coefficient of elasticity also varies with the amount of sinking as described in the figures 1, 3.

These curves show that the coefficients by which the mechanical properties of soil under considerations are characterised vary during the bearing test.

We assumed that these coefficients are constant, then these facts are to be interpreted as follows, i.c. the average values of these coefficients if these are constant from the beginning of the test to the instant under consideration, vary with the amount of the sinking of the plate.

From these curves we conclude that the resistance of the soil by elastic properties is considerable when the test was begun but shortly this resistance falls down when the sinking of the plate attained some value, say Δ_E and then it decreases slowly.

Contrary to this, the resistance of soil by viscous properties increases steadily and attains its maximum when the sinking of the plate is, say, Δ_V. When the soil is not stiff the viscous resistance decreases when the sinking of the plate passes Δ_V.

The stiffness of the soil is characterised by the value of Δ_E and Δ_V. The stiffer soil has larger Δ_E, Δ_V and $\Delta_V - \Delta_E$

From the above considerations we may take the value of Δ_E or Δ_V as the critical amount of sinking, as the case may be.

If we compare these values with that obtained by Mr. Housel from the consideration that the amount of sinking of the plate is critical when K2 attains its maximum, we conclude that his value is approximately equal to our Δ_V.

Figure 5 shows the results calculated from the data of the test performed by Mr. Housel in 1933, for the foundation consisting of two layers of soil. The curve shows that the resistance of the lower layer is growing after the upper layer loses its power of resisting.

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I e 3

THE STABILITY OF SLOPES ACTED UPON BY PARALLEL SEEPAGE

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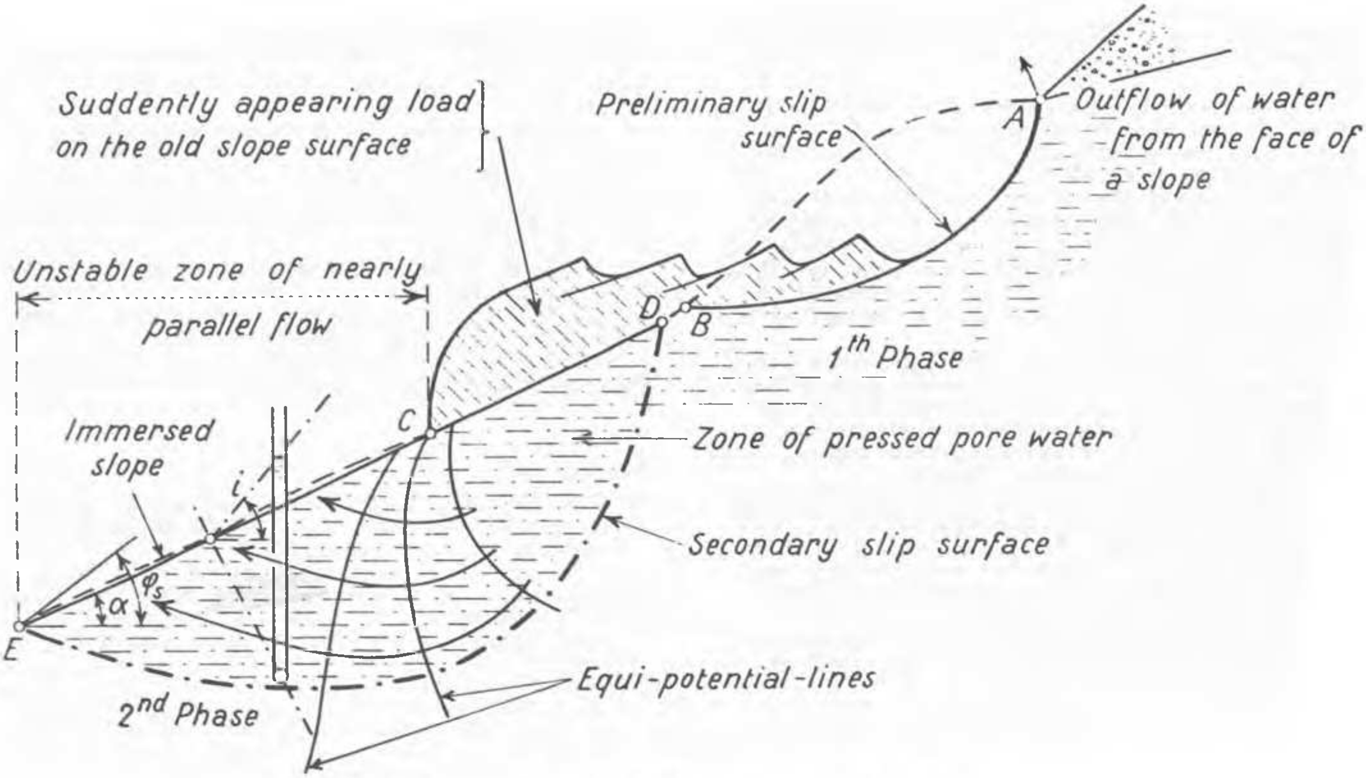
I. INTRODUCTION.

