I e 4

LIMITATION OF THE VALIDITY OF APPLICATION OF THE FORMULAS FROM PRANDTL-BUISMAN AND FROM ANDERSEN FOR THE ULTIMATE BEARING CAPACITY OF THE SOIL UNDERNEATH FOOTINGS.

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The formula of Prandtl gives the value of the ultimate bearing capacity of an incompressible material with a cohesion c and an angle of friction ϕ, not subjected to gravity and loaded at its surface. The surface of failure has the shape of a logarithmic spiral situated between two straight lines (fig. 1). Buisman

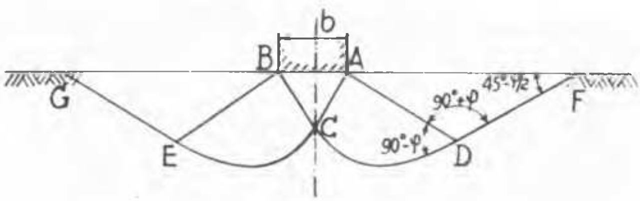

FIG. 1

first completed the formula of Prandtl for the case of an uniform overload p_b on the surface of the material beside the loaded area. The existence of such an overload doesn't alter the shape of the surface of failure.

Going further, and with the purpose to take into account the own weight γ of the material, Buisman assumed that, even if the material is subjected to gravity, the shape of the surface of failure is not altered; thus it becomes easy to find an expression for the part of the ultimate bearing capacity produced by the own weight of the soil located above the surface of failure.

With the assumption that the most dangerous surface of sliding is a logarithmic spiral with two straight extensions, Prandtl-Buisman found for the ultimate bearing capacity d_g the following expression:

$$d_g = V_b p_b + V_c c + V_g \gamma b \qquad (1)$$

where

- d_g = the ultimate bearing capacity (t/m^2)
- p_b = the overload existing beside the footing (t/m^2)
- c = the cohesion (t/m^2)
- γ = that part of the volume-weight of the soil, which is to be taken into account for the computation of the effective stresses.
- b = the width of the footing.
- V_b, V_c and V_g = three functions of the angle of friction ϕ

$$V_b = e^{\pi tg\phi} tg^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right) \qquad (2)$$

$$V_c = e^{\pi tg\phi}\,\frac{2\cos\phi}{1-\sin\phi} + \left(e^{\pi tg\phi}-1\right)\cot\phi \qquad (3)$$

For V_g the following analytical expression was found by Prof. Raes:

$$V_g = \frac{1}{8}\left[\frac{1+tg^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)}{1+9tg^2\phi}\left\{\left[3tg\phi\, tg\left(\frac{\pi}{4}-\frac{\phi}{2}\right)-1\right]e^{\frac{3}{2}\pi tg\phi} + 3tg\phi + tg\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\right\} + 2e^{\frac{3}{2}\pi tg\phi}\, tg^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right) + -2tg\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\right] \qquad (4)$$

When the foundation is established at a depth h underneath the surface of the soil, the phreatic level being located at the surface of soil or at a great depth underneath the foundation level, one gets

$$p_b = \gamma h \qquad (5)$$

thus

$$d_g = V_b \gamma h + V_c c + V_g \gamma b \qquad (6)$$

The formula (6) is the expression of the

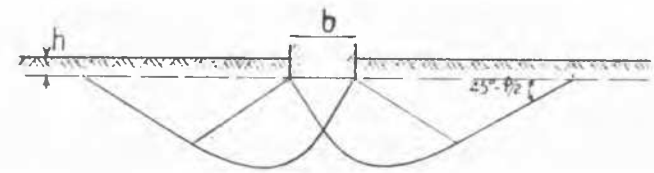

FIG. 2

ultimate bearing capacity for the case of fig. 2. The only arbitrary assumptions are that the material is incompressible and that the most dangerous surface of sliding is a logarithmic spiral with 2 straight extensions.

