I c 2

HEAT-FLOW TOWARDS THE FLOOR OF COLD STORES, SITUATED IN THE GROUND LEVEL, AND CALCULATION OF THE INSULATION OR THE HEATING SYSTEM TO PREVENT FROST PENETRATION IN THE GROUND

ROBERT F.X. RUCKLI, Dr. ès sc.techn.Ing.
Inspecteur fédéral des travaux publics, Berne

1. PROBLEM.

The floor of cold stores has to be designed to prevent frost penetration in the ground and its effect, the ice segregation and the uplift of the superstructure. The protection of the basement against frost may be attained by insulation or heating of the floor. The usual methods of calculation assume the ground-temperature to be constant in a fixed depth below the basement (ground water level or 10 m). 1) and 6). Data obtained in England and Russia 2) and 8) from examination of cold stores, where frost heave damage has occurred show, that this assumption is not just. The writer proposes a method of calculation, which conforms more to nature and which demonstrates the distribution of the heat-flow from the subground over the area of cold stores.

2. COLD STORES WITH CIRCULAR AREAS.

a) Analogy with the Artesian well.

The groundwater-flow and the heat-flow are governed by the same differential equation. In case of the same formal boundary conditions, the solution of the hydro-dynamic problems may be transferred to the thermal problems.

Such an analogy exists between the Artesian well with a plane bottom lying in the subplane of the impermeable toplayer and the circular cold store situated on the ground-level if the exterior heat conductibility between the open air and the open ground outside of the store is neglected.

The following are corresponding values: Fig. 1

well:	cold store:
hydrostatic pressure, H	initial ground temperature, ϑ_{II}
piezometric level on the equi-potential-curve defined by b, h	temperature on the isothermal defined by b, ϑ
lowered level during pumping, h_r	floor temperature in the cold room ϑ_I
flowing-off f by pumping Q	loss of heat, Q_W
permeability, k_D	heat-conductivity of the ground λ_e

r_o radius

b) Calculation of the heat-flow to the basement of circular cold store.

The formula for the Artesian well, given by Forchheimer 4),

$$H-h=\frac{Q}{2\pi k_D r_o}\cdot \operatorname{arctg}\frac{r_o}{b} \qquad (1)$$

wherein b is the small axis of the rotation ellipsoid, may be applied in our case by changing the letters as indicated above 2a). Therefore

$$\vartheta_{II}-\vartheta=\frac{Q_W}{2\pi\lambda_e r_o}\cdot \operatorname{arctg}\frac{r_o}{b} \qquad (2)$$

and for the whole difference of temperature

$$\left.\begin{aligned}\vartheta_{II}-\vartheta_I&=\frac{Q_W}{4\lambda_e r_o}\\ Q_W&=(\vartheta_{II}-\vartheta_I)\,4\lambda r_o\end{aligned}\right\} \qquad (3)$$

eq.(3) in(2) substituted results in

$$\vartheta=\vartheta_{II}-\frac{2}{\pi}\left(\vartheta_{II}-\vartheta_I\right)\operatorname{arctg}\frac{r_o}{b} \qquad (4)$$

We get the position b of the zero isothermal in the vertical axis by setting

$$\vartheta=0$$

$$b_{\vartheta=0}=\frac{r_o}{\operatorname{tg}\frac{\pi\vartheta_{II}}{2(\vartheta_{II}-\vartheta_I)}} \qquad (5)$$

In a meridional section the heat stream-lines are confocal hyperboles and the isothermals are confocal ellipses.

The heat-flow through the floor, rising from the ground at a point at the distance x from the middle of the cold store, $x < r_o$ may be calculated by considering two neighbouring hyperboles, whose asymptotes have the inclination β and $\beta+d\beta$ respectively fig. 2. Between these two stream-lines the heat-flow is constant owing to the condition of continuity. The heat, passing through the ring formed by rotation of the element dx round the vertical axis is

$$2\pi x dx\cdot q_x = 2\rho\cos\beta\,.\,\pi\,.\,\rho d\beta\,.\,q_\rho$$

if q_ρ is the specific heat-flow at the great distance of ρ from the origin and q_x the heat-flow at the point x. At the distance ρ from the centre, the isothermal may be considered as a semi-sphere. Therefore we can write