The stability and conditions of inclination of natural and built-up slopes are influenced in a definite manner by the conditions of flow of the water in the pores. Then we have to deal either with steady percolating flow or with the known unsteady hydrodynamic tension phenomena of the pore water. In close connection with this stand above all the slipping phenomena taking place in different phases on hill-sides saturated with water.

Fig. 1 illustrates diagrammatically the two first phases of such a slipping phenomenon. It has been assumed that a source of water on the slope at A has first of all caused a slipping of the steep slope between A en B on a shell-shaped sliding surface. The material fallen down causes a sudden loading on the adjoining zone of slope downwards C - D. Under the action of this load a region of pore water subjected to compressive stressing starts in loam soil which has a very small permeability. A seepage, directed towards the free slope C - E on the valley side, exists in parallel with the gradual expanding of the pore water, and in the neighbourhood of the slope may be regarded approximately as parallel flow. The flow pressures thereby occurring cause a disturbance in the equilibrium of the lower parts of the slope in the form of a fresh sliding as second phase of a process which, by a repetition of such sliding phenomena, progresses downwards step by step and may attain formidable proportions, depending on morphological and geological conditions. The sequence of slip-ends in the form of wave peaks, and sagging zones as wave troughs, gives that greatly cut relief which is characteristic of hill-sides that are in danger of sliding and creeping.

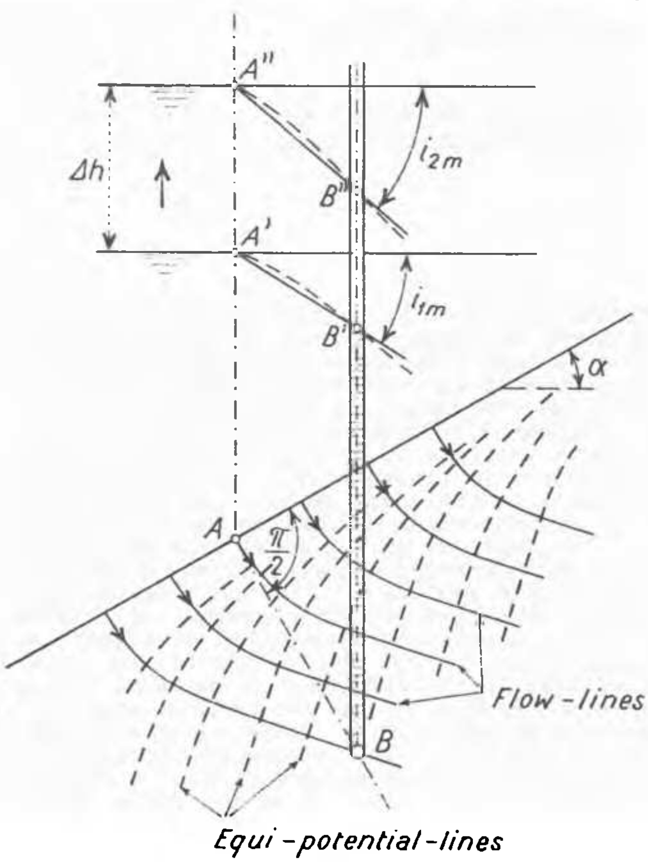
In contrast to the above case of a movement of pore water directed outwards and disturbing the equilibrium, is the case of an percolating flow directed inwards, which acts not as a disturbing factor, but as a stabilising factor on the equilibrium of the slope in question. If, for instance in an earth dam as sketched in Fig. 2, the percolation gradient along a stream line is increased by the damming of the water, this causes an increased lift in the zone of the dam freshly submerged, and with this a relieving of the slope is connected.

