SUB-SECTION I d

STRESS-STRAINS RELATIONS; CONSOLIDATION

I d 1

PROBLEMS OF SOIL-SETTLEMENT

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I. HYDRODYNAMIC SETTLEMENT.

1) The settlement of homogeneous soil of which the voids are filled with water is ruled by the differential equation: $\frac{\partial^2 w}{\partial x^2} = \frac{a}{k} \cdot \frac{\partial w}{\partial t}$

in which:
w = the increment of pressure in the water, caused by the increment if loud, x = the depth on which the named w appears, a = the coefficient of compressibility, K = the coefficient of permeability and T = the time passed since the increment of load is carried on.
In the formula the specific gravity of water = 1
By supposing: $t = \frac{k}{a} T$, the equation is simplified to $\frac{\partial^2 w}{\partial x^2} = \frac{\partial w}{\partial t}$

2) For the case that a layer of soil, continuous till endless depth, of homogenous composition, is limited on the upper side by a horizontal flat plane and is loaded by an equal devided load p, which is stretched unlimited, the boundery conditions for this linear case can be defined as follows:

$w = p$ when $t = 0$ at each value of x;
$w = p$ when $x = \infty$ at each value of t;
$w = 0$ when $x = 0$ at each value of t,

except when $t = 0$.
The solution of the equation is formed by:
$W = f(u)$ in which $u = \frac{x}{2\sqrt{t}}$ By introducing u the partial differential equation becomes the ordinary differential equation:

$$\frac{d^2 f}{du^2} = -2u \frac{df}{du} \qquad \text{suppose} \qquad \frac{df}{du} = f'$$

$$\text{then is} \quad \frac{df'}{du} = -2uf'$$

$$\text{or} \quad \frac{df'}{f} = -2u\,du \text{ . Then} \quad \frac{df}{du} = C_1 e^{-u^2}$$

$$\text{and} \quad f = F(u) = w = C_1 \int e^{-u^2} du$$

Then substitution of u the boundery conditions can be taken together to two specimen:

$w = p$ when $t = 0$
$w = p$ when $x = \infty$ to: $w = p$ when $u = \infty$
and $w = 0$ when $x = 0$; to: $w = 0$ when $u = 0$

The integral $\int e^{-u^2} du$ cannot be expressed in elementary functions; tables exist for the definite integral:

$$\frac{2}{\sqrt{\pi}} \int_0^u e^{-u^2} du = E(u)$$

$E(u) = 1$ when $u = \infty$ and $E(u) = 0$ when $U = 0$.
We can write:

$$\int e^{-u^2} du = \frac{1}{2}\sqrt{\pi}\left\{E(u) + C_2\right\} \qquad \text{so}$$

$$w = C_1 \frac{1}{2}\sqrt{\pi}\left\{E(u) + C_2\right\}$$

The substituation of the boundery conditions gives:

$$C_1 = \frac{2}{\sqrt{\pi}} p \text{ and } C_2 = 0 \,, \qquad \text{so } w = p.E(u)$$

It is preferred to expand the function E (u) in the form of a series:

$$\int e^{-u^2} du = \frac{1}{2}\sqrt{\pi}.\,E(u) = u - \frac{1}{1!} \cdot \frac{u^3}{3} + \frac{1}{2!} \cdot \frac{u^5}{5} - \frac{1}{3!} \cdot \frac{u^7}{7} + \frac{1}{4!} \cdot \frac{u^9}{9} - \text{------ a.s.o}$$

