The Stability of Slopes of Dams Composed of Heterogeneous Materials

Stabilité des talus dans les barrages hétérogènes

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Summary

The stability analysis of slopes is generally based on cylindrical sliding surfaces. If a dam consists of two materials of different properties the failure will occur along the line of minimum resistance which is shown as a sharp break at the border between these materials. This result has been obtained in investigations carried out by the author on models consisting of a cohesive core supported by a bank of non-cohesive material.

Based on the results of tests the author suggests a new method of analysis for the stability of earth dams in heterogeneous materials.

Sommaire

L'étude sur la stabilité des talus est basée en général sur la supposition de surfaces de glissement cylindriques. Si un barrage en terre se compose de deux matériaux de caractéristiques différentes, la rupture se produira le long de la ligne de résistance minimum qui présente un point de discontinuité à la limite de deux matériaux. Ce résultat a été obtenu par des essais que l'auteur a exécutés sur des modèles comportant un noyau en matériau cohésif, soutenu par un talus en matériau non cohésif.

Se basant sur les résultats obtenus dans ses essais l'auteur propose une nouvelle méthode d'analyse de la stabilité des digues en terre à coupe hétérogène.

Generality

The failure of a slope generally occurs along a continuous curved sliding surface. The analytical computation of the position of the sliding surface and of the stability is possible only with certain assumptions making this computation simple and easy.

The problem can be solved, in a much simpler way by the *Swedish* method, and by adopting further simplifications to compute the equilibrium of forces on the sliding body as a whole. The material is assumed to be homogeneous, the sliding surface circular, and the sliding body rigid.

These approximate suppositions and simplified computations give results accurate enough for the practical work but are justified only where there is a homogeneous material in which more or less circular sliding surfaces occur and where large movements are possible without causing deformations of the sliding body. As such it can also be applied to non-homogeneous sections, if the differences of strength and elasticity between the layers are not great, so that the sliding surface is approximately circular or spiral.

Large earth dams are often composed of different materials, the core being of cohesive material, and the rest of gravel or stone. When the differences in strength are small, the sliding surface is more or less continuous, and the analysis can be carried out according to the *Swedish* method. But when the differences between the materials are great, the sliding surface will be formed along the line of minimum resistance, often with a sharp break at the border between the two different materials. In this case sliding is possible only with the deformation of the sliding body, which influences the play of forces competent for the stability analysis.

There are various proposals for the application of the *Swedish* method to such cases, but they are all based on empirical assumptions which do not take into account the deformation of the sliding body. According to this, *Clarke* (1946) gives a simplified scheme for the stability analysis of non-homogeneous sections with circular sliding surfaces. *Daehn* and *Hilf* (1951) give also circular sliding surfaces for the analysis of the stability of large non-homogeneous earth dams in the U.S.A. It can be stated that for the stability analysis of non-homogeneous dams the continuous circular sliding surface is used.

It is shown in Fig. 1 that the sliding surface at the border between the clay core and the gravelly or stony retaining body must change its direction, because the resistance to movement along the dotted circle *B–C* in the non cohesive material would be greater then along the straight surface *B–C*. *Ehrenberg* (1931) takes these facts into consideration and proposes the analysis shown in Fig. 2. The sliding body is divided into two separate parts by the vertical planes A–B. The weight and other forces acting on the part ABC are the displacing forces, the passive pressure of the part ABD and the shear strength along the sliding surface AC are the stabilising forces. The slope is safe if there is an equilibrium between the displacing and stabilising forces (the resultant R is within the circle $r \sin \varrho + a$).

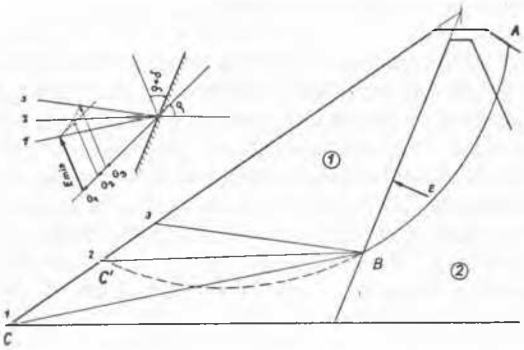

Fig. 1 Sliding Surfaces in Non-Homogeneous Section
1 Bank of Rock
2 Clay Core
A–B–C Possible Sliding Surface
Surface de glissement en section non homogène
1 massif en roches
2 noyau en argile
A–B–C surface de glissement possible

There is no theoretical explanation for such a partition of the sliding body. The deformations that occur in the sliding body while sliding along the surface D–A–C will be competent for the determination of the relationship of forces in the moment of the disturbance of the equilibrium.