Closer investigation of the conditions of stability of a slope with seepage shows that the problem may be regarded as purely static and without separation between solid and liquid phases. In the first place the question of the hydraulic gradient i along a normal to the surface of the slope comes into consider-



Slides and parallel flow. Seepage from the inside to the surface. $i>0$; $\alpha<\varphi_s$.

FIG. 1



Slope under water, seepage from the outside to the inside ($i<o$); $\alpha_{lim}>\varphi_s$.

FIG. 2

ation. As a case for comparison we use the part of the slope under water through which there is no flow, where normal hydrostatic conditions are to be found with vertically directed lifting forces. We construct the ratio between the limiting inclination of the slope with parallel seepage and the slope with no seepage. If this quotient is greater than 1, the seepage has a stabilising effect; if on the other hand it is less than 1, the effect will be to disturb the equilibrium.

Quite a similar problem has been solved by Bernatzik by the introduction of the slope circle 1). A more general treatment of the static conditions of slopes with seepage has been effected by Meyer-Peter, Favre & Müller 2) and 3).

II. CONSIDERATIONS OF EQUILIBRIUM.

The slope of unlimited height, built up with uniform material, is subjected to a parallel flow in any desired direction. To be found is the limiting inclination α_{lim} of the slope with seepage, in which unstable equilibrium has been reached.

The calculation below is based on the following assumptions: The angle φ_s of the apparent internal friction of the material is taken as being identical with the angle of the natural slope of the loose sediment without seepage. Strictly speaking this assumption is only fulfilled in material without any cohesion for which the angles of apparent and of true internal friction are identical. But our theory may be applied with an approximation also to coherent loose sediment, in so far as its compacting corresponds in all points to the locally changeable conditions of tension. In this special case it is also possible to reckon with a constant angle φ_s of the apparent internal friction, which is practically identical with the angle of the slope.

With regard to the pressure conditions in the pore water, the only assumption holding good is that the lines of uniform water pressure (equi-pressure lines), which are not identical with the known lines of uniform potential or piezometer conditions (equi-potential lines) must run parallel to the surface of the slope. This condition is only fulfilled with parallel flows when the surface of the slope is free. If the slope lies under water, as for instance in Fig. 2, the following theory is not applicable. On the other hand the assumption made regarding the run of the lines of uniform water pressure with free surface of slope is as a rule fulfilled even in the case of bent parallel flow (Fig. 7a).

For solving the present problem we first of all consider the equilibrium of a prismatic element coinciding with the surface of the slope, the element consisting of solid and liquid phases and being completely saturated (Fig. 3).

The base dF of the mentioned element dV forms a possible sliding surface running parallel to the surface of the slope, and is at the same time a surface of uniform water pressure. In the limiting case of equilibrium, the resulting tension p on the surface dF makes with the normals to the surface the angle φ_s, i.e. the following hold good:

$$\frac{\tau}{\sigma} = \operatorname{tg} \varphi_s \quad \Big| \quad \alpha_{lim} + \delta = \varphi_s \tag{1}$$

The following forces act on the element dV: Its own weight dG, the symmetrical lateral forces dE which mutually cancel each other, and the lift dP. From the partial forces dG and dP the tensions σ and τ acting on the surface dF can be calculated as follows:

Weight : $dG = \gamma_e' \, dV$

Lift : $$dP = dp \cdot dF = \frac{dp}{dy} \cdot dV \tag{2}$$

According to Fig. 2 the pressure gradient $\frac{dp}{dy}$ is determined first of all for any desired angle $\alpha < \alpha_{lim}$

$$\left.\begin{aligned} \frac{dp}{\gamma_w} &= dy \cos\alpha + dy \sin\alpha \operatorname{tg} i \\ \frac{dp}{dy} &= (\cos\alpha + \sin\alpha \operatorname{tg} i)\, \gamma_w \end{aligned}\right\} \tag{3}$$

Substituting equation (3) in equation (2) we have:

$$dP = (\cos\alpha + \sin\alpha \operatorname{tg} i)\, \gamma_w \, dV \tag{4}$$

By introducing this value of dP into the equilibrium conditions (2), the tensions τ and σ are obtained, whose quotient in the limiting case according to equation (1) gives the limiting angle α_{lim} which was sought.

$$\sigma dF = dG \cos\alpha - dP = (\gamma_e' \cos\alpha - \cos\alpha \, \gamma_w - \sin\alpha \operatorname{tg} i \, \gamma_w)\, dV$$