On the other hand prof. Andersen of the University of Minnesota established an analytical expression for the ultimate bearing capacity of a material c, ϕ, γ, underneath a footing with a breadth b established at a depth h, starting from the arbitrary assumption that the most dangerous surface of sliding is composed by two circles with radius r and r + h (fig. 3).

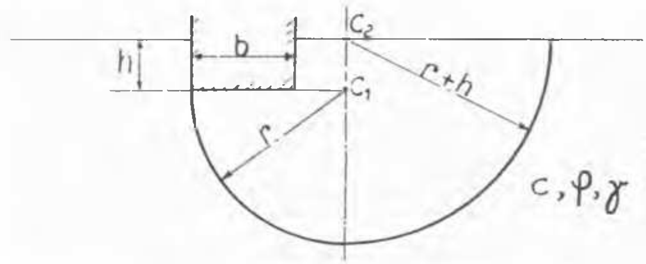

FIG. 3

The radius r of the most dangerous circular surface, and the value of the ultimate bearing capacity corresponding to circular surfaces d'_g, are given by the following system of equations:

$$2r^2 tg\phi + r\left[(2tg\phi+1)h + \frac{\pi c}{\gamma}\right] + \frac{2tg\phi+1}{2}h^2 + \frac{\pi c}{2\gamma}h = \frac{bd'_g}{4\gamma}\left(2-\frac{\pi}{2}tg\phi\right) \qquad (7)$$

$$\frac{2\gamma}{3}\left[r^3(2tg\phi-1)+(r+h)^3(2tg\phi+1)\right] + \pi c\left[r^2+(r+h)^2\right] = bd'_g\left[r\left(2-\frac{\pi}{2}tg\phi\right)-b\right] \qquad (8)$$

The formulas of Prandtl-Buisman, as well as those of Andersen are based on an arbitrary assumption: namely the shape of the most dangerous surfaces of sliding. Thus it is necessary to compare in each particular case the values obtained by the 2 methods. Indeed, in this way can be found wether of the two values d_g or d'_g is the smaller for the given case, and thus has to be retained.

Here is considered the case of a material without cohesion, $c = 0$, and for this case the limits of the zones of applicability of the formulas of Buisman and Andersen are determined.

Eliminating r between the equations (7) and (8) for $c = 0$, an equation of the following shape is obtained:

$$b^2 - m_1 \frac{\gamma h^3}{d'_g} + m_2 bh - m_3 \frac{\gamma h^4}{d'_g b} + m_4 h^2 + m_5 \frac{\gamma^2 h^6}{d'^2_g b^2} + \\ - m_6 \frac{d'_g b}{\gamma} = 0 \qquad (9)$$

where

$m_1, m_2 \ldots\ldots\ldots\ldots m_6$ = 6 functions of the angle ϕ.

Instead of considering the ultimate bearing capacity for the unit of area d'_g, the computations become in this case more simple, when the ultimate bearing capacity per unit of length D'_g is introduced.

$$D'_g = b\, d'_g \qquad (10)$$

Thus (9) becomes

$$b^2 - m_1 \frac{\gamma}{D'_g} bh^3 + m_2 bh - m_3 \frac{\gamma}{D'_g} h^4 + m_4 h^2 + m_5 \frac{\gamma^2}{D'^2_g} h^6 + \\ - m_6 \frac{D'_g}{\gamma} = 0 \qquad (11)$$

When the width b is taken as an abscissa, and the depth h as an ordinate, it is found that by varying parameter D'_g/γ, the curves represented by the equation (11) are in the first quadrant ($b > 0$, $h > 0$) nearly straight parallel lines going through the points

