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SUB-SECTION I c

PHYSICAL AND PHYSICO-CHEMICAL PROPERTIES OF SOIL

I c 1

CONTRIBUTION TO THE THEORY OF SHRINKING

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I. INTRODUCTION.

After the theory of soil mechanics under the guidance of Terzaghi had conquered its own special territory of civil engineering 1), it has in recent years penetrated other branches with fruitful results. In addition to the science of snow and glacier mechanics, mention should be made here particularly of the testing of moulding sand and of work in connection with the clay industry, with whose problems our Institute had to occupy itself very closely during World War II 2) and 3).

Experience shows that the process of shrinking is of decisive importance for the quality of the products of the clay industry, particularly in the manufacture of bricks and tiles, since the undesired "structures" produced by the moulding process are later influenced to a considerable degree by the shrinking process. In order to investigate more closely the peculiar behaviour of the clay during shrinkage, exhaustive tests were carried out in our laboratory to the order of the Swiss Association of Brick and Tile Makers and Buhler Brothers Ltd., engineers, Uzwil. These tests dealt chiefly with threedimensional shrinking. The test pieces used were principally cubes with sides 2 cm long and the corners cut off (Fig. 1). Four different kinds of clay, whose principal characteristics are shown comparatively in table 1, were used as material to be tested. The tests were carried out and evaluated with the collaboration of W. Eng, engineer. For valuable support and suggestions we have also to thank Dr. A. Stutz, director of the Brick and Tile Works Passavant-Iselin & Cie. A.-G., Allschwil, Basle.

by a group of curves in which the preparing-water content plays the part of parameter. The higher the preparing-water content - i.e. the water content present at the beginning of the compacting - the greater will be the water content remaining for a given compacting pressure, although the compacting is in itself more intense.

An analogous phenomenon may be noted also during the shrinking of clay. This is not surprising when consideration is taken of the relationship between the shrinking process on the one hand and the compacting under the action of external forces on the other hand. The shrinking process is indeed nothing else than a compacting taking place under the action of capillary pressure which increases in accordance with theoretical principles, instead of under the action of external mechanical forces.

As K. von Terzaghi and O.K. Fröhlich have already determined, two phases must be distinguished during the shrinking process 4 : In the first phase the evaporating of the water takes place on the surface of the sample. Hand in hand with that, a reduction in size of the pores takes place, caused by a steady increase in capillary pressure, until finally the maximum is reached, which Terzaghi designates shrinking pressure. Here the second phase begins, in which the evaporating surface withdraws into the sample. The change in volume of the sample is comparatively small during this period; the capillary pressure remains approximately constant. Nothing definite can exactly be stated with regard to the actual magnitude of the shrinking pressure. On the other hand

TABLE 1

<u>Characteristics of the material tested</u>							
No. Laboratory	Limits of consistency			Coefficient of compressibility % (of 5)	Coefficient of permeability k_{10}^0 ... cm/sec	Void ratio based on $\sigma = 1 \text{ kg/cm}^2$	
	f %	a %	p %				
1 1632	55,7	22,9	32,8	9,12	$2,0 \cdot 10^{-8}$	0,965	
2 B ₁	76,6	34,7	41,9	6,53	$3,7 \cdot 10^{-8}$	1,27	
3 1406	56,0	20,1	35,9	8,80	$6,1 \cdot 10^{-9}$	0,940	
4 1407 (vacuum)	54,0	18,9	35,1	9,37	$7,0 \cdot 10^{-9}$	0,923	

II. EXTERNAL CHARACTERISTICS OF THE SHRINKING PROCESS.

To-day, it is a known fact that the water content or the void ratio of a saturated, fine-grained loose sediment cannot be expressed as a simple function of the pressure. The dependance in question can only be illustrated

the equivalent compacting pressure (shrinking pressure equivalent) nevertheless allows the order of magnitude of the effective forces to be determined. By this is understood the compacting pressure which produces in the test sample in an oedometer the same water content as the shrinking process does. 5)