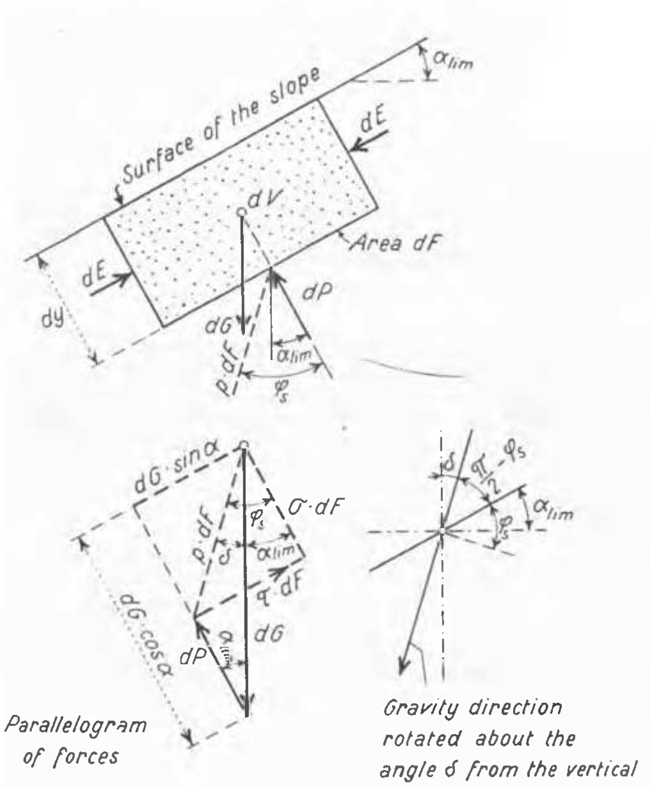
$$\sigma dF = [\cos\alpha(\gamma_e' - \gamma_w) - \sin\alpha \operatorname{tg} i \, \gamma_w]\, dV \tag{5}$$

$$\tau dF = \gamma_e' \sin\alpha \, dV \tag{6}$$

$$\frac{\sigma_{lim}}{\tau} = \left(1 - \frac{\gamma_w}{\gamma_e'}\right) \operatorname{ctg} \alpha_{lim} - \frac{\gamma_w}{\gamma_e'} \operatorname{tg} i = \operatorname{ctg} \varphi_s \tag{6a}$$

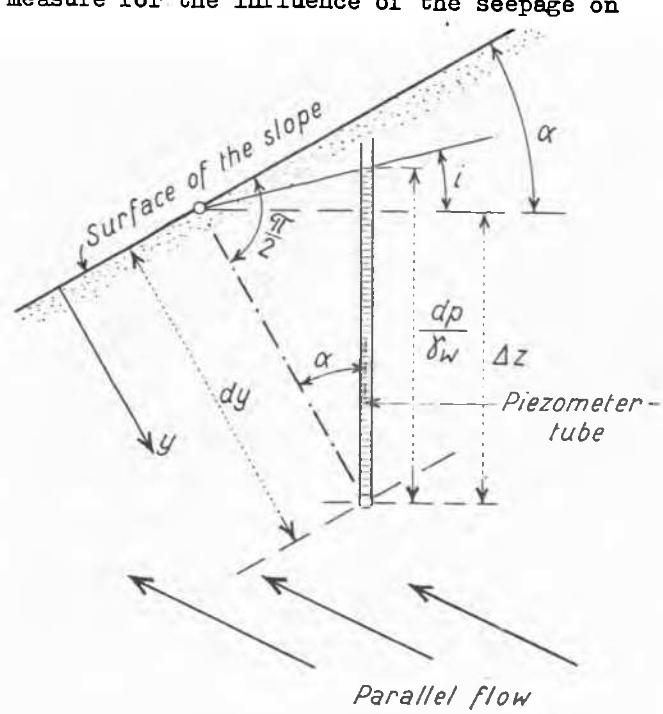
$$\operatorname{tg} \alpha_{lim} = \frac{\gamma_e' - \gamma_w}{\gamma_e' \operatorname{ctg} \varphi_s + \gamma_w \operatorname{tg} i} \tag{7}$$

The coefficient factor ζ serves as a measure for the influence of the seepage on



Considerations of equilibrium of a prismatic element.

FIG. 3



Definition of the decisive hydraulic gradient tg i.