Experiments

These deficiencies are the reason for the uncertainty in the stability analysis of large dams. For the design of a rock dam with a clay core, 50 m height, it was decided to investigate by models the kinematics of the failure of the combined section and, on the basis of the results, to determine the method of computation. The models were made in a glass box 16 cm wide.

The first series of model tests were carried out to determine the behaviour of the retaining body of rock under different deformations.

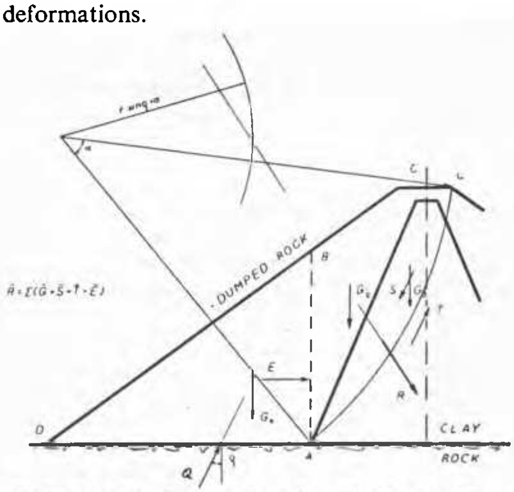

Fig. 2 Stability Computation after *Ehrenberg* in Scheme Rock—Clay—Dumped Rock Fill
Schéma de l'étude de stabilité d'après *Ehrenberg*
Roche – argile – massif en roches

The retaining body in the model was made of quartz sand. By moving in different directions the inclined support, which represented the core, the position and the shape of the sliding surfaces formed were determined. Controlling analytically the results obtained using only the earth pressure theory it was found that the sliding surfaces act in accordance with the earth pressure theory, and thus the pressure of the retaining body acting on the support (core) can be computed using the same theory.

The second series of investigations on models was carried out on a "mechanical" dam model with a non-homogeneous section consisting of a core with a retaining body. The sliding surface in the core was predetermined by a solid greased cylindrical surface. The models were made of uniform sand of grain size 0.2–0.5 mm, and the boundary between the core and the supporting body was fixed by a thin layer of grease which was differently placed in the different models. A model of homogeneous sand was also examined. It is seen in Fig. 3 that the discontinuities of the deformations occur at the boundary between the core and the retaining body, while these are continuous in a homogeneous section. According to this, the discontinuity of deformations recorded on the models of non-homogeneous section does not occur because of the break in the sliding surface, but because of the non-homogeneity of the section. The discontinuity occurs at the boundary between

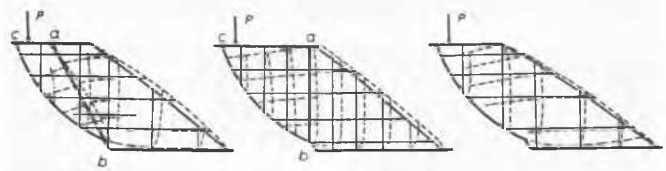

Fig. 3 Deformations of "Mechanical" Models of Non-Homogeneous Sections
a–b Greased Layer
b–c Greased Sliding Surface
P Load
Déformations de modèles «mécaniques» à sections non homogènes
a–b couche graissée
b–c couche graissée de glissement
P charge

the different materials. Consequently it could be concluded that the non-homogeneous section is divided into two parts along the boundary between the different materials. The active forces in the core produce the primary deformations which activate the passive pressure of the whole retaining body.