The compression which a part of a soil-layer with thickness D undergoes, is:

$$Z_D = p.a.D - a\int_0^D w\,dx \quad \text{in which} \quad x = 2u\sqrt{t}$$

$$dx = 2\sqrt{t}\,du$$

and for that reason
By performing the integration we get:
$Z_D = p.a.D(1-A)$ in which:

$$A = \frac{2}{\sqrt{\pi}} \cdot \left\{\frac{1}{2} u_D - \frac{1}{1!}\frac{1}{3}\frac{1}{4} \cdot u_D^3 + \frac{1}{2!}\frac{1}{5}\frac{1}{6} \cdot u_D^5 - \frac{1}{3!}\frac{1}{7}\frac{1}{8} u_D^7 \cdots\right\}$$

$$\text{and} \quad u_D = \frac{D}{2\sqrt{t}} = \frac{1}{2} \cdot D \cdot \frac{1}{\sqrt{T}} \cdot \sqrt{\frac{a}{k}} \qquad \text{Suppose}$$

$$\alpha = \frac{1}{4} \cdot \frac{1}{u_0^2} \qquad \text{then} \qquad T = \alpha . \frac{a}{k} . D^2$$

When α =	0	0.25	1	4	25	∞
A =	1.-	0.486	0.270	0.140	0.055	0.000

With the help of these values the time-settlement-curve can be designed (fig.1); it proves

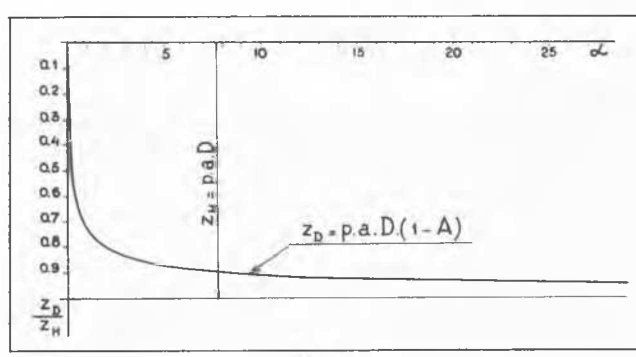

FIG. 1

to exist a final-value $z_h = p.a.D.$, which is reached after an endless time.
The variation of the waterpressure at the depth is for a special time t represented by a dotted curve of fig. 2; these dotted curves are named "Isochrones"

3) A clay-layer, thickness D, on upper- and lower side bordered by a layer of large-

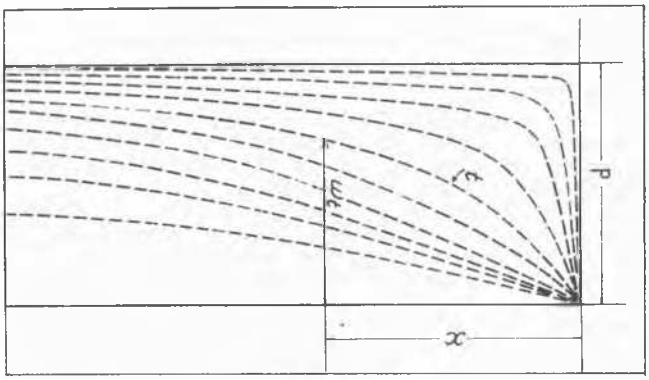

FIG. 2

grained sand (or by porous stone) will show isochrones of symetric shape of which the maximum lies at $x = \frac{1}{2} D$ and for which $w = 0$ when $x = 0$ and when $x = D$ (fig. 3). The cal-