FIG. 4

the angle of inclination of the slope in the limiting position of equilibrium:

$$\zeta = \frac{tg\alpha_{lim}}{tg\varphi_s} = \frac{\gamma_e' - \gamma_w}{\gamma_e' + \gamma_w \, tgi \, tg\varphi_s} = \frac{\frac{\gamma_e'}{\gamma_w} - 1}{\frac{\gamma_e'}{\gamma_w} + tgi \; tg\varphi_s}$$

$$\zeta = \frac{\lambda' - 1}{\lambda' + u}; \; \lambda' = \frac{\gamma'_e}{\gamma_w} \quad u = tgi \; tg\varphi_s \qquad (8)$$

This equation shows that, with a given material which is characterised by its unit weight at saturation (γ'_e) and its angle to the apparent internal friction φ_s, the limiting angle α_{lim} of the slope with seepage depends solely and alone on the hydraulic gradient i along a normal to the slope. Amongst others the following special cases are of interest (Fig. 4) :

a) The limiting angle of the slope with seepage reaches a right angle, i.e.

$$\alpha = \frac{\pi}{2} \text{ and } \zeta = \infty$$

Condition: $u = -\lambda'$; $tgi = -\frac{\gamma_e'}{\gamma_w} \, ctg\varphi_s$

b) The limiting angle α_{lim} of the slope with seepage is equal to that of the slope without seepage, i.e.

$$\alpha_{lim} = \varphi_s; \; \zeta = 1$$

Condition: $u = -1$; $tgi = -ctg\varphi_s$; $i = \varphi_s - \frac{\pi}{2}$

In this case the slope has seepage vertically from above.

c) The flow is parallel to the surface of the slope. Consequently there cannot be any flow gradient at right angles to the slope, i.e. i becomes = 0 and thereby $u = 0$. Equation (8) then passes over into the condition derived at another place (4).

$$\zeta = \frac{\lambda' - 1}{\lambda'} = \frac{\gamma_e' - \gamma_w}{\gamma_e'} = \frac{\gamma_e''}{\gamma_w} \qquad (9)$$

d) With a gradual increasing of the hydraulic gradient i (Fig.4), the limiting angles α_{lim} become theoretically always smaller until finally the horizontal is reached ($\alpha_{lim} = 0$ for $i = \infty$). This extrapolation, however, does not hold good whenever the gradient i is exceeded at which hydraulic rupture of the soil occurs. If the gradient i is increased beyond this critical value, it is no longer the danger of slipping but the danger of soil rupture which is decisive for stability. Equation (8) then becomes of no importance. In order to find the position of transition from the region in which sliding conditions hold good to that of the usual soil rupture condition, we calculate the respective limiting angle α_K by introducing as first approximation in equation (8) the critical gradient

$i_k = \frac{\gamma_e''}{\gamma_w}$ for material without cohesion.

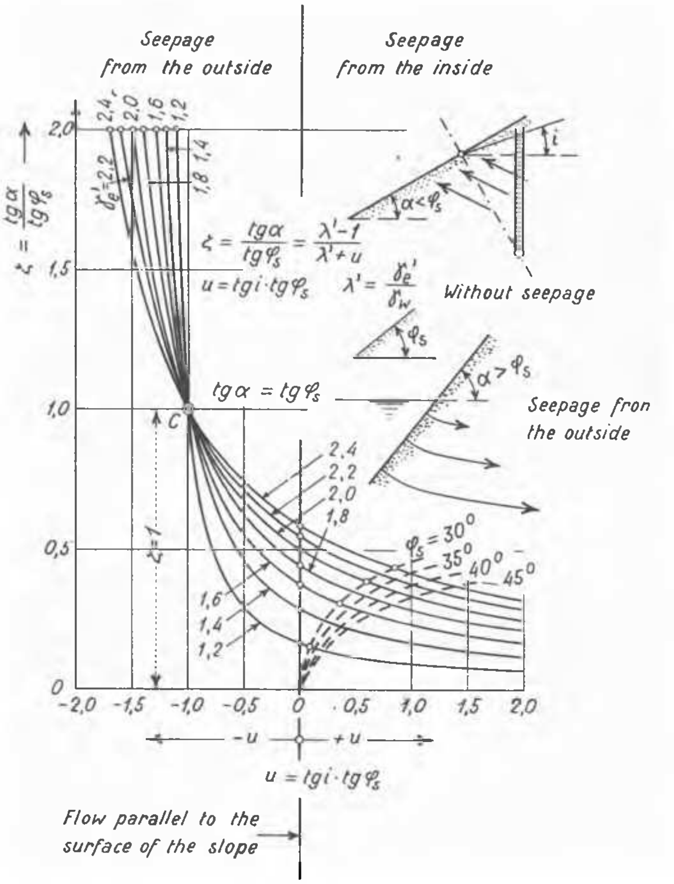
$$\zeta_K \cong \frac{\gamma_e''}{\gamma_e' + \gamma_w \frac{\gamma_e''}{\gamma_w} \cdot tg\varphi_s} = \frac{1}{\frac{\gamma_e'}{\gamma_e''} + tg\varphi_s} \qquad (10)$$