Since the analysis of the stability can be based on such a relation of forces, an attempt was made to verify the results on "physical" models of a non-homogeneous section, composed of two different materials in which, under the action of a load, the sliding surface could freely be formed. Models were built with very different material properties for the core and for the retaining body. The core was formed of grease with an angle of internal friction $\varrho = 0°$, cohesion $c = 6$–12 g/cm^2. The retaining body was built of sand (0.2–0.5 mm), with an angle of internal friction $\varrho = 40°$. The top of the model was loaded with a uniformly distributed load which was gradually increased at a speed of $^1/_{10}$ of the bearing capacity per minute, until the bearing capacity was exceeded.

Figs. 4 and 5 show the sliding surfaces which occurred in some of the models. The sliding surface is approximately circular in the core and straight in the retaining body, with a sharp break at the boundary between the two materials. The vectors of deformation show also a sudden change of direction on the transition from the core to the retaining body, a proof that each of the materials deforms according to its own laws, and not as an individual body. The retaining body of the models is most deformed on the top, while at the lower end it remains in its position even after failure. In the model 34, Fig. 5, the

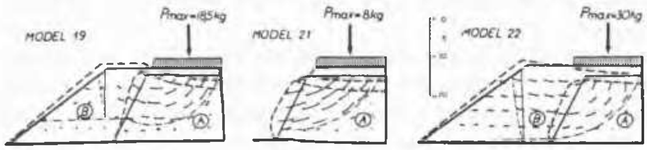

Fig. 4 Deformations and Sliding Surfaces of "Physical" Models of Non-Homogeneous Sections
A Grease B Sand
—— Boundaries before Sliding
– – – Boundaries after Sliding
–·–·– Sliding Surfaces
Déformations et surfaces de glissement des modèles «physiques»
A graisse B sable
—— limites avant glissement
– – – limites après glissement
–·–·– surface de glissement

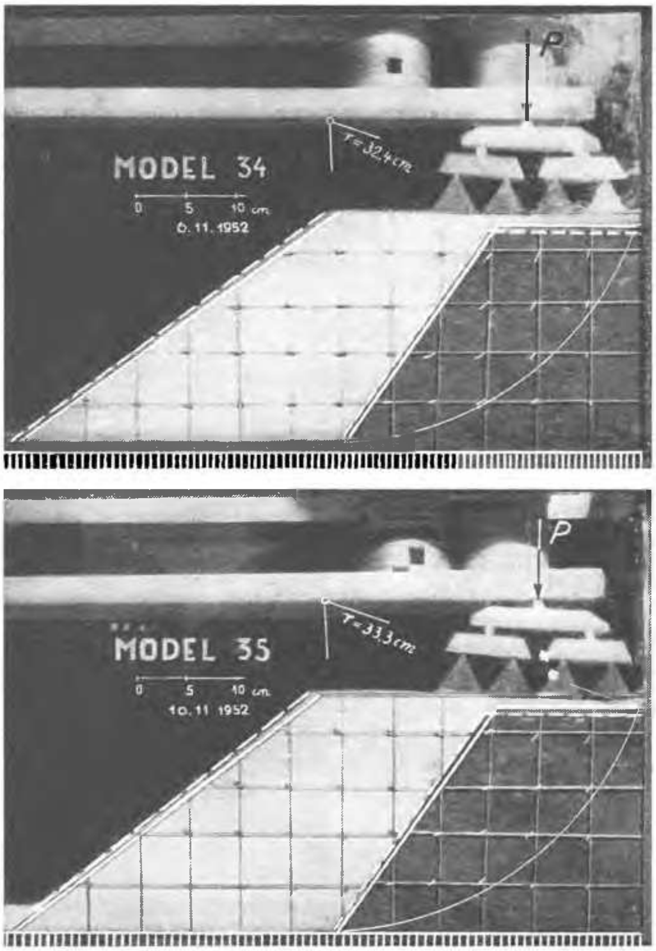

Fig. 5 Deformations of Models Nos. 34 and 35
P Load
—— Boundaries before Sliding
– – – Boundaries after Sliding
Déformations des modèles Nos 34 et 35
P charge
—— limites avant glissement
– – – limites après glissement

retaining body lies on a thin layer of grease. The retaining body is only slightly deformed, but removed as a whole in the direction of the slope in accordance with the smaller shear strength at the bottom of the retaining body. We may conclude that the deformations of the core, resulting from the applied load, are transmitted to the retaining body along the contact plane and that the retaining body counteracts according to its elastic properties.