$$q_\rho=\frac{Q_W}{2\rho^2\pi} \qquad (7) \qquad 2\rho^2\pi \text{ area of the semi-sphere}$$

eq. (7) substituted in eq. (6) produces

$$q_x=\frac{Q_W\cdot\cos\beta\,.d\beta}{2\pi\,.\,x\,dx} \qquad (8)$$

On a hyperbole the following relations exist

$$\cos\beta=\frac{x}{r_o}$$

$$\beta=\arccos\frac{x}{r_o}$$

$$\frac{d\beta}{dx}=-\frac{1}{\sqrt{1-\frac{x^2}{r_o^2}}}\cdot\frac{1}{r_o} \qquad (9)$$

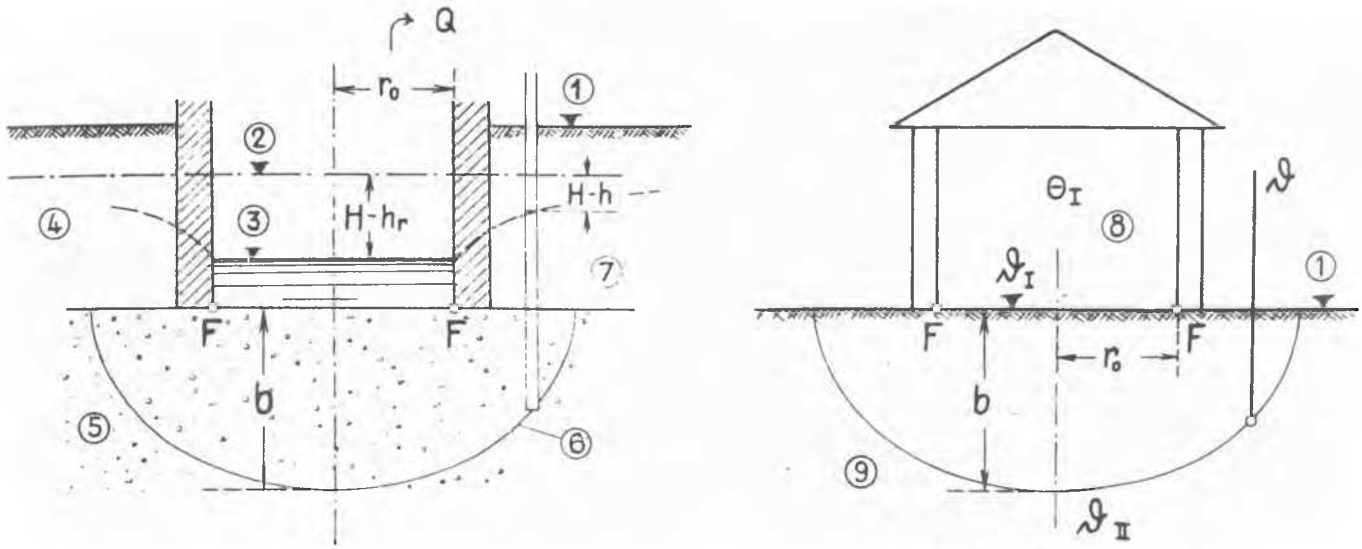
With this value the equation (7) becomes

$$q_x=\frac{Q_W}{2r_o^2\pi\sqrt{1-\frac{x^2}{r_o^2}}} \qquad (10)$$

or

$$q_x=\frac{2\lambda_e(\vartheta_{II}-\vartheta_I)}{r_o\pi\sqrt{1-\frac{x^2}{r_o^2}}} \qquad (10^a)$$

This is the repartition of the heat-flow over the area of the circular cold store.