where γ_e'' means the specific gravity under water, i.e. $\gamma_e'' = \gamma'_e - \gamma_w$.

From equation (10) it also follows:

$$tg\alpha_K \cong \zeta_K \; tg\varphi_s = \frac{1}{1 + \frac{\gamma_e'}{\gamma_e''} ctg\varphi_s} \qquad (11)$$

Since the deriving of the critical gradient for soil rupture is with reference to a flow directed vertically from below to above, whilst the normal to the slope in which the gradient i is measured deviates from the vertical by the angle α_k, equations (10) and (11) hold good only approximately.
In addition they can only be adopted for material without cohesion, since we have taken the tensile strength of the material = 0 when deriving the critical gradient i_k. 5).



Ratio ζ of the natural stability of the slope with and without seepage in function of the decisive hydraulic gradient tg i.

FIG. 5

In Fig. 5 the coefficient factors ζ are represented as parameter as a function of the gradient i for different unit weights γ_e'. This group of curves shows that seepage from within requires a considerable reduction of the limiting angle α_{lim} with respect to the angle φ_s of the natural slope. As transition between seepage from within to percolation from the outside, the case $u = 0$ or $i = 0$ appears, for which the flow is directed parallel to the surface of the slope. From the point $u = 0$ the ζ curves lie steeply to the common point of intersection C, which is given by the ordinate $\zeta = 1$ and the abscissa $u = -1$. In this case the seepage - which takes place here from the outside - has no influence on the stability on the slope ($\alpha_{lim} = \varphi_s$).
After closer consideration it is found that we are dealing here with a percolation of the material in the vertical direction from above to below. To the left of C the curves approach their corresponding vertical asymptotes in a steep rise, the position of the asymptotes being given by the abscissa sections $u = -\lambda'$

The corresponding limiting angle amounts to $\alpha_{lim} = \frac{\pi}{2}$ which corresponds to a vertical slope. At the righthand side of the diagram the validity of the group of curves is limited by the region of validity of the soil rupture condition, whose extent also depends on φ_s. As an example the limiting lines for φ_s = 30°, 35°, 40° and 45° have been plotted in chain-dotted lines. Beyond these limiting lines the gradient i cannot be increased, even with a horizontal material surface, without the occurrence of rupture of the ground.

III. CONCLUSIONS.

As can be seen clearly from the above theory, in determining the stability of a slope saturated with water, the velocity of flow plays no part; it depends exclusively on the pressure ratio in the pore water, or on the run of the lines of uniform water pressure (equi-pressure lines) which are not to be confused with the lines of uniform potential (equi-potential lines). Of course this holds good only under the supposition that no fine particles are swept away by the percolation process