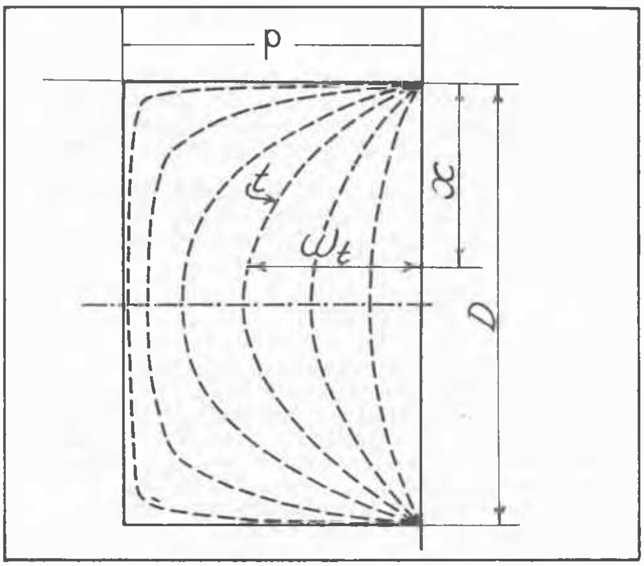

FIG. 3

culation of w and therefore of z in this case can be connected with the proceding case, by the addition of two equal but removed, reflected images of the isochrones of fig. 2 and by subtracting off the function w_D, we got in this way. In fig. 4, in which this construction is carried out can be read:

$$w = w_x + w_{(D-x)} - w_D$$

The settlement amounts to:

$$Z_D = p.a.D - a\int_0^D w\,dx = p.a.D +$$

$$-a\left\{\int_0^D w_x\,d_x + \int_0^D w_{(D-x)}\,dx - \int_0^D w_D\,dx\right\}$$

From fig. 4 is further concluded:

$$\int_0^D w_x\,dx = \int_0^D w_{(D-x)}\,dx$$

while too $\int_0^D w_x\,dx = p.A.D$

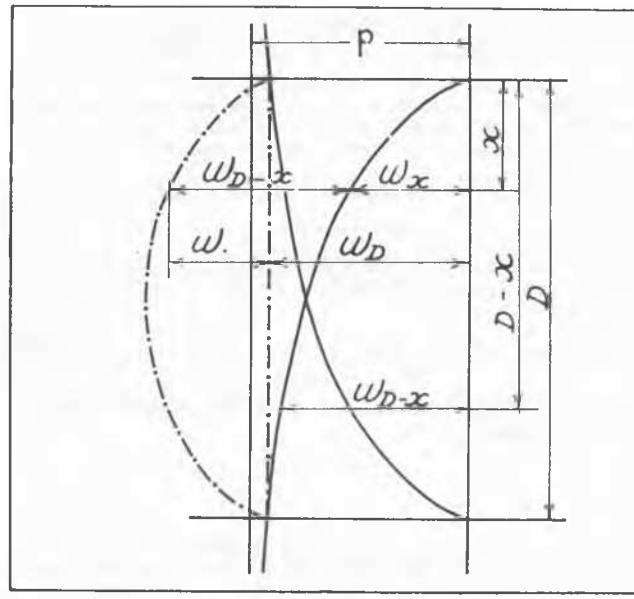

FIG. 4

It follows that $Z_D = p.a\,D(1-B)$ in which

$$B = \frac{2}{\sqrt{\pi}}\cdot\left\{\frac{1}{2!}\cdot\frac{u_D^3}{3} - \frac{2}{3!}\cdot\frac{u_D^5}{5} + \frac{3}{4!}\cdot\frac{u_D^7}{7} - \frac{4\,u_D^9}{5!\,9} + \ldots\text{a.s.o}\right\}$$

If $\alpha = \frac{1}{4}\cdot\frac{1}{u_D^2}$ some values can be calculated :

When α =	0	0.0625	0.25	0.56	1.-	4.-	∞
is B =	1.-	0.190	0.118	0.047	0.022	0.003	0

With the help of these values the time - settlement-curve again can be plotted (fig.5).