In Fig. 2 is illustrated on the one hand the

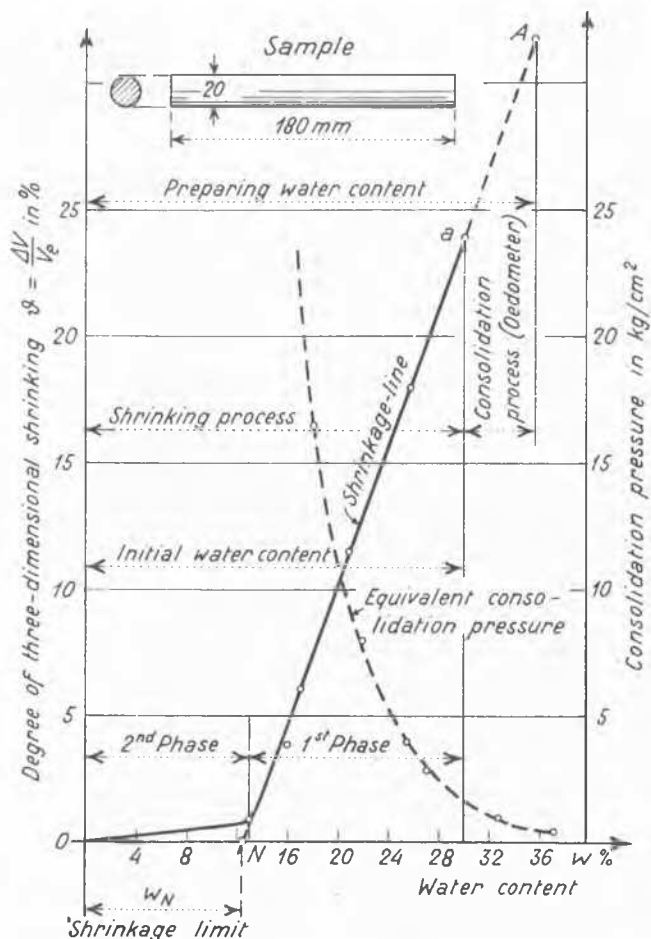


Shrinkage measuring.

FIG. 1

dependence of the degree of three-dimensional shrinking on the water content (curve 1), and on the other hand the relation between the water content possessed by the sample immediately after it has been prepared, whilst the latter means the water content when the shrinking process begins. Quite exceptionally we shall have $w_A = w_a$. First of all we assume that, after the sample has been prepared, a compacting process is first performed in the oedometer until a certain consolidation has been reached. Secondly, after that the shrinking process starts and continues until the sample in the drying cupboard at 105°C no longer gives off any water. In this case the initial water content differs from the preparing-water content; in fact it is smaller. It has now been found that for a given clay the compacting, or the unit weight reached after completion of the shrinking process, depends only on the preparing water content. The higher the latter was chosen, the less was the final compacting, in other words, the dry unit weight of the respective test material was also less. From this the above-mentioned relationship between the compacting process in the oedometer on the one hand, and the shrinking process on the other hand, can be seen very clearly. Strictly speaking we have here fundamentally one single

process taking place in two different forms. This particular behaviour of clay during the consolidating process can be explained by the fact that there is a quite definite texture corresponding to each preparing water content, and that during the consolidation a change certainly occurs in the texture in the sense of a compacting without the principle features of this texture being lost. A sudden alteration in the inner structure may only be caused by intense shaking or mechanical disturbance. This change is termed breakdown of structure.



Shrinkage-line. Three dimensional shrinking of a clay cylinder as a function of the water content. Clay No. 1632.

FIG. 2

process taking place in two different forms.

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III. ANALYSIS OF THE SHRINKING PROCESS

(1st phase).

In the following theoretical investigation we deal only with the first phase of the shrinking process, which we presume to last until the straight line of shrinking shown in Fig. 2 and 3 intersects the axis of the abscissa at the point N. The corresponding section of the axis w_N is in the following termed the shrinkage limit.