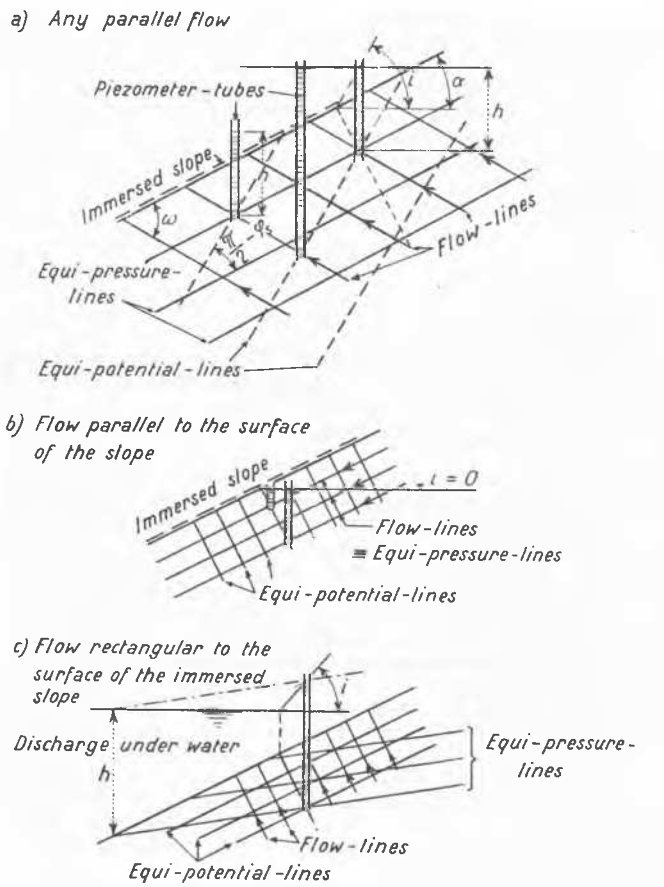


FIG. 6

or are chemically dissolved.

In the case of little-pervious fine-grained slopes percolation can very often be scarcely perceived, the evaporation on the surface of the slope being sufficient to remove the issuing water; this is very frequently the case in dams. Although in this case the velocities of flow are practically = 0, the slope in question is nevertheless subjected to unfavourable conditions of stability for the material through which the flow occurs, since it just depends on the run of the equi-pressure lines which, like the flow picture, are known to be independent of the coefficient of permeability and thereby of the speed of percolation.

In order to make clear the difference between the equi-pressure and the equi-potential lines, three different cases of parallel flow are illustrated in Fig.6. Fig.6a shows a flow from inside directed at an angle ω to the slope, where the equi-pressure lines, which here run parallel to the surface of the slope, enclose with the equi-potential lines the angle $(\frac{\pi}{2}-\omega)$. Fig. 6b shows a flow parallel to the surface of the slope, where the flow and equi-pressure lines coincide, whilst the equi-potential lines are at right angles to the slope. In these two cases a) and b) the surface of the slope will be slightly flooded over. Case c) shows a parallel flow at right angles to the slope and directed outwards, as is normally observed below the surface of the water. Here the equi-potential lines are parallel to the slope, but the equi-pressure lines on the other hand are directed obliquely to it. Our theory is therefore not applicable in this case. Only if the discharge takes place on the free slope, with-

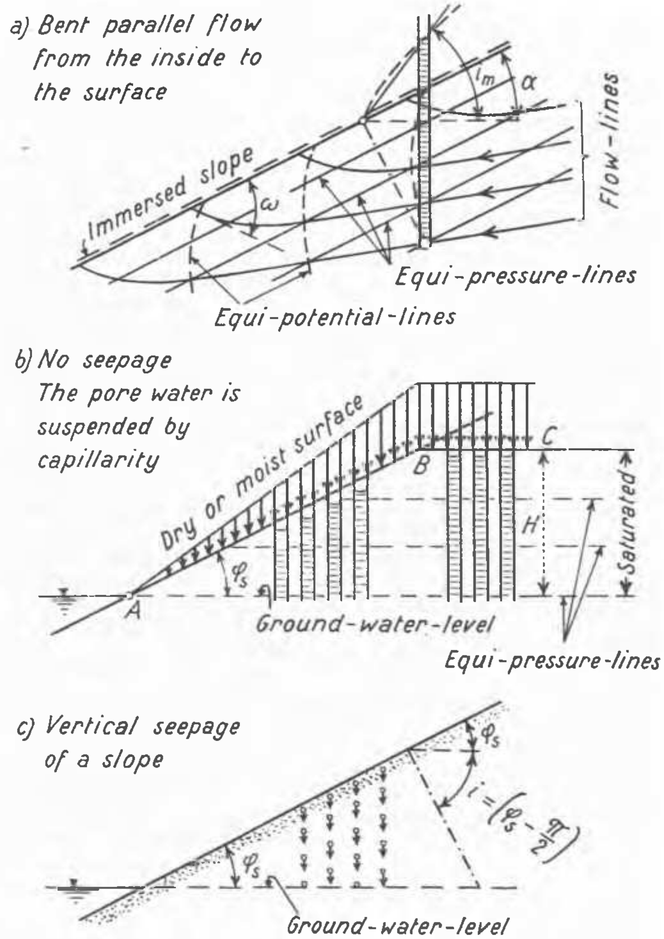


FIG. 7

out being submerged, so that the slope is only slightly flooded over, the lines of uniform potential and uniform pressure will also in case c) be identical and parallel to the surface of the slope and thereby fulfil the assumptions of the theory.