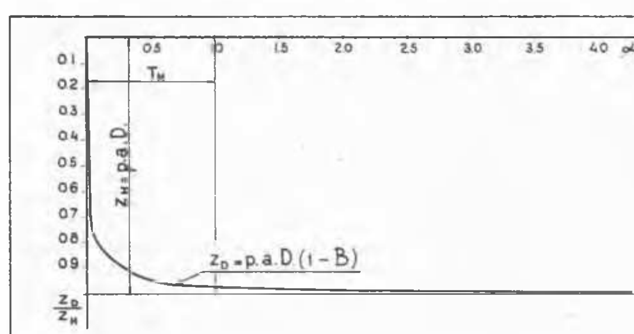

FIG. 5

For $\alpha = 1$ the settlement is almost completed; the corresponding time-interval $T_h = \frac{a}{k}\cdot D^2$ is called the "hydrodynamic period"

4) The formula :$Z_D = p.a.D\,(1-B)$ can be approximated by an orthogonal hyperbole:

$$Z_D = Z_r\left(F\frac{t}{T_h} + 1\right)$$

in which Z_r is the remaining settlement on the time t, ZD is the eventual settlement, T_h = the hydrodynamic period and F is a constant.
A value F = 30 gives $Z_r = 0{,}032\ Z_D$ when $t = T_h$

so that after the hydro-dynamic period the settlement would be fineshed for nearly 97%

5) Suppose D_u = the thickness to which a soil-layer of thickness D at the end can be compressed until there exist no more voids; then the voids-ratio n, defined as the volume of voids per unit volume equals $\frac{D-D_u}{D}$ or

$$n = 1 - \frac{D_u}{D}$$

If the layer is hydro-dynamically compressed by an increment of load p from n_o till n_h then is

$$Z_r = \left(\frac{1}{1-n} - \frac{1}{1-n_h}\right) D_u \qquad Z_D - \left(\frac{1}{1-n_o} - \frac{1}{1-n_h}\right) D_u$$

therefore the formula of the time-settlement curve changes in:

$$\frac{1-n}{n-n_h} \cdot \frac{n_o - n_h}{1-n_o} = F \frac{t}{T_h} + 1$$

6) Kinds of soil exist which show the settlement as may be expected from the mentioned theory. Such soils are defined as "singular soils", they show an ultimate settlement, while at the end of the hydrodynamic period by means of waterpressure-meters can be established that the increase of pressure in the water has disappeared.

II. SECULAR SETTLEMENTS.

1) There too exist kinds of soil which will be named "composed soils" by which the settlement still continues during a long time after the pressure of the water is reduced to the original value.
This "secular settlement" passes very slowly. It cannot be caused by the flow of the "free water" (the water, of which the pressure can be measured in a water-pressure-meter) because the influence hereof is charged already by the calculation of the hydrodynamic-settlement and because as is mentioned too, during the secular settlement there is no increase in pressure concluded in this free water. If a flow of fluid is responsible for this secular settlement this has to be a flow of a liquid with a great viscosity, by which the long duration of the secular settlement becomes acceptable.
Clay-investigations have laid to the opinion that water molecules in the presence of mineral particles of clay by these are aimed and electrically tied; near the mineral particles the density of the absorbed water seems to be so high and the molecules so hard movable that this water behaves as a very sticky fluid.
By supposing that in composed soils there are micro-complexes between the bigger grains, composed of very little mineral-particles close one to another, then the voids of such a complex will be filled by sticky water. This tied water will show a stark retardation in flow by increase of load to the micro-complex; in consequence of this the micro-complex undergoes a stark retarded hydro-dynamic-settlement.
The addition of these micro-settlements delivers the secular-settlement of the soil.
For the limited space we cannot go farther into the subject of the phenomena of different microstructures which are imaginable. No more we can go farther into the calculation of the settlements that follows from here. However it may be evident that the relation between the secular settlement and the time can be approximated by a similar formula as is distracted for the hydro-dynamic settlement; approximately the secular time-settlement-curve can be represented by an orthogonal hyperbole.