First of all the relations between degree of shrinking and water content shall be determined. The fact that the sample remains saturated with water during the first phase of the shrinking process justifies us in putting the change in water content equal to the change in volume of the sample. Accordingly

the following holds good:

$$V = V_s + V_w = G_s \left(\frac{1}{\gamma_s} + \frac{W}{\gamma_w} \right) \quad (1)$$

$$dV = \frac{G_s}{\gamma_w} \cdot dW; \quad G_s = V_e \cdot \gamma_w \quad (2)$$

$$\frac{dV}{V_e} = d\vartheta = \frac{\gamma_e}{\gamma_w} dW = \text{tg } \alpha \cdot dW \quad (3)$$

Where V = volume of the sample

V_s = volume of solids

V_w = volume of water portion

V_e = volume of dry sample (after the shrinking process)

γ_e = unit weight of dry sample.

Neglecting the change in volume due to the second phase - the total extent of shrinking being considered as the result of the first phase, - the equation of the straight line of shrinking is obtained from equation 3) in the following form:

$$\vartheta = \frac{\Delta V}{V_e} = \frac{\gamma_e}{\gamma_w} (W - W_N); \quad \text{tg } \alpha = \frac{\gamma_e}{\gamma_w} \quad (4)$$

The inclination $\text{tg } \alpha$ of the shrinkage-line is consequently identical with the unit weight ratio $\frac{\gamma_e}{\gamma_w}$ (cf. Fig. 3). The relation between the three-dimensional shrinking and the linear degrees of shrinking ($\partial x, \partial y, \partial z$) in the direction of the three axes in space is:

$$\vartheta = \partial_x + \partial_y + \partial_z + \partial_x \partial_y + \partial_x \partial_z + \partial_y \partial_z + \partial_x \partial_y \partial_z \quad (5)$$

Now if shrinkage tests are made with a given material with different preparing water contents and the results are plotted graphically, a corresponding number of shrinkage-lines with different inclinations and different shrinkage-limits will be obtained. Fig. 4 shows two such idealised shrinkage-lines with different preparing water contents. Between these two straight lines there are interesting relations which we formulate as follows. First of all the following holds good for three-dimensional shrinking:

$$\vartheta = \frac{V - V_e}{V_e} = \frac{V}{V_e} - 1 = (1 + W) \cdot \frac{\gamma_e}{\gamma_e'} - 1 \quad (6)$$

where γ_e' represents the unit weight of the material with water content w , whilst γ_e represents the unit weight of the dry material.

Solving equation(6) with respect to γ_e , we obtain:

$$\gamma_e = \frac{1 + \vartheta}{1 + W} \cdot \gamma_e' \quad (7)$$

The unit weights of the dry material in two parallel tests with different preparing water contents w_A , but with the same water content w , are therefore in the following relation to each other:

$$\frac{\gamma_{e1}}{\gamma_{e2}} = \frac{1 + \vartheta_1}{1 + \vartheta_2} \quad (\text{cf. Fig. 4}) \quad (8)$$

For the change of the shrinkage limit due to a change in the preparing water content, - which is also to be seen in Fig. 4), - the following relation can be obtained between the axis sections w_N and the corresponding unit weights of the dry material:

$$W_{N2} = W_{N1} + \frac{\gamma_w}{\gamma_{e2}} - \frac{\gamma_w}{\gamma_{e1}} \quad (9)$$

$$W_{N2} = W_{N1} + \frac{\vartheta_1 - \vartheta_2}{1 + \vartheta_2} \cdot \frac{\gamma_w}{\gamma_{e1}} \quad (10)$$

Since $\gamma_{e1} > \gamma_{e2}$ and $\vartheta_1 > \vartheta_2$, then $w_{N2} > w_{N1}$. Since on the other hand w_{N2} belongs to the shrinkage test with the higher preparing water content, it is seen clearly from equations(9) and(10) that the shrinkage limit becomes greater with increasing preparing water content.