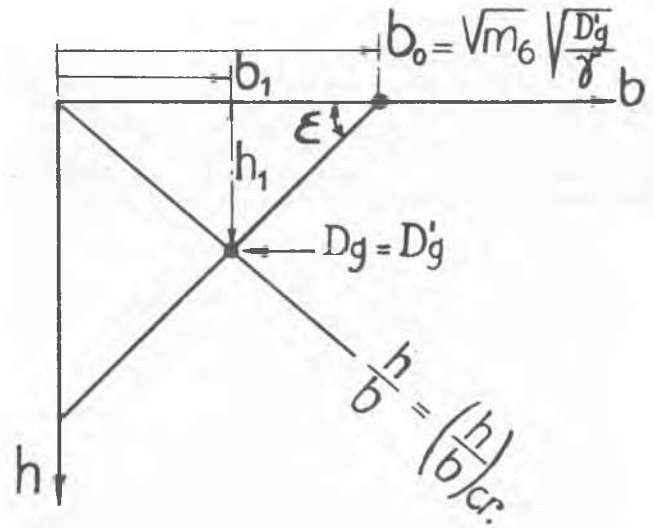

FIG. 4

$$h_0 = 0 \qquad b_0 = \sqrt{m_6}\sqrt{\frac{D'_g}{\gamma}} \text{ (see fig 4)} \qquad (12)$$

For different values of the angle ϕ the values of m_6 and $\sqrt{m_6}$ are given in the table I. The values of $\sqrt{m_6}$ are also given in the diagram of fig. 6.

The relation (12) gives the possibility to draw next the scale of b, a scale of the para-

TABLE I

φ	5^0	10^0	15^0	20^0	25^0	30^0	35^0	40^0
m_6	4,10306	1,61166	0,81641	0,44473	0,24262	0,12568	0,05786	0,020997
$\sqrt{m_6}$	2,0256	1,2695	0,9036	0,6669	0,4926	0,3545	0,2406	0,1449
$1:m_6$	0,24372	0,62048	1,22487	2,24855	4,12167	7,95671	17,28310	47,62585
$\sqrt{V_g}$	0,2476	0,7233	1,641	3,452	7,163	15,190	33,868	81,748
V_g	0,498	0,8505	1,281	1,858	2,676	3,897	5,82	9,0414
V_b	1,568	2,302	3,939	6,396	10,654	18,384	33,258	64,109
n_1	-0,000974	-0,0867	-0,91485	-6,3777	-37,8599	-209,75739	-1100,70379	-4787,9311
n_2	0,35998	0,96469	-0,8645	-17,12899	-110,48116	-570,19586	-2599,2943	-9232,43796
n_3	4,82986	10,8424	18,89922	29,26405	6,99743	-195,4788	-1086,29635	-2999,91759
n_4	22,09533	32,91564	57,77811	124,62738	256,4690	542,50587	1205,81212	+3056,75091
n_5	44,9478	35,7021	51,50323	99,18038	188,61499	379,14743	781,81827	1568,17301
n_6	36,2148	12,59132	7,90396	4,66799	1,3051	-3,60823	-11,73941	-25,17112
n_7	3,41013	0,68499	0,19658	0,04634	0,00217	0,00926	0,05234	0,13016
tg ε	0,16	0,317	0,472	0,633	0,79	0,96	1,231	1,748
$(\frac{h}{b})$crit	0,003	0,053	0,185	0,404	0,695	1,025	1,453	1,82