2) In the following is considered the settlement of a layer of composed soil that, from the point of time of coming into existence, is influenced by a constant load p.
Let the voids-ratio without load be n_K (characterestic voids-ratio); the load p carried on at the time t = o makes the voids-ratio go down hydro-dynamically till n_m (virgin-voids-ratio). The settlement from n_K till n_m happens in a hydro-dynamic period T_h, which is neglected with respect to the geological period, in which the secular settlement takes place. Under influence of p the soil then becomes secular compressed from n_m till n_s: the secular voids-ratio, which represents the final situation to which the soil comes finally under the influence of p.
This secular course from n_m till n_s may be represented by the formula

$$\frac{1-n}{n-n_s} \quad \frac{n_m - n_s}{1-n_m} = G \frac{\tau}{T_s} + 1$$

completely analogic to the formula derived for the hydro-dynamic settlement. In the formula G is a constant, T_s = the secular period, while n_m and n_s may be taken as yet unknown functions of p.
Plotting this formula as a straight line, by which t is plotted on linear scale, the n-scale becomes a projective scale, which is fixed by the alignment-equation:

$$R = \mu \cdot \frac{1-n}{n-n_s}$$

(in which μ is the chosen lenght.unit)
Of this projective scale the zero falls in n = 1 (for n = 1 when E = 0) while the point n = n_s falls in the infinite (for n = n_s when R = ∞) The scale is dependent on n_s, that is to say that to every n_s, and therefore to every load, belongs a separate n - scale.
The plotting of different scales of various settlement-curves can be escaped in the way as is shown in fig. 6. Besides it is consider-

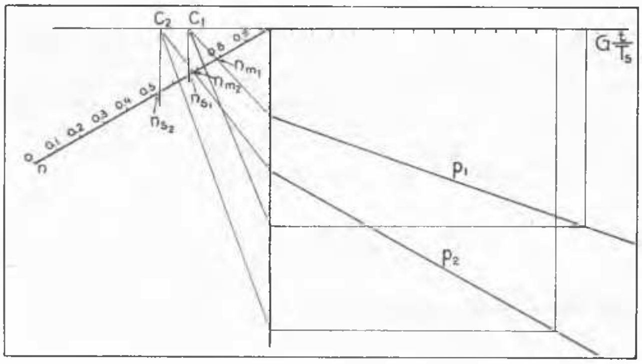

FIG. 6

ed that every projective-scale can be found by projecting from a fixed point a linear scale on a straight line, which cuts this scale. In fig. 6 consequently n is read on the inclined linear scale by using the projection-point C, that belongs to the conserning time-settlement-curve. Consequently every settlement-curve has its own reading-point C, that lays vertically above the corresponding n_s on the n-scale.

3) If at a fixed moment t_h the load p_1 is increased till p_2, the soil will at first obtain a hydro-dynamic settlement from n_1 till n_2, by which n_1 is the voids-ratio that is reached at the moment t_h under influence of p_1, in consequence of the secular line for p_1. From the moment t_h the settlement will take place (T_h is to be neglected again) in consequence of the secular line for p_2, however in such a way that its point n_2 falls in t_h. The original secular line for p_2 thus moves parallel to its own over t_v in the direction of the time-axis. The projection-point C_2 does not change by this movement; the part of the moved line between $t = 0$ and $t = t_h$ becomes fictive. The total course of settlement is given by the drawn line in fig. 7.

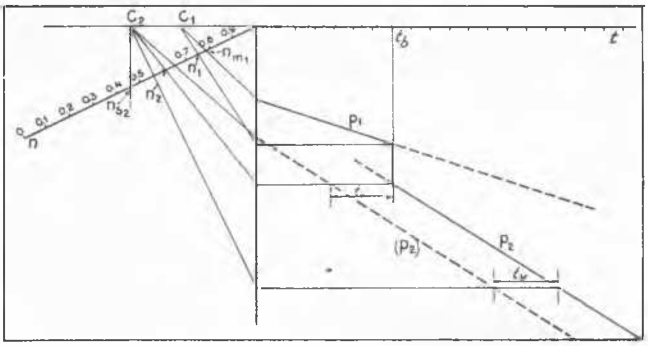

FIG. 7

4) As a projective scale is defined by three points, it is possible to appoint n_s by a fixed load if on three points of time is known the n, that the soil has obtained under this load. If at the points of time t_1, t_2 and t_3 the respective observed voids-ratios are n_1, n_2 and n_3, the construction of n_s can take place in consequence of fig. 8. In this the