It is now necessary to determine the position of intersection S of the two shrinkage lines of the same material. First of all the ordinate of this point may easily be calculated as follows from equation(8) and Fig.4:

$$\frac{1 + \vartheta_1}{1 + \vartheta_2} = \frac{\text{tg } \alpha_1}{\text{tg } \alpha_2} = \frac{D_1}{D_2} = \frac{-\vartheta_s + \vartheta_1}{-\vartheta_s + \vartheta_2}$$

$$\vartheta_s = -1 \quad (11)$$

Substituting this value in equation(4), the abscissa of the point S is finally obtained:

$$\vartheta_s = (W_s - W_N) \cdot \frac{\gamma_e}{\gamma_w} = -1 \quad (12)$$

$$W_s = W_N - \frac{\gamma_w}{\gamma_e}$$

or direct from Fig. 4) in another form:

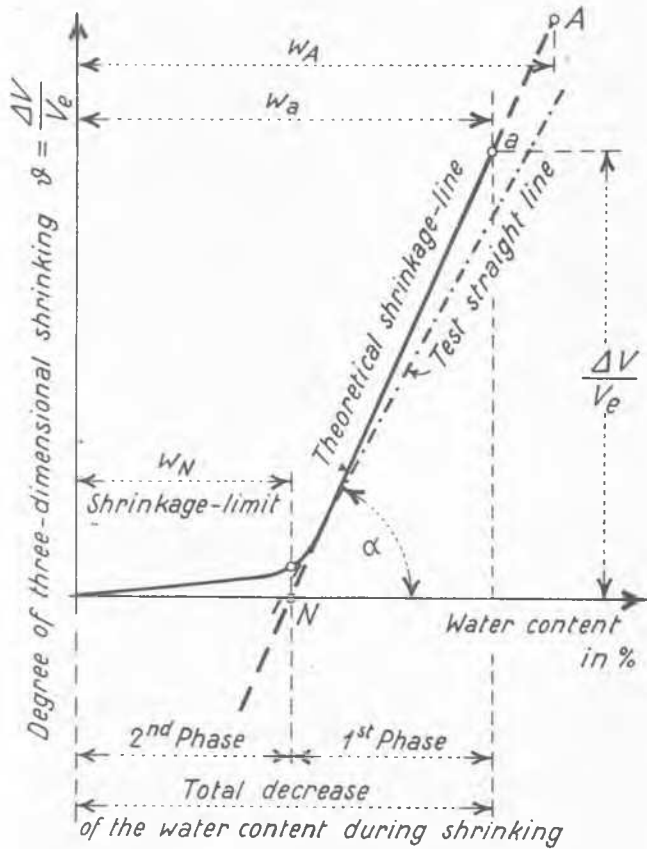
$$W_s = W_1 - (1 + \vartheta_1) \frac{\gamma_w}{\gamma_{e1}} \quad (13)$$

The final equations(11),(12) and(13) may be interpreted as follows:

From equations(12) and(13) it is evident that the abscissa w_s of the point S depends only on the elements of one single straight line. But this is only possible if all straight lines of shrinkage of one and the same material intersect at one common point S. Since w_N is normally less than $\frac{\gamma_w}{\gamma_e}$, w_s is as a rule negative. On the other hand, from equation(11) it is evident that the ordinate ϑ_s of the point S has the same value for all materials, namely $\vartheta_s = -1$. Consequently the points S for all materials lie on a horizontal line at a distance $\vartheta_s = -1$ below the origin A.