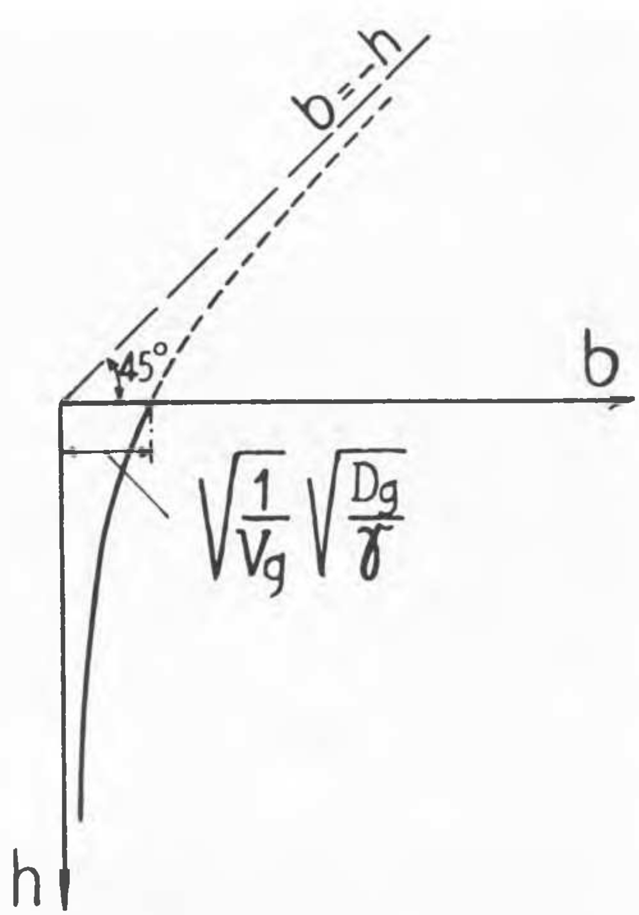

FIG. 5

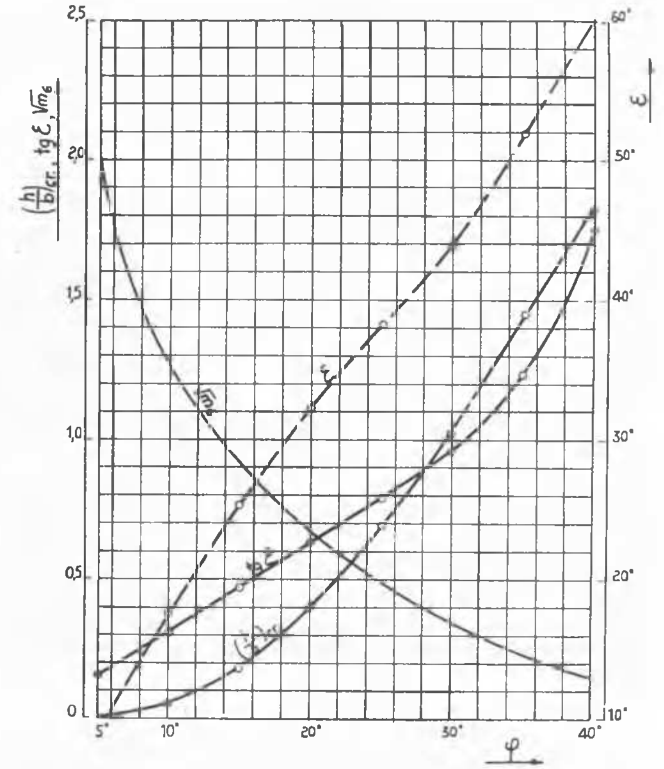

FIG. 6

meter D_g/γ , so that the value of D'_g/γ corresponding to an arbitrary line of the net can instantaneously be found. On the other hand, putting $D_g = b\, d_g$ (13), the equation (6) gives:

$$\frac{D_g}{\gamma} = V_b\, bh + V_g\, b^2 \qquad (14)$$

For each value of D_g/γ , the equation (14) represents an hyperbola, with the asymptotes b = 0 and b = - h (see fig. 5). The hyperbolas cut the axis of the b's at the points

$$b = \sqrt{\frac{1}{V_g}}\sqrt{\frac{D_g}{\gamma}} \qquad (15)$$

With the relation (15) it is possible to draw, next to the scale of the b's, a scale of the parameter D_g/γ; thus the value of D_g/γ corresponding to a given hyperbola of the net, can be found. The values of V_g and $\sqrt{V_g}$ are given in the table 1.

Let us now determine the locus of the points (b,h), for which the formulas of Andersen (11) and of Buisman (14) give the same value of the ultimate bearing capacity $D_g = D'_g$.

Eliminating $D_g = D'_g$ between the equations (11) and (14) one gets:

$$n_1 b^6 + n_2 b^5 h + n_3 b^4 h^2 + n_4 b^3 h^3 + n_5 b^2 h^4 + n_6 b h^5 + n_7 h^6 = 0 \qquad (16)$$

where n_1, n_2 .... n_7 = seven functions of the angle ϕ .