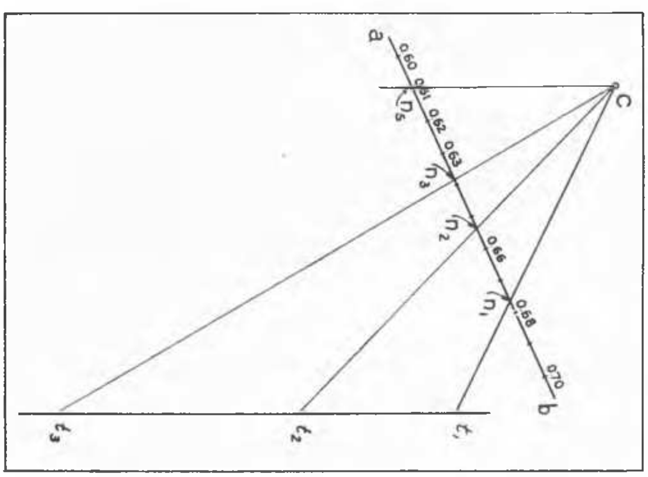

FIG. 8

distances t_3-t_2 and t_2-t_1 are in mutual good proportion but on arbitrary scale put on the vertical axis, while the point C in chosen arbitrarily. Then a linear scale is laid on the figure in this way, that its points n_1, n_2 and n_3 fall on the lines Ct_1, Ct_2 and Ct_3; the ultimate voids-ratio then vertically lays beneath C. The fixing of n_s is the more exactly as the timeperiods between t_1, t_2 and t_3 are longer.

5) The results of some tests of undisturbed claysamples were checked; the results provisionally seem to show that between n_3 and the load p exists the relation:

$$C n_s = \log \frac{M}{p+p_c}$$

in which C is a constant number, M is a modulus and p_c a (very little) pressure. In consequence, for the eventual settlement too the logarithmic relation according to the load-settlement-curve of Von Terzaghi seems to be true. It has been obvious to calculate C and M from the results of experiments.
It is not possible to calculate p_c with any exactness; because, when $p = c$, $n_s = n_k$ and therefore $Cn_k = \log \frac{M}{p_c}$, it was difficult too to fix n_k with sufficient accuracy.
When $n_s = 0$, $\log \frac{M}{p+p_c} = 0$, therefore $M = p + p_c$, or, because p_c is very small $p = M$. Consequently M can be imagined as the load, that is necessary to compress the soil ultimately to a solid stone.

6) As follows from the preceding pages it is possible to prophesy approximately the course of settlement of a ground layer, under influence of different load-increments.
It is not so simple to unravel the prehistory of a groundlayer. If the geological age of a layer is known and if the n_s - curve does not show an angle, n_m (fig. 7) can be obtained for the ground load. In this way the original thickness of the layer is approximated. If the relation between p and n_m was known, perhaps herefrom could be calculated n_m for the groundload, and therefore the geological age of the layer.
If the n_s -curve shows a bob, these calculations become very unsafe. Truely it is possible to know the bigness of the load which formerly burdened the layer;
nevertheless there is not yet found a method to fix the points of time at which this increment of load did begin or end. No more it has been possible to find out whether the layer formerly has been under the influence of a load that is smaller than the present. In general a bob in the load-settlement-curves troubles the prehistory considerably.