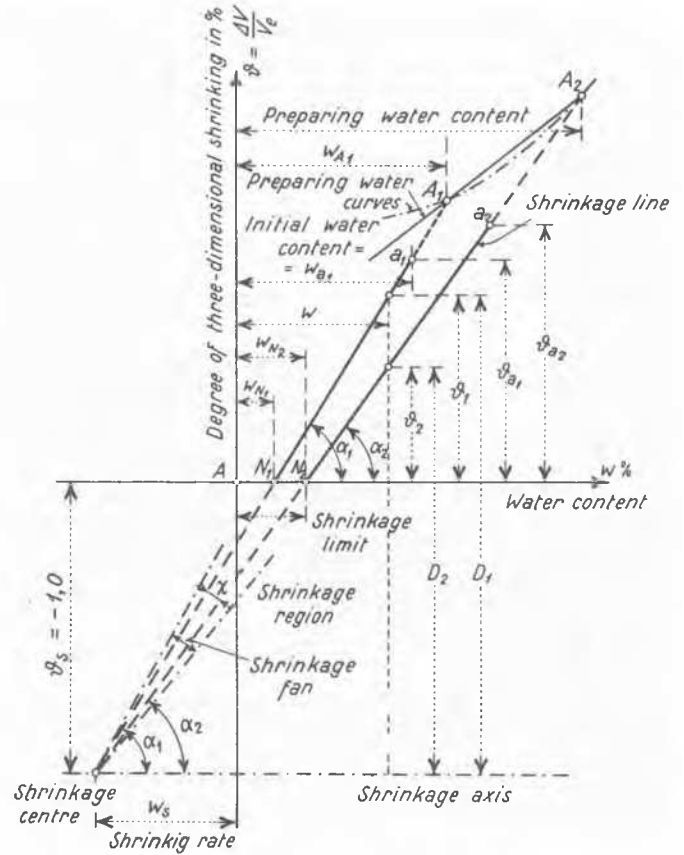
These results may be expressed in the following terms and definitions:

- 1) The point of intersection S of all shrinkage-lines for one and the same material is defined as the shrinking centre.
- 2) The shrinking centres of all materials lie on the horizontal line of the equation $\vartheta_s = -1$, which is defined as the shrinkage axis.
- 3) The abscissa w_s of a given shrinking centre is termed the shrinking rate of the material in question. The behaviour of the loose sediment during the shrinking process in question is to a large extent characterised by this single magnitude.
- 4) The total of all physically possible straight lines of shrinking of one and the same material lies within the so-called shrinkage fan, whose apex is the shrinking centre. With increasing preparing water content the shrinkage-lines rotate clockwise round the shrinking centre within the shrinkage fan. Accordingly also the shrinkage limit w_N increases with increasing preparing water content.
- 5) The angle χ of opening of the shrinkage fan is termed the shrinkage region. To determine it, at least two tests are required: A first test with as high a preparing water content as possible gives the shrinking-line with the greatest slope, and a second test



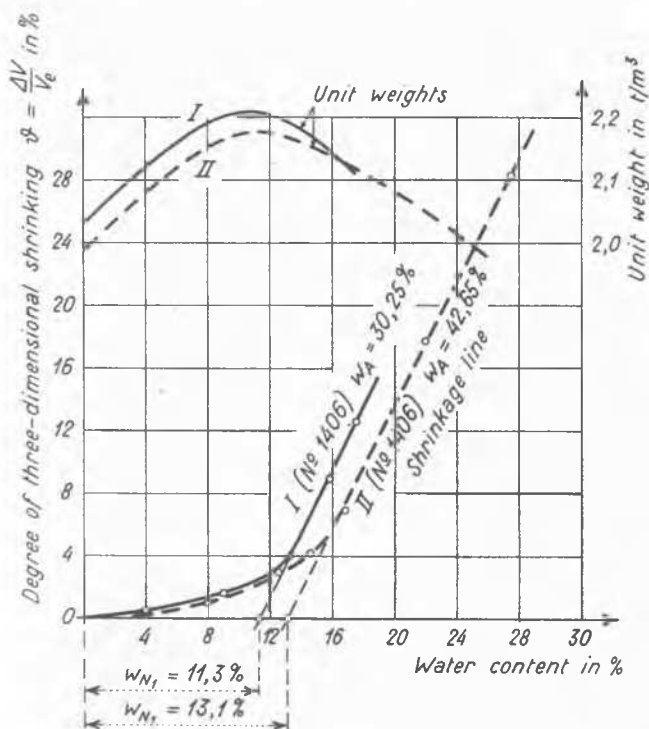
Shrinkage-line (diagrammatic)

FIG. 3



Degree of three-dimensional shrinking for different preparing water contents (diagrammatic).