The equation (16) is homogeneous and of the 6th degree. It can be comomposed in the equation of real or imaginary straight lines going through the origin (b = 0 h = 0). In the first quadrant there is but one real straight line. The equation (16) can be written:

$$n_1 + n_2\frac{h}{b} + n_3\left(\frac{h}{b}\right)^2 + n_4\left(\frac{h}{b}\right)^3 + n_5\left(\frac{h}{b}\right)^4 + n_6\left(\frac{h}{b}\right)^5 + n_7\left(\frac{h}{b}\right)^6 = 0 \qquad (17)$$

The values of the coefficients n_1 till n_7 are given in function of ϕ in the table I. The equation (17) has been solved by trying; in this way the values h/b given in the table I were found.

Finally in a diagram b,h, the zones of applicability of the formulas of Andersen and of Buisman are separated by a straight line going through the origin. For small foundation depths h, combined with large footing widths b, the formula of Andersen gives smaller values and therefor is determining; on the contrary for high values of the ratios h/b the formula of Prandtl-Buisman must be retained.

The critical values of the ratio h/b are given in the table I, and also in the diagram of fig. 6. These critical values increasing with the angle ϕ , it means that the range of applicability of the formula of Buisman becomes narrower when the value of the angle ϕ is increasing. With the data of the table I it will be possible to verify rapidly in each real case in what zone it is located and what formula has to be applied.

For practical purposes it is sufficiently correct to assimulate the curves of Andersen D'_g/γ = cte to straight lines. These straight lines are determined by the point ($h_o = 0$ $b_o = \sqrt{m_6}\sqrt{\frac{D_g}{\gamma}}$) and by their angle ε (fig. 4). The value of the angle ε can easily be found from the consideration that the curve of Andersen cuts the straight line h/b = $(h/b)_{crit}$ in the point $b_1 h_1$ where $D_g = D'_g$. This point is defined by

$$\left.\begin{aligned} \frac{h_1}{b_1} &= \left(\frac{h}{b}\right)_{crit} \\ \frac{D'_g}{\gamma} &= V_b h_1 b_1 + V_g b_1^2 \end{aligned}\right\} \qquad (18)$$

$$h_1 = \frac{\sqrt{\frac{D'_g}{\gamma}}}{\sqrt{V_b\left(\frac{b}{h}\right)_{crit} + V_g\left(\frac{b}{h}\right)^2_{crit}}} \qquad (19)$$

$$b_1 = \frac{\left(\frac{b}{h}\right)_{crit}\sqrt{\frac{D'_g}{\gamma}}}{\sqrt{V_b\left(\frac{b}{h}\right)_{crit} + V_g\left(\frac{b}{h}\right)^2}} \qquad (20)$$

Fig. 4 gives $\quad tg\,\varepsilon = \frac{h_1}{b_0 - h_1} \qquad (21)$

The equations (12) (19) (20) and (21) give:

$$tg\,\varepsilon = \frac{1}{\sqrt{m_6}\sqrt{V_b\left(\frac{b}{h}\right)_{crit} + V_g\left(\frac{b}{h}\right)^2_{crit}} - \left(\frac{b}{h}\right)_{crit}} \qquad (22)$$

The values of tg ε are given in function of ϕ in the table I and in the fig. 6.

For the cases $\phi = 20^\circ$, 25° and 30°, the fig. 7, 8 and 9 give the straight laws of Andersen $D'_g/\gamma = c^t$ and the hyperbolas of Buisman $D_g/\gamma = c^t$, and also the straight separation between the respective zones of applicability.