Fig. 6 shows the settlement measured in the middle of the loaded section for a few models of different shape. The deformations, resulting from the first loading, increase gradually until the exceeding of the bearing capacity. When unloading the model, the deformations diminish elasticly, and by reloading it, up to approximately 70% of the bearing capacity of the model, the slope of the line of deformation is equal to that of the unloading deformation line. The elastic behaviour of the model, when unloaded or reloaded, results from the action of the passive pressure of the deformed retaining body acting even after the unloading until a new state of equilibrium is reached, on account of the diminished deformation and with a diminished passive pressure. The very slight elastic deformation of the model 21, consisting of a grease core only, proves the correctness of this observation.

From the pictures of deformations and settlement diagrams it can be concluded that at the boundary between the core and the retaining body, in a loaded dam, the earth pressure at rest becomes active thus causing corresponding stresses in the core. When the core is loaded it starts to deform, the earth pressure at rest on the contact surface gradually increases to the full value of the passive earth pressure of the retaining body, the deformations increase rapidly, and the bearing capacity is exhausted. For the stability analysis, which always represents a certain approximation, it is sufficient to determine the force that the retaining body can take over along the contact plane with the core, and that is the *passive pressure*.

The *magnitude* and the *point of application* of the passive pressure of the retaining body depend on the deformations of the contact plane with the core. It can be seen from the picture of deformations in the test models that the minimum deformation occurs at the lower end of the sliding surface in the core, then increases and can thus be represented by a straight line as in Fig. 4, which corresponds to the *Coulomb* case of earth pressure. The magnitude of the passive earth pressure can be determined according to *Coulomb*, and the point of application assumed in the lower third point of the height which is fairly correct considering the deformations.

The *direction* of the action of the passive pressure can be

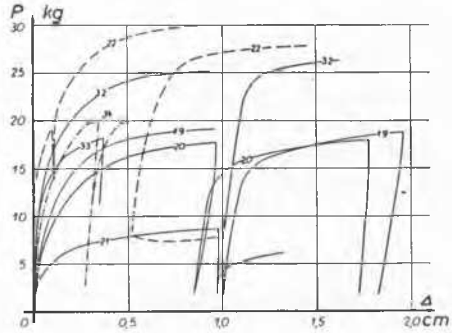

Fig. 6 Settlements of Loaded Models
P Load *Δ* Settlement
Tassement des modèles chargés
P charge *Δ* tassement

taken approximately at a right-angle to the contact plane. This supposition is mostly proved incorrect. The direction of the passive pressure is a function of the mutual displacement on the contact face, which shows in dams the tendency to divert the resultant downward; consequently the security is greater than estimated.

This fact justifies the assumption of the right-angle direction of the passive pressure action.

With these suppositions the bearing capacity of the models was controlled, as shown in Fig. 7 for the model No. 33. The core carries the corresponding part P_1 of the load P. The

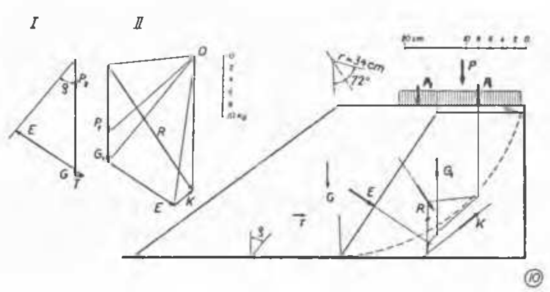

Fig. 7 Computation of the Bearing Capacity of Model No. 33
I. Polygon of Forces for Determination of Passive Pressure E
II. Polygon of Forces for Equilibrium of Core
Calcul de la force portante du modèle No 33
I. Dynamique pour la détermination de la résistance passive E
II. Dynamique et funiculaire pour l'équilibre du noyau

superimposed load P_1 and the weight of the sliding body G_1 give the moment of displacement $M_1 = G_1 \cdot r_1 + P_1 \cdot r_2$. The cohesion K in the sliding surface, and the passive pressure of the retaining body also act on the body and produce the stabilising moment $M_2 = E \cdot r_3 + K \cdot r_4$.