FIG. 4



Results of two shrinkage tests with the same clay for different preparing water contents.

FIG. 5

with the minimum preparing water content gives the steepest straight line of shrinking.

6) The locus of all preparing water contents, i.e. the line connecting the points A, we define as the preparing water curve. For certain kinds of clay this line is practically straight within the shrinkage fan, for other kinds it is bent. Theoretically it is for the moment not possible to make any more accurate statement with regard to the shape of this line.

The greater the possible variations of the material in question with respect to texture formation, the greater should be the shrinking region. The shrinking region will be caused to vary by additions which, as for instance electrolytes, affect the coagulation and consequently the structure formation to a great extent. It should further be noted that the formulae derived above are not confined to the shrinking operation, but hold good quite generally for any process of consolidation which corresponds to the assumptions made: Continuance of complete saturation with water during the whole process, and identity between the volume of water given up and the corresponding change in volume of the material.

IV. EXPERIMENTAL RESULTS.

In order to check the correctness of the theoretical assumptions, two samples of one and the same clay with different preparing and initial water contents were submitted to the shrinking process. The water content and also the corresponding degree of shrinking, was then determined at different phases of the

shrinking process and plotted graphically (Fig. 5). From the measured unit weights γ_{e1} and γ_{e2} of the dry material, the total degree of shrinkage v_{a1} and v_{a2} and also the known initial water content w_{a1} and w_{a2} , the coefficients w_N , w_S and v_S given in Fig. 2, were calculated on the basis of the equations derived in section 3; their significance can be seen from Fig. 5.

in reality withdraw to a certain extent into the interior of the sample during the first phase of the shrinking process; this was neglected in the theory also the assumed identity between the volume of water given off and the corresponding change in volume is not exactly realized. The measured shrinking centre moves somewhat more to the left in comparison with the calculated centre. But since a satis-

TABLE 2

Evaluation of the test results								
Clay	Test	w_A %	w_a %	γ_e t/m ³	θ_a %	w_N %	w_S %	θ_S %
		← measured →				← calculated →		
1406	I	30,25	17,55	2,023	12,6	11,3	- 38,0	- 100
1406	II	42,5	27,4	1,976	28,3	13,1	- 38,0	- 101

In conjunction with Fig. 5, these results lead to the following conclusions:

- The measured intermediate points of the same shrinkage curve actually lie on a straight line during the first phase of the shrinkage process. The existence of the shrinkage-line is thus proved experimentally.
- The slope of the straight line of shrinking II with higher preparing water content is somewhat less than that of the straight line of shrinking I, and this is in agreement with the theory.
- Test II with the higher preparing water content gives, in agreement with the theory, the lower unit weight γ_e of the dry material but the greater shrinkage limit w_N , than test I.
- The abscissa values w_S of the shrinkage centre S, as calculated independently for each separate degree of shrinking, are practically identical; as anticipated by theory.
- The values ($v_S = -1 = 100\%$) of the ordinate of the shrinking centre, as calculated for each separate degree of shrinking, agree practically completely with the theoretical values.
- The curve of unit weights γ_e obtained when passing over from the first into the second phase of the process, becomes a maximum. This also agrees with our assumptions, since the weight-diminishing influence of the air makes itself perceptible in the second phase.

The experimental proof of the applicability of the theoretical laws has thus been demonstrated fundamentally. Complete accordance between theory and measurement could indeed not be reached. As a rule the slope of the measured shrinkage-lines was always somewhat less than the slope calculated theoretically from the unit weight of the dry material. This can be explained by the fact that the meniscus

factory accordance between theory and measuring has otherwise been determined. The theoretically accepted laws may be adopted as a basis for calculating the shrinking process in the region of the first phase. On the other hand, further explanation 6) and 7) is required for the behaviour of the linear degrees of shrinking, which depend to a very great extent on the orientation of the finest clay laminae and thus bring about the anisotropy of the consolidated material.

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