Artesian Well

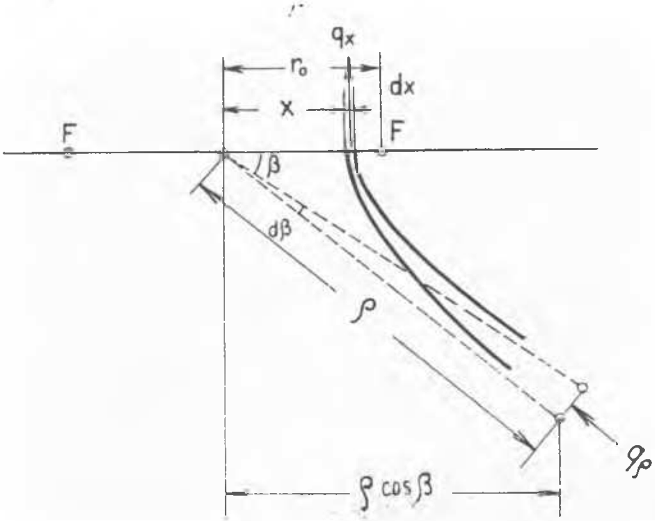
Circular Cold Store

Form of the equipotential faces for an Artesian well and form of the isothermals under a cold store, situated in the ground level.

1) ground level
2) hydrostatic pressure
3) lowered water level in the well
4) impermeable toplayer
5) ground-water stratum
6) equipotential faces, rotation ellipsoid
7) boundary face
8) cold room
9) isothermal, rotation ellipsoid

Q: flowing off
F: foci
Θ: room temperature
ϑ_I: floor temperature
ϑ: soil temperature in the cooling zone
ϑ_{II}: undisturbed soil temperature

FIG. 1



Heat flow rising from the ground towards the basement of a cold store.

FIG. 2

c. Calculation of the insulation.

The insulation of the floor must be designed in a manner to keep the temperature under the basement at $+0°C$. The heat-flow through the basement construction is governed by the equation (5)

$$q = -\frac{\Theta - \vartheta_o}{\frac{1}{\alpha_{fl}} + \frac{S_1}{\lambda_1} + \frac{S_i}{\lambda_i} + \frac{S_2}{\lambda_2}} = -\frac{\Theta - \vartheta_o}{\frac{1}{k}} \quad (11)$$

wherein:

Θ : air temperature of the cold room
ϑ_o : temperature under the basement construction
α_{fl} : surface emissivity or exterior conductibility of the floor (cal/cm^2 h°C)
S_i : thickness of the insulation
S_1, S_2 : thickness of the structural concrete slabs over and under the insulation
$\lambda_i \lambda_1 \lambda_2$: the respective heat conductibilities

$$\frac{1}{k} : \quad \frac{1}{\alpha_{fl}} + \frac{S_1}{\lambda_1} + \frac{S_i}{\lambda_i} + \frac{S_2}{\lambda_2} \quad (11^a)$$

$\frac{1}{k}$: resistance against heat transition
k : ratio of heat transition

The resistance needed against heat transition $\frac{1}{k}$ to keep the zero isothermal in the subface of the basement arises from the equation (11) and (10a);

$$\frac{1}{k} = \frac{\vartheta_o - \Theta}{q_x} = \frac{\pi r_o (\vartheta_o - \Theta)}{2 \lambda_e (\vartheta_{II} - \vartheta_o)} \cdot \sqrt{1 - \frac{x^2}{r_o^2}} \quad (12)$$

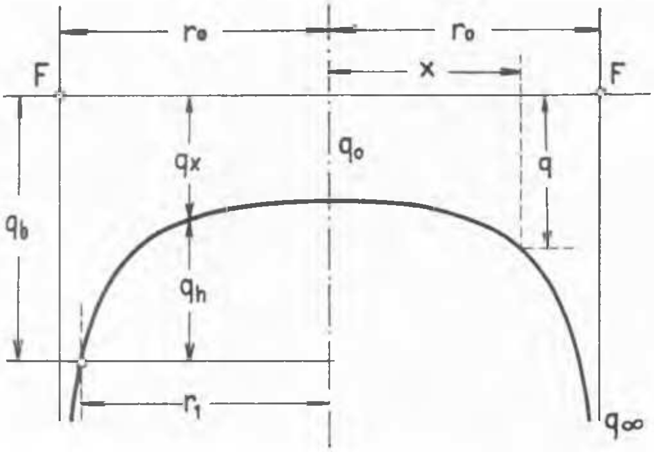
By $\frac{1}{k}$ the thickness S_i of the insulation is determined if the thickness of S_1 and S_2 of the structural concrete slabs over and under the insulation and their respective conductibility. $\lambda_i . \lambda_1 . \lambda_2$ and α_{fl} are given.

d. Calculation of the floor-heating to prevent the penetration of the zero isothermal into the ground.

