $\left(\dfrac{T_0}{T}\cdot\dfrac{r}{2\lambda}\right)^2$ against unity.

With all this we obtain finally: $Q_p = Q_m e^{-\frac{T}{T_0}}$ and $C = \dfrac{Q_m}{2\pi KD}$

Let be $h_x$ the fall of the level within the pumped-well, then we obtain:

$$h_x = -\frac{Q_m}{4\pi KD} e^{-\frac{T}{T_0}} E_i(-u_x^2) \tag{10}$$

and this solution holds good if a pumping will be maintained according to

$$Q_p = Q_0 e^{\frac{T}{T_0}} \tag{11}$$

This condition seems to be inconvenient, for we don't know the value $T_0 = \dfrac{\lambda^2}{\varepsilon_s}$ (indeed we try to estimate it by our pumping-test) and so it seems impossible to set up a pumping scheme in accordance with this condition (11). This difficulty, however, is only apparent, because we apply the formulae only in the non-steady period, during which $T \ll T_0$. Therefore $Q_p$ will be a constant during the first hours of pumping. Our solution therefore is a useful approximation.

*Jacob* (1946) derived the exact formulae for this case. The calculation according to his formulae however is a very intricate one, especially in the initial period. In practice we prefer therefore our above-mentioned formulae.

A calculated example follows here.

Let be: $\lambda$ = 500 m, $\varepsilon_s$ = 6,25.10⁴ m²/day and $r$ = 0.20 m, $T_0$ = 4 days.

$Q_p$ reaches its maximum after $T = \tfrac{1}{2}\dfrac{r}{\lambda} T_0 = 1.15$ min.

When $T = 1 \quad 2 \quad 4$ and $8$ hours,
the quotient $\frac{Q_b}{Q_0} = 0.99 \quad 0.98 \quad 0.96$ and $0.92$.

The values $\frac{h_x}{q_0}$ $\left(\text{wherein } q_0 = \frac{Q_m}{4\pi KD}\right)$ for: $x = 0.2$ m (pumping well), $x = 25$ m, $x = 50$ m and $x = 100$ m are given in the next table.

| *T (in hours)* | *x = 0,2 m* | *x = 25 m* | *x = 50 m* | *x = 100 m* |
|---|---|---|---|---|
| ¼ | 10.60 | 1.08 | 0.23 | — |
| ½ | 11.30 | 1.86 | 0.59 | 0.06 |
| ¾ | 11.70 | 2.03 | 0.95 | 0.14 |
| 1 | 12.00 | 2.30 | 1.07 | 0.23 |
| 1.5 | 12.40 | 2.68 | 1.41 | 0.42 |
| 2 | 12.70 | 2.96 | 1.66 | 0.59 |
| 3 | 13.10 | 3.35 | 2.03 | 0.86 |
| 4 | 13.40 | 3.70 | 2.30 | 1.08 |
| 6 | 13.80 | 4.04 | 2.68 | 1.41 |
| 8 | 14.08 | — | 2.96 | 1.66 |

From observations the value of $\varepsilon_s$ can be determined in a easy manner. We remember, that

$h_x = -q_0 Ei(-u^2)$ and $u = \frac{x}{2\sqrt{\varepsilon_s T}}$ ; therefore

$\log h_x - \log q_0 = \log\{-E_i(-u^2)\}$ and

$\log u = \log x - \log 2\sqrt{\varepsilon_s T}$

Build up an $U$-diagram, by plotting $-E_i(-u^2)$ against $u$, both on a logarithmic scale.

Build up, on a transparency, an $\chi$-diagram by plotting for several values of $T$ the observed $h_x$ against $x$, both on a logarithmic scale too. Each $\chi$-line corresponds with one value of $T$.

Put the transparency down upon the $U$-diagram and let one of the $\chi$-lines coincide with the $U$-line, the axis of both diagrams being parallel to each other.

Then the values: $\log q_0$ and $\log 2\sqrt{\varepsilon_s T}$ can be determined from the displacements of the origin. As $Q_m$ and $T$ are known, $KD$ and $\varepsilon_s$ can be determined.

By moving the $\chi$-diagram in a direction parallel to the $\chi$-axis it is possible to bring another $\chi$-line in coincidence with the $U$-line, $q_0$ being a constant. This manipulation has to bring out the same value for $\varepsilon_s$.