7) According to the method Buisman the secular-settlement is set out on a linear and the time on a logarithmic scale, if the zero of the log-scale falls at the time t = 1 (day) the secular-time-settlement-curve becomes a straight line.
The maximum period over which this linear course has been found at a sample is about one year.
Plotting the formula $\frac{1-n}{n-n_s} \cdot \frac{n_m - n_s}{1-n} = G \frac{\tau}{T_s} + 1$

in this way a curve comes into existence as shows fig. 9. Here out would be concluded that the straight line, as follows from the method Buisman, after some years bends and goes more steep, finally reaching a horizontal asymptote. By extra-polation, according to the method Buisman, over some decades so there could be forsaid too small a settlement. Therefore it is important that settlements of constructions are observed over longer periods to ascertain how long for different soils and voids-ratios, is the time-period in which the linear course may safely be extrapolated.

8) During the hydro-dynamic-period the increment of the grain pressure increases grad-

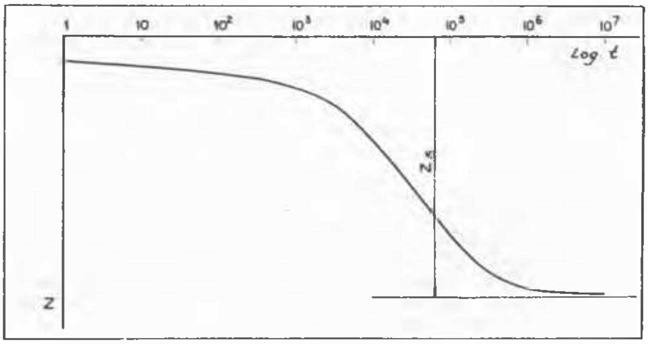

FIG. 9

ually from zero till maximum. The increment of the grainpressure is considered for the secular phenomena as the increment of load; so the load which causes the secular settlement, increases during the hydro-dynamic-period gradually from zero till maximum. By this the secular-settlement is rotarded and the secular-settlement curve deformed. During this period the total settlement then is not obtained by addition of the hydrodynamic and the undeformed secular-curve.
Incidentally may be point out the fact that constructing fig. 8 the moment t is to be chosen outside the area in which this deformation still has a noticeable influence.
The hydro-dynamic settlement shows a layer-thickness-effect (T_h is proportional to the square of the layer thickness) the secular-settlement does not show such an effect.
The deformation of the secular-curve during T_h therefore will be important longer in a layer then in a sample. Interpreting an observed settlement-curve of a layer this has to be considered; in general this curve is not directly comparable with the one, observed by the sample.

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I d 2

PRECISE DETERMINATION OF PRIMARY CONSOLIDATION

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SUMMARY.

In research on Secondary Consolidation of Clays the accurate determination of the end point of theoretical primary consolidation is essential. The accuracy of existing methods of determination is open to question. An improved method has therefore been developed. The paper commences by reviewing the existing methods of curve fitting. Using the new method the theoretical initial and final readings and the coefficient of consolidation can be determined. Examples of the application of the new curve fitting method are given and the accuracy of the results obtained is discussed.

In the analysis of settlement-time results obtained from the consolidation test it is necessary to fit the theoretical relation between percentage consolidation and time factor to the experimental readings. There are at present two methods of carrying out this curve fitting, due respectively to A. Casagrande and D.W. Taylor. 1)

The authors have found in general that these methods give results which are only rarely in agreement or correct. The errors, while not great, become of importance when amounts of secondary consolidation are being measured. Table I shows typical results from

TABLE I

Initial and Final Values of Primary Consolidation by various methods

Test	Material	Load tons/ft^2	Initial Values			Final Values		
			Taylor	Casagrande	New Method	Taylor	Casagrande	New Method
C.14	Recent alluvium	0.285	1940	1940	1952	1151	1166	1161
C.2	Boulder clay	1.60	968	978	968	728	674	723
HC.1	Lias clay	4.00	1650	1648	1650	1360	1338	1332
C.12	Peat	1.50	2690	2710	2631	1485	1449	1449
C.24	Bentonite	0.80	1940	1938	1860	855	854	830