Instead of a thick insulation it is possible to prevent the freezing of the subsoil by heating the subface of the basement construction as it was indicated by several authors 1),2) and 6).

We stipulate, that the zero isothermal

remains on the subplane of the floor construction (no frost penetration). With a given construction of the floor and a fixed room temperature the heat q_b passing upwards through the floor is determined. If it is greater than the heat q_x rising from the subground, artificial heat q_h must be added, for: Fig. 3



Repartition of the heat flow q_x over the area of a circular cold store.

FIG. 3

$$q_h = q_b - q_x. \qquad (13)$$

At a distance $r < r_o$, from the centre of the cold room we find by the use of the equations (10a), (11) and (13)

$$q_h = k(\vartheta_o - \Theta) - \frac{2\lambda_e(\vartheta_{II} - \vartheta_o)}{r_o \pi \sqrt{1 - \frac{x^2}{r_o^2}}} \qquad (14)$$

and the total heat which is to be added (15)

$$Q_{w_h} = \int_o^{F_1} q_{h_x} dF = r_1^2 \pi k(\vartheta_o - \Theta) - 4\lambda_e(\vartheta_{II} - \vartheta_o)\left[r_o - \sqrt{r_o^2 - r_1^2}\right]$$

r_1 is determined as follows,
F_1 is the circular area of the radius r_1 to be heated artificially, in other words over which the heat-flow q_h is to be integrated; outside the circle r_1, the heat rising from the subground is great enough to prevent the penetration of the zero isothermal without addition of artificial heat. r_1 is determined by the equation (12)

$$\frac{1}{k} = \frac{\pi(\vartheta_o - \Theta)}{2\lambda_e(\vartheta_{II} - \vartheta_o)} \cdot \sqrt{r_o^2 - r_1^2}$$

and

$$r_1 = \sqrt{r_o^2 - \left(\frac{2\lambda_e(\vartheta_{II} - \vartheta_o)}{\pi k(\vartheta_o - \Theta)}\right)^2} \qquad (16)$$

Example:

With $r_o = 10$ m $\quad \Theta = -15°$ $\quad \vartheta_{II} = +10°$
$s_i = 0{,}3$ m
$\alpha_{fi} = 6$ kcal/m2h°C $\quad \lambda_i = 0{,}06$ kcal/mh°C
$\lambda_e = 1{,}5$ " "

we find
$r_1 = 9{,}45$ m
$r_o - \sqrt{r_o^2 - r_1^2} = 6{,}71$ m
$Q_h = 412$ kcal/h $\quad$ or for electric heating
$E_h = 0{,}48$ kW

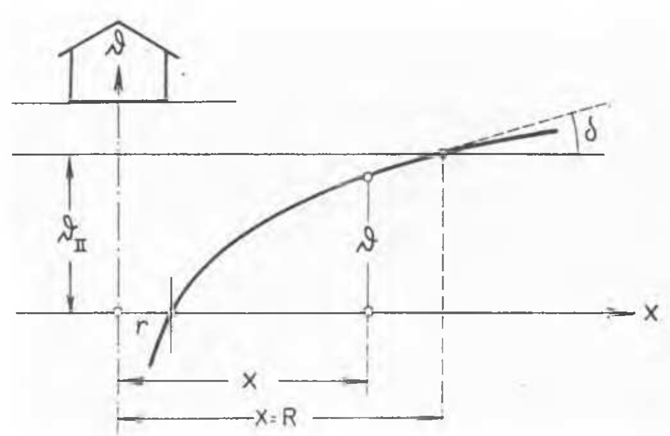
This example shows, that the comsumption of energy is very small.