## The Piezometer Test

With the help of a soil auger a pipe of small diameter $R$ is driven into the soil. At the lower end of the pipe, the auger drills out a small, spherical cavity with diameter $r$ $(r > R)$. After some pumping (in order to remove impurities) the installation is left to itself for some hours and the soil water is allowed to rise to its former piezometric head. Then the water within the pipe is pumped out as quickly as possible, whereafter the soil water is allowed to rise in the pipe. During this rise the pumping ceases. The rate of rise is determined with the aid of stopwatches and the like. And now the problem arises, how to derive some characteristic hydrological constant from these observations.

In a field of flow, with a boundary, that does not change its *shape*, and in which *Laplace*'s equation $\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2} = 0$ is available, the flow $Q$ through some surface is proportional to some determinate difference of head: $y$. Dealing with a well in phreatic groundwater we choose for $y$: the difference between the phreatic surface and the level within the well, and for $Q$: the flow into the well. In that case: $Q = \varepsilon_b y$; $\varepsilon_b$ being the constant of proportion.

The level within the well rises with a velocity $-\frac{\delta y}{\partial T}$, so $Q = -\pi R^2 \frac{\delta y}{\partial T}$, and therefore:

$$-\pi R^2 \frac{\delta y}{\delta T} = \varepsilon_b y \quad .. \tag{12}$$

Integrating this equation (and minding the condition: when $T = 0$ is $y = y_0$) we obtain

$$\varepsilon_b = \pi R^2 \frac{\lg \frac{y_0}{y}}{T} \tag{13}$$

It can easily be seen that when the well is placed in a distance $h$ from the phreatic surface or the impervious base ($h \gg r$), the presence of the phreatic surface or of the impervious base cannot affect practically the flow into the well. We can therefore compute the coefficient $\varepsilon_b$ supposing that the well is placed in an unlimited medium

$$\varphi = \frac{C}{\varrho}:$$

$$\varphi_0 = \frac{C}{r}$$

The flow into the well $Q_0 = -4\pi r^2 K \left(\frac{\partial\varphi}{\partial\varrho}\right)_{\varrho=r}$

or $Q = 4\pi KC$.

We know $\varepsilon_b = \frac{Q_0}{\varphi_0}$ ; therefore $\varepsilon_b = \frac{4\pi KC}{C\left(\frac{1}{r}\right)}$

and from (V 2) we derive: $\pi R^2 \frac{\lg\left(\frac{y_0}{y}\right)}{T} = 4\pi K r$

$$K = \frac{R^2}{4r} \cdot \frac{\lg\left(\frac{y_0}{y}\right)}{T} \tag{14}$$

By plotting $\frac{y_0}{y}$ on logarithmic scale against $T$ on linear scale we obtain a straight line. So it seems very simple to determine $K$ (or at least: $r^2K$) from observing the rate of rise of the level within the pipe.

On page 221 we found, that, dealing with a pumping test, we have to wait until the steady flow has established itself, in order to avoid errors, due to the compressibility of the soil. We found too, that these errors will occur to a significant extent, if we measure within a short time after the beginning.

Surely, our piezometer-test is a pumping test; surely we are not able to wait the required time period; surely we are dealing with a non steady flow and surely we are measuring here within a very short time after the beginning.

Obviously we have to fear, that our Formula (14) *cannot be true*, except if applied to soils, whose compressibility is negligible. And therefore, we have to derive a new formula, taking into account the compressibility of the soil.

From page 220 we learned, that the decrease of head, which occurs within a relatively short time is negligible at relatively great distances. Therefore the supposition seems to be allowed, that in our case the decrease of head at the phreatic surface is negligible during the whole period of our test. This supposition allows to get rid of all intricate boundary-conditions. It will be sufficient to calculate the flow towards a spherical cavity within a homogeneous and isotropic soil, which is unconfined in every direction.

The flow field is described by the partial differential equation

$$\frac{\partial^2 \varphi}{\partial \varrho^2} + \frac{2}{\varrho}\frac{\partial \varphi}{\partial \varrho} = \frac{\partial \varphi}{\partial t} \quad .. \tag{15}$$

wherein $t = \varepsilon_s T$ and $\varepsilon_s = \frac{KE}{\gamma_w}$

The solution has to satisfy the boundary conditions:

(1) when $\varrho = r$ is $\pi R^2 \varepsilon_s \left(\frac{\partial \varphi}{\partial t}\right)_{\varrho = r} = 4\pi K r^2 \left(\frac{\partial \varphi}{\partial \varrho}\right)_{\varrho = r}$