Three further cases of practical importance are illustrated in Fig. 7. Fig. 7a shows a bent parallel flow issuing on the slope at an acute angle ω, whilst in Fig. 7b the circumstances are indicated which arise under the influence of capillary forces with no flow and horizontal ground-water level. The fact is important that the equi-pressure lines in this case run horizontally, so that the lifting forces, which are also effective in the closed capillary region, remain vertical. The equilibrium of such a slope may therefore be investigated in the usual manner with bent sliding surfaces. The weights of the solid and liquid phases may then be inserted in the calculation either separately, by introducing the capillary forces as an external loading; or such a separation is not adopted, the unit weight γ_e' of the saturated material above water being introduced into the calculation, a proceeding which is much simpler. If the water evaporates on the surface of the slope, a certain percolating movement directed upwards takes place in consequence of the capillary rise above the groundwater level. No change is thereby made in the external static conditions of the slope, since the frictional forces that occur neutralise each other mutually as action and reaction in the interior of the static system. As polar opposite of the capillary rise of the water, the case of vertical parallel flow from above to below is illustrated in Fig. 7c ; this may occur for instance with rain of with the melting of snow. The vertical percolation of the slope is here characterised by the pressure of the pore water at all points, and therefore also along a normal to the slope, being everywhere equal to atmospheric pressure, i.e. $i = \varphi_s - \frac{\pi}{2}$ and $u = -1$

This case thus corresponds to the point of intersection C in Fig. 5, for which, in accordance with what has been said already, the limiting angle α_{lim} becomes identical with φ_s.

The above comparisons show that in general the slight flooding of the surface of the slope has an unfavourable influence on its stability. Account must be taken of this fact especially when slopes are being drained. Certainly, even if there is no flooding, seepage flow directed down the slope may occur within the closed capillary region, and this has an adverse effect on the equilibrium, but it will take place as a rule with a very small gradient i. On the other hand, it may frequently be observed that the evaporating surface withdraws into the interior of the slope and an evaporating crust is formed through drying, and this acts as a stabiliser. Also the latest attempts to improve the stability of slopes with seepage by adopting electro-osmotic methods 7) should be noted in this connection.

If it is desired to investigate the conditions of equilibrium of a slope with an angle of inclination α and parallel seepage, this slope being in a stable condition, two different methods may be adopted with a given gradient i: Either the limiting inclination $tg\alpha_{lim}$ is calculated according to equation(7) and the degree of safety N is found in the form:

$$N = \frac{tg\alpha_{lim}}{tg\alpha} \qquad (12)$$

or the angle δ is calculated, which is included by the parallel directed resultants pdF of own weight and lift with the verticals (cf. Fig. 3 . This angle δ therefore gives particularly valuable conclusions, since the influence of the parallel flow on the equilibrium of the slope may be regarded as a field of gravity rotated about the angle and somewhat reduced in intensity. The criterion for the equilibrium of the slope is then:

$$\alpha \leqq \varphi_s - \delta \qquad (13)$$

(cf. Fig. 13)

The angle δ is easily calculated from equation (6') by substituting the sum of the angles $\alpha + \delta$ for φ_s ; then we obtain:

$$tg\,\delta \quad \frac{\overbrace{1 + tg\,i \;\; tg\alpha}^{u}}{\frac{\gamma_e'}{\gamma_w} tg\alpha + \frac{\gamma_e''}{\gamma_w} ctg\alpha - tg\,i} \qquad (14)$$

From this equation the rapid increase of δ with increasing gradient i can be clearly seen.

Finally it should once again be pointed out that the above theory, derived in the first place for cohesionless material, is only applicable to a coherent material if the cohesion increases from 0 at the surface of the slope proportionally to the depth, so that the a-line of the shear diagram remains valid at all points of the slope, corresponding to a constant angle φ_s of the apparent internal friction. Such cohesion conditions may for instance arise with the compacting of fresh sediment under its own weight, also the modulus of plasticity then increasing in proportion to the depth 5) and 6). If on the other hand a loose sediment of another kind of cohesion conditions - for instance with constant cohesion - as considered the stability of the slope with seepage may be examined only with the help of known methods by adopting bent sliding surfaces.

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