3. COLD STORES WITH INFINITELY LONG RECTANGULAR AREAS (TWO-DIMENSIONAL HEAT-FLOW.)

The solution found for the circular cold store can also be transferred to the cold store with an infinitely long rectangular area (two-dimensional heat-flow), if we take as the hydro-dynamic analogon the artesian ditch of the constant width 2r, with a plane bottom lying in the subplane of the impermeable toplayer. Neglecting again the exterior heat conductibility between the open air and the ground surface outside the cold store, the heat stream-lines of the plane heat-flow are confocal hyperboles and the isothermals are confocal ellipses. By an analogous development as shown under 2 we find for the heat-flow to the subface of the floor of the infinitely long cold store per unit length 3)

$$\bar{q}_w = -\lambda_e(\vartheta_{II} - \vartheta_o)\frac{\pi}{\frac{2R}{r}} \qquad (17)$$

The steady heat-flow is only possible if we assume, that at the distance R of the axis of the cold store, the soil temperature of the ground surface is not influenced by the cold store. Fig. 4. Theoretically R is infinitely



Temperature in the ground level beside an infinitely long cold store, plotted against the distance from the axis of the store.

FIG. 4

great; but for practical purposes it is allowed to assume, that for a great extent of R the influence of the cold store is negligible. R may be estimated with the assumption, that at the distance R the heat-flow q_{w_k} is, owing to the cold store, only a part n of the geothermic heat q_{w_g} rising from the earth because of the geothermic gradient G;

$$q_{w_k} = n\, q_{w_g} \qquad n < 1 \quad \text{e.g. } \frac{1}{2}$$

By a short mathematical development we find

$$R = \frac{\vartheta_{II}}{n.G.w} \qquad (18)$$

in which

$$w + \ln w = \ln \frac{2\vartheta_{II}}{nGr}$$

The repartition of the heat-flow from the subground to the basement is determined by the equation

$$q_x = \frac{\lambda_e(\vartheta_{II} - \vartheta_o)}{r \ln\frac{2R}{r}\sqrt{1-\frac{x^2}{r^2}}} \qquad (19)$$

and the insulation, needed to prevent the penetration of the zero isothermal in the subground

$$\frac{1}{k} = \frac{r(\vartheta_o - \Theta)}{\lambda_2(\vartheta_{II} - \vartheta_o)} \cdot \ln\frac{2R}{r} \cdot \sqrt{1-\frac{x^2}{r^2}} \qquad (20)$$

If the penetration of the frostline is to be prevented by artificial heating, the heat to be added is at a point x

$$q_{h_x} = k(\vartheta_o - \Theta) - \frac{\lambda_e(\vartheta_{II} - \vartheta_o)}{r \ln\frac{2R}{r}\sqrt{1-\frac{x^2}{r^2}}} \qquad (21)$$

and the heat per unit length of the cold store

$$\bar{q}_h = \int_0^{x_1} 2 q_{h_x} dx = 2k(\vartheta_o - \Theta)x_1 - \frac{2\lambda_e(\vartheta_{II} - \Theta)}{\ln\frac{2R}{r}} \cdot \arcsin\frac{x_1}{r} \qquad (22)$$

wherein

$$x_1 = \sqrt{r^2 - \left(\frac{\lambda_e(\vartheta_{II} - \vartheta_o)}{k(\vartheta_o - \Theta)\ln\frac{2R}{r}}\right)^2} \qquad (23)$$

Example:

r = 10 m Θ = -15° ϑ_{II} = 10° S_i = 0,3 m

λ_i = 0,06 kcal/mh°c

λ_e = 1,5

α_{fl} = 6 kcal/m2h°C.

With n = 0,5 R = 185 m

$\ln\frac{2R}{r} = \ln 37$ = 3,61

x_1 = 9,9 m

and $\bar{q}_h$ = 45,6 kcal/m^1.h

$\bar{e}_h$ = 0,053 kW/m^1.

We compare the heat, needed for the circular cold store, and that needed for a part L of an infinitely long cold store of the same area and the same diameter.

$$r_o^2\pi = 2rL \;.\; r_o = r\;;\; L = \frac{r_o\pi}{2}$$

With r_o = 10 m L = 15,7 m

Heat needed for the: -

1) circular cold store example 1, spatial heat-flow E_1 = 0,48 kW

2) infinitely long cold store, plane heat-flow part 15,7 m long L . e = 15,7 .0,053 E_2 = 0,83 kW

$$\frac{E_2}{E_1} = \frac{0,83}{0,48} = 1,73.$$

Owing to the special heat-flow, the consumption of energy is smaller in the case of a circular cold store, than in a part of an infinitely long cold store of the same diameter and area.