When computing the passive pressure E (polygon of forces I), the friction between the sand and the glass walls was also taken into consideration with $\varrho = 30°$.

The model is stable if $M_1 = M_2$. The ultimate value of the force P with which the model is still stable, was found graphically (the polygon of forces and the funicular polygon II). The following table shows the results of these computations for a few models. The bearing capacity computed with the *Ehrenberg* supposition is also given: (Table 1).

From the table mentioned above, it can be seen that the results of the computed bearing capacity based on *our supposition coincide fairly well* with the ultimate load of the model, whereas the bearing capacity computed according to the proposal of *Ehrenberg* is considerably *smaller* then the actual one. This fact confirms that, for practical purposes, our supposition reflects fairly well the play of forces at the moment of failure of the slope.

It is seen from these investigations that for the stability of a slope of a non-homogeneous dam the passive pressure of the retaining body in front of the core is competent. The computation of the stability is to be conducted in the following way:

(*a*) A cylindrical sliding surface in the core, and a straight sliding surface in the retaining body, corresponding to the minimum passive pressure, are chosen.

(*b*) The displacing forces acting on the core are determined: the weight, the seepage forces (if existing) and other loads.

(*c*) The stabilising forces acting on the core are determined, i.e. the passive pressure of the retaining body and the shearing strength in the sliding surface in the core.

The section remains stable as long as the passive pressure of the retaining body is not surpassed, although the stress in the sliding surface already reached the shear strength. Greater deformations, and thus failure, are not possible until the retaining body is pushed away *regardless* of the magnitude of the stress in the sliding surface of the core.

Based on the fact that the slope is stable until the passive pressure of the retaining body is exceeded, the factor of safety can be expressed with:

$$S = \frac{E_m}{E_p}$$

E_p = passive pressure of the retaining body necessary for the maintenance of equilibrium;
E_m = maximum possible value of passive pressure.

This expression of the factor of safety is in accordance with the mechanics of failure and with the conditions of the deformations established on the models. Only if the pressure at rest of the retaining body at the border with the core is sufficient for the maintenance of equilibrium, the core is not deformed under an increased load for the restoration of the equilibrium of forces, and the stress in the sliding surface in the core does not attain the limit shear strength. The design of the retaining body with such a high value of pressure at rest would not be economically justified, and furthermore it is unneccessary for the stability and security of the dam. In order to keep the deformations within reasonable boundaries it is necessary that the factor of security be $s = 2$.

Good compaction of the retaining body is of great importance for the unchangeability and thus for the stability of such dams. The equilibrium of forces for the additional load of the core of a dam with less compacted banks, is attained after greater

Table 1

Model No.			*18*	*19*	*20*	*21*	*22*	*32*	*33*	*34*
Dimensions, see Fig. 7	(*a*)	cm	21	21	21	21	21	29	25	24
	(*b*)	cm	30	30	30	30	25	25	30	31
	(*c*)	cm	30	30	30	30	35	28	30	33
Cohesion of grease		g/cm²	6	6	6	6	6	6	8	12
Unit weight of sand		g/cm³	1.28	1.28	1.17	–	1.30	1.39	1.32	1.40
Load at the beginning of flow		kg	8	5	8.6	4	14	15	13	8.5
Deformation at the beginning of flow		mm	1.1	0.7	0.9	1.0	1.0	0.7	0.6	0.4
Bearing capacity		kg	20	18.5	17.5	8	30	26	18	20
Deformation at failure		mm	6	7	8	4	6	6	4	4
Computed bearing capacity		kg	17	17	16	8	27	27	17	22
Bearing capacity after *Ehrenberg*		kg	–	–	–	–	–	–	12.7	15.7

deformations than those with well compacted banks. The ultimate bearing capacity differs only very slightly (see the diagram of deformations for the loading and reloading of the models, Fig. 6).

Such a stability computation is certainly still an approximation, but it is based on results obtained with model investigations. For that reason the method can be regarded as a step forward as compared with the *Ehrenberg* computation or other applications of the *Swedish* method.

Further investigations on models for the determination of the distribution, the point of application and the direction of the action of the passive pressure, are in progress. We hope that these investigations will throw more light on these unsolved questions.

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