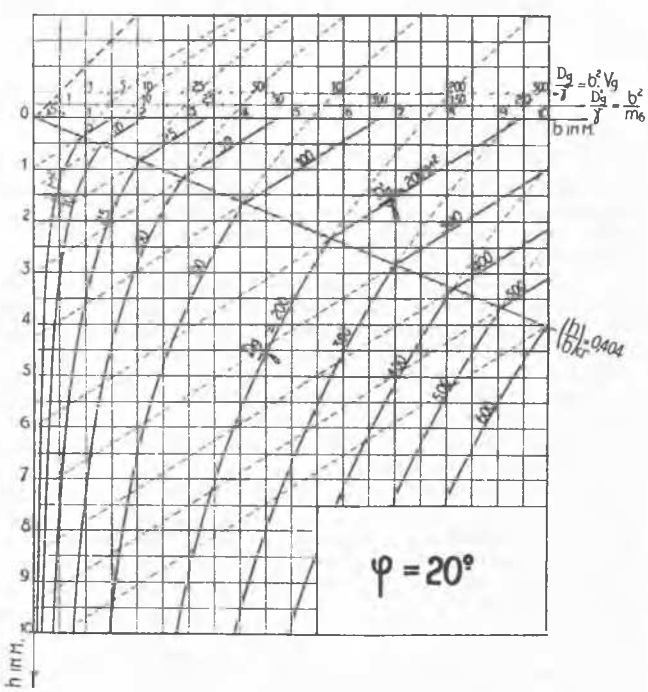

FIG. 7

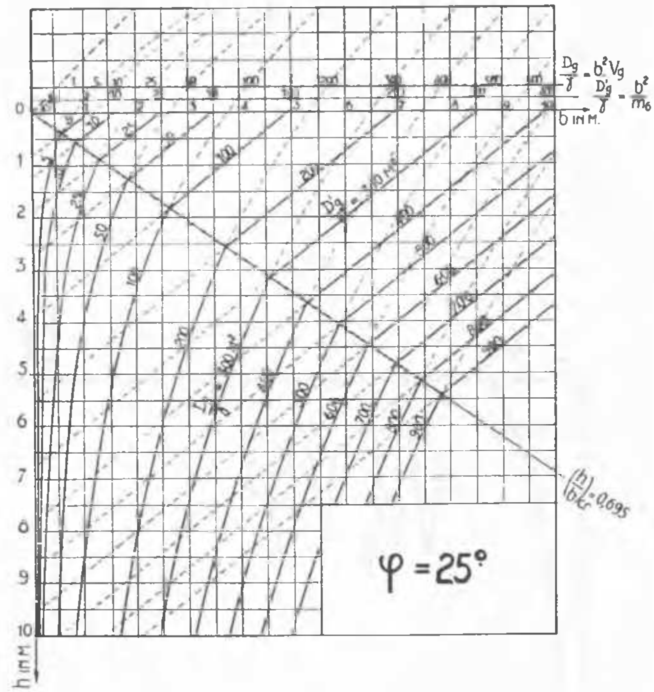

FIG. 8

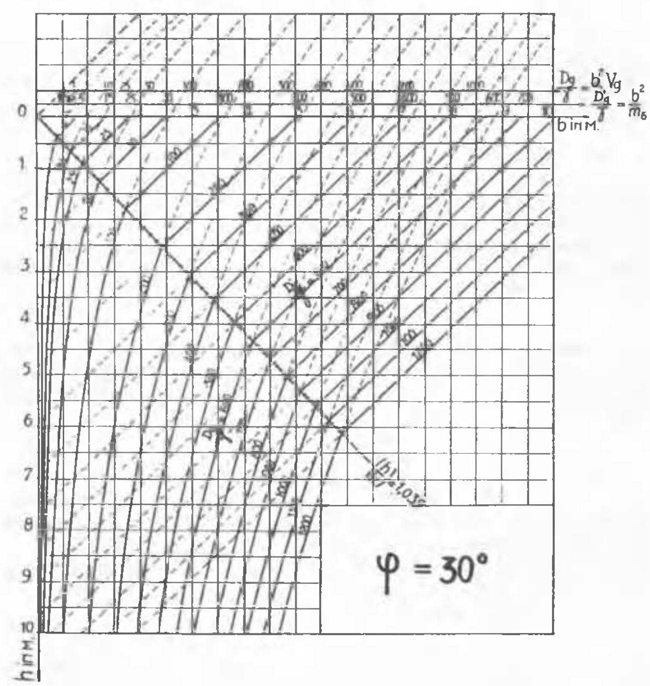

FIG. 9

The use of the fig. 7, 8 and 9 is direct, and doesn't need any other explanation.