4. MODEL TEST TO EXAMINE THE BOUNDARY CONDITIONS.

Fixing the boundary conditions two extreme assumptions concerning the exterior conductibility or surface emissivity ground-air (α) are possible:

1) $\alpha = 0$: No exterior heat conduction between the ground surface and the open air (outside the cold store).

2) $\alpha = \infty$: Outside the cold store the ground surface has the constant temperature of the air.

The results of a rough model test, made by the writer to examine the usefulness of the method developed above, are shown in fig. 5. Contrary to nature, the temperature difference between the store and the ground was for practicable reasons not produced by refrigeration but by heating the store. The ground was represented by dry sand and the circular store by an electric heated pot, filled with boiling water.

Fig. 5 shows that the assumption 1)(α = 0), which served as the basis of our theory, leads to a form and position of the isotherm under the basement, which is nearer reality than those of assumption 2) ($\alpha = \infty$).

Neglecting α, in other words, setting α = 0, the calculated heat rising from the ground is a little too small; this fact guarantees a higher degree of security.

The heat rising from the subground is in the çentre, which is the critical point, of the circular cold store

1) assumption $\alpha = 0$: $q_o = \frac{2\lambda_e(\vartheta_{II} - \vartheta_o)}{r_o\pi}$ (24)

2) assumption $\alpha = \infty$: $q_o = \frac{4\lambda_e(\vartheta_{II} - \vartheta_o)}{r_o\pi}$ (25)

The value of α has no great influence on the calculated heat-flow. Fig. 6.

Equation (25) corresponds formally to the formula for the spatial groundwater-flow against a circular ice-lens, given by the writer in another paper of this collection 7).

SUMMARY.

For cold stores of circular and of rectangular infinitely long areas, situated on the ground surface, formulas are given to calculate the

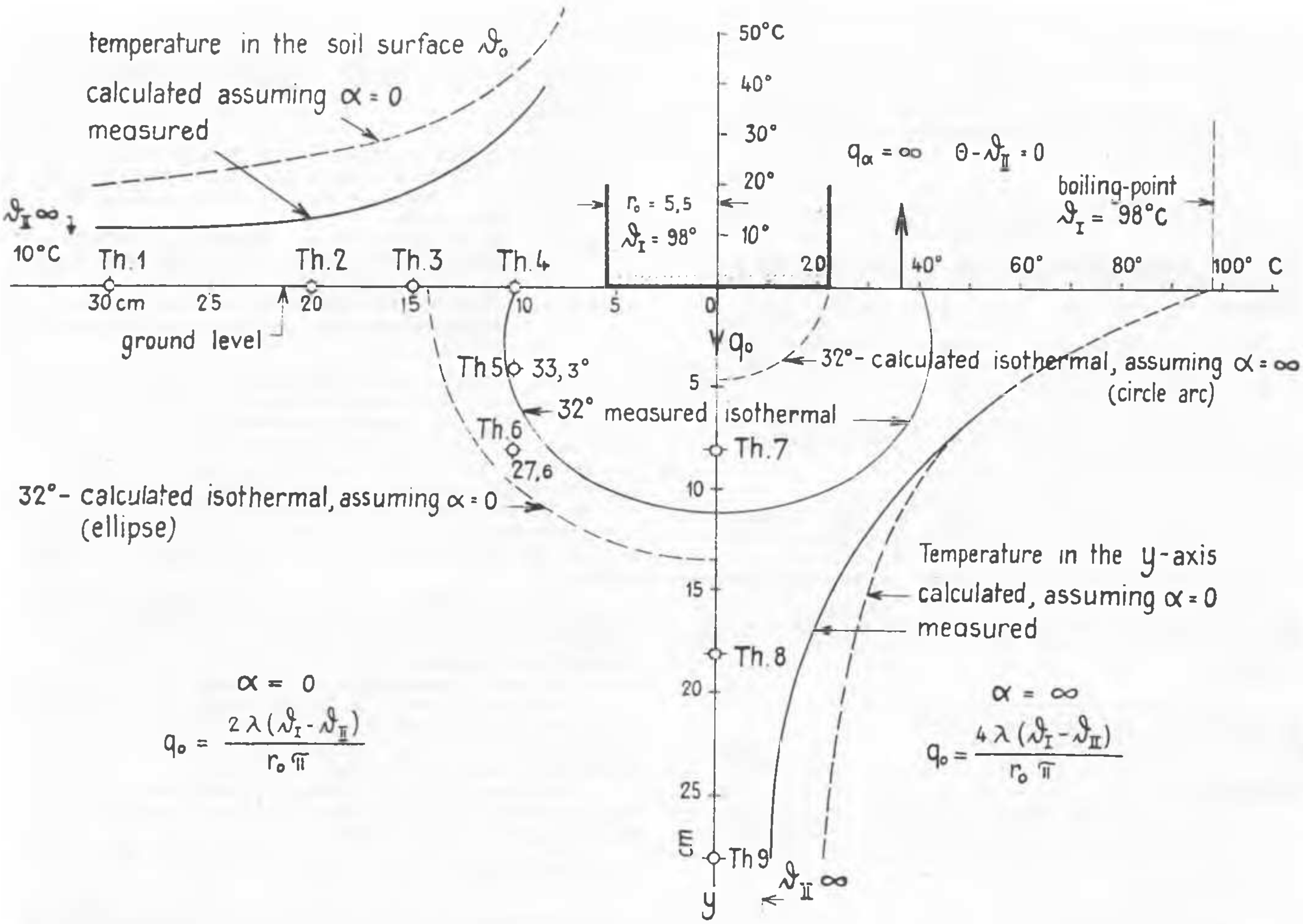
repartion of the heat-flow from the subground to the basement of the store;
the insulation, needed to prevent the frost penetration into the ground;
the heating energy, needed to prevent the frost penetration into the ground for a given basement construction.