(2) when $\varrho = \infty$ is $\varphi = 0$

and the initial condition: at $t = 0$ is (1) $\varphi = y_0$ when $\varrho = r$
(2) $\varphi = 0$ when $\varrho > r$

The solution which satisfies these conditions may be omitted here; we found for the head $\varphi_r = y$ within the weel the following intricate double-series:

$$\begin{aligned}\frac{y}{y_0} = \Bigg[ & 1 - \tfrac{1}{2}(1 - \tfrac{1}{2}P)z + \frac{1}{\varrho}\left\{(1 - \tfrac{1}{2}P)^2 - \tfrac{1}{2}P\right\} z^2 - \\ & - \frac{1}{48}\left\{(1 - \tfrac{1}{2}P)^3 - \tfrac{1}{2}P(3 - P)\right\} z^3 + \\ & + \frac{1}{8 \times 48}\left\{(1 - \tfrac{1}{2}P)^4 + \tfrac{1}{4}P^2(1 - \tfrac{1}{2}P) - P(P + \tfrac{3}{2})\right\} z^4 - \\ & - \ldots \text{a.s.o.}\Bigg] - \sqrt{\frac{Pz}{\pi}}\Bigg[1 - \frac{2 - \frac{1}{2}P}{3} z + \\ & + \frac{\{(1 - \frac{1}{2}P)^2 + 2 - P\}}{3 \times 5} z^2 - \frac{\{(1 - \frac{1}{2}P)^3 - \frac{1}{2}P(1 - \frac{1}{2}P) + 3\}}{3 \times 5 \times 7} z^3 + \\ & + \ldots \text{a.s.o.}\Bigg] \text{ wherein } P = \frac{8r^3}{R^2}\frac{\gamma_w}{E} \text{ and } z = \frac{8rK}{R^2}T.\end{aligned}$$

Because always $\frac{E}{\gamma_w} \gg \frac{4r^3}{R^2}$ it is allowed to neglect $\frac{1}{2}P$ against unity. Thus we obtain:

$$\frac{y}{y_0} = e^{-\frac{1}{2}z} - \sqrt{\frac{r}{\pi}\frac{\gamma_w}{E}} \sum_{1}^{\infty} \frac{(-2)^n (n+1)(n!)}{(2n+1)!} z^{n+\frac{1}{2}} \text{ or}$$

$$\frac{y}{y_0} = e^{-\frac{1}{2}z} - A\,R(z) \quad .. \tag{16}$$

$A = 0$ when $\frac{E}{\gamma_w} = \infty$ (incompressibility) and then: $\frac{y}{y_0} = e^{-0{,}50z}$

$A = 0.02$ when $\frac{E}{\gamma_w} = 12.7$ m (and $r = 0.10$ m), and then: $\frac{y}{y_0}$ is about $e^{-0{,}51z}$.

In nature, $\frac{E}{\gamma_w}$ will probably never reach such a low value as 12.7 m, and it will be evident that practically in (16) the term $A\,R(z)$ may be neglected against $e^{-\frac{1}{2}z}$ for the whole area of existing soils therefore:

$$\frac{y}{y_0} = e^{-\frac{1}{2}z} \text{ or } K = \frac{R^2}{4r}\frac{\lg \frac{y_0}{y}}{T} \quad .. \tag{16}$$

The two equations (14) and (16) prove to be identical, though their derivation started from an absolutely different point of view. Our fear that the compressibility of the soil might disorder the results of the piezometer-test proves not to be true. From this somewhat amazing result, we may conclude that the shape of the flow-field at greater distances has practically no influence, and that obviously only the radial flow-field within the nearest proximity of the spherical cavity matters. So the value of $K$, which is determined by our piezometer-test, will be the $K$-value of the soil at the base of the pipe. Therefore we fully agree with *J. N. Luthin* and *Don Kirkham* (1949), who point out that the advantage of the piezometertest is that the permeability of any layer in the soil can be measured. In this paper we may have proved their statement even in another, more extensive respect.

## References


*de Glee, G. J.* (1930): Over grondwaterstromingen bij wateronttrekking door middel van putten. Dissertatie Delft.

*Jacob, C. E.* (1946): Radial Flow in a Leaky Artesian Aquifer. Transactions American Geophysical Union, Vol. 27, No. 11, April.

*Luthin, J. N.* and *Don Kirkham* (1949): A Piezometer Method for Measuring Permeability of Soil in situ Below a Water Table. Soil Science, Vol. 86, No. 5, November.