It is worthwhile to note that the straight lines $D'_g/\gamma = c^t$ intersect the hyperbolas $D_g/\gamma = c^t$ under a certain angle; this would correspond to an abrupt change in the law of variation of the ultimate bearing capacity.

As this capacity is a physical phenomena such an abrupt change is excluded. Thus the indicated combination of the formulas of Andersen and Buisman is itself but an approximation of the problem.

The circular shape adopted by Andersen for the surface of failure is only strictly valid for very small values of h/b; on the other hand the surface of rupture consisting of a logarithmic spiral with two straight extensions, is only valid for very high values of h/b; for values of h/b approaching the critical value, the surfaces of failure must have a more complicated shape, intermediate between that of circular surfaces and that of the spiral surfaces; the shape of the surface changing gradually, there will be a smooth junction between the two laws $D_g/\gamma = c^t$ and $D'_g/\gamma = c^t$ in the neighbourhood of the critical line. This junction is necessarely located beyond the point A (fig. 10), and will have the shape of the dotted line. In the neighbourhood of the critical line the formulas of Andersen and of

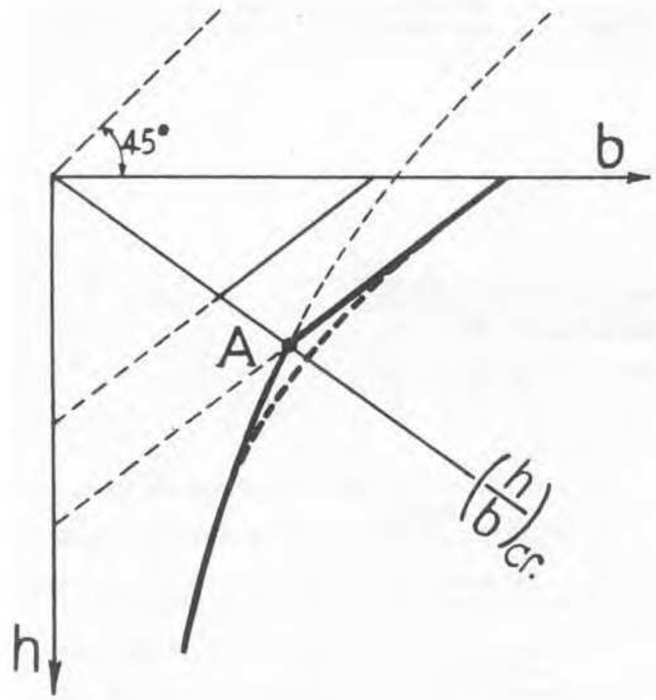

FIG.10

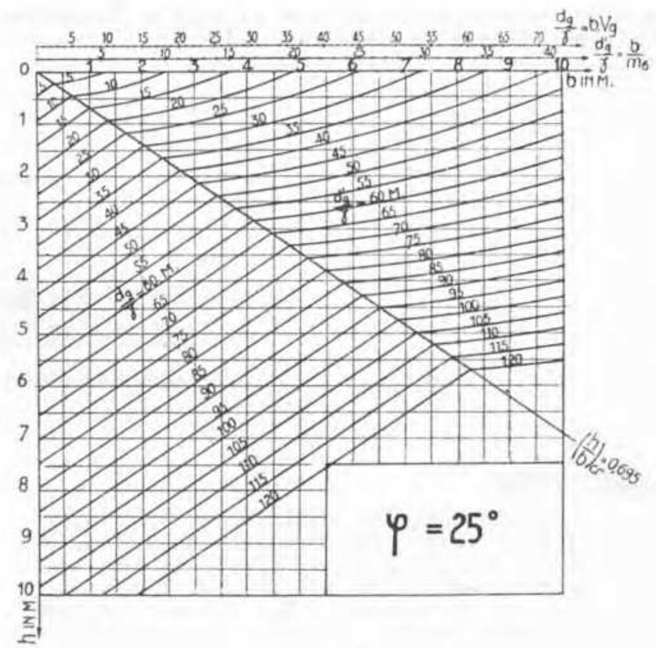

FIG.12

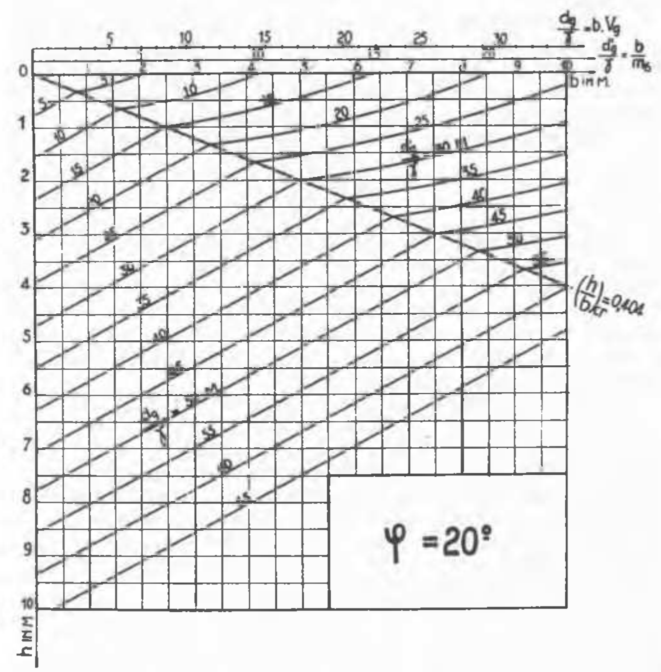

FIG.11

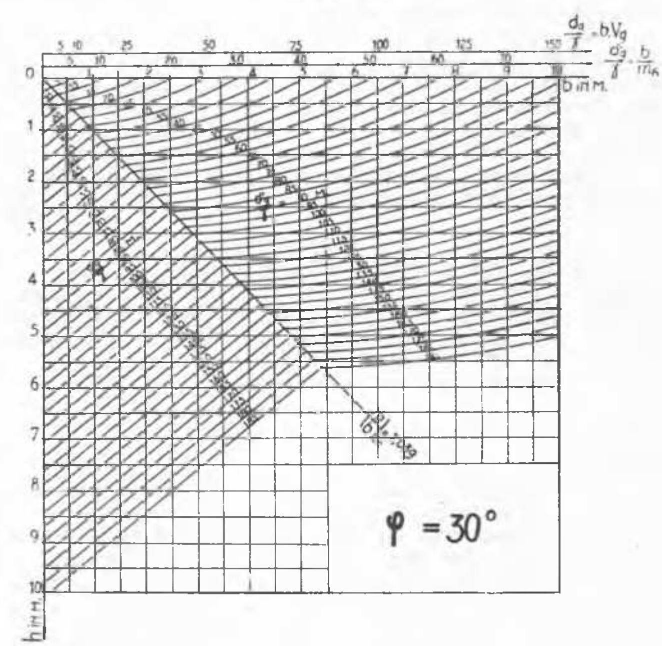

FIG.13

Buisman both will give too small values; thus it is indicated to take this into account by the choice of the factors of safety.

REMARKS.

1) Instead of considering the ultimate bearing capacities per linear meter D_g and D'_g (t/m), the ultimate bearing capacities per unit of area d_g and d'_g (t/m^2) can also be used.

Then the equation (9) is maintained, but this equation doesn't represent straight lines, but more complicated curves.

The equation (6) becomes $d_g = V_b \gamma h + V_g \gamma b$ (23)

In the diagram b,h the equation (23) represents a net of parallel straight lines.

The elimination of the parameters $d_g/\gamma = d'_g/\gamma$ between the equations (9) and (23), gives again the equation of a straight line.

The advantage to consider the parameter d_g/γ (t/m^2) instead of D_g/γ (t/m) consists in the fact that the formula of Buisman is now represented by a straight line, what is easier when the problems to solve are frequently located in the range of applicability of this formula. For this reason the fig. 11, 