The results of a model test are reported to demonstrate that for the calculation of the heat-flow the surface emissivity of the ground outside the cold store can be neglected (setting α = 0).

This report represents an extract of the writer's book "Der Frost im Baugrund" (The Frost in the Sub-soil), which will probably be published in autumn 1948, by the firm "Springer, Vienna".

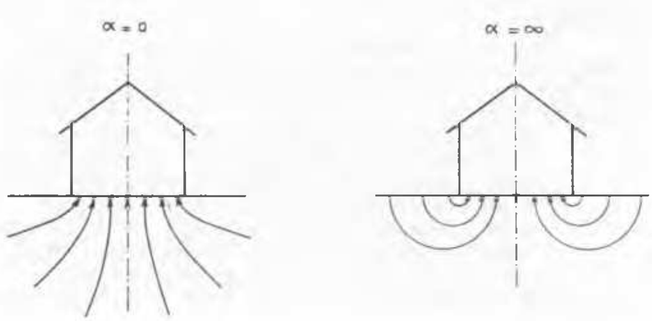
REFERENCES.

1) Cammerer, J.S.: Wärmeschutz mit elektrischer Hilfsheizung. Schweiz. Bauzeitung, 1937, Bd.109. Nr.3. S.34.



Model test for determinating the form of the isothermal under the basement of a cold store.

FIG. 5



Heat stream lines under the basement of a cold store, assuming $\alpha = 0$ and $\alpha = \infty$, respectively.

FIG. 6

2) Cooling u. Ward: Damage to Cold Stores to Frost-Heaving. The Institute of Refrigeration, Session 1944-45.
3) Dachler,R.: Die ebene Grundwasserströmung. Springler, Wien 1936.
4) Forchheimer, Ph.: Hydraulik. III. Aufl. 1930.
5) Hottinger,M. und Imhof, A: Wärmetechnische Grundlagen und Technologie der Isolierstoffe. Schweizer Druck- und Verlagshaus,Zürich 1945.
6) Linde, S.F.: Das Unterfrieren von Kühlhäusern. Wärme- und Kältetechnik. Bd. 44,1942, H.2/3.
7) Ruckli, Rob.: Two- and three-dimensional groundwater-flow towards the ice-lenses formed in the freezing ground. Report to the Second internat. Conf. of soil Mec.and Found. eng. Rotterdam, 1948.
8) Wärmegänge der Kühlhausgründungen. Die Bautechnik, 1936. H.38. S.547.

-o-o-o-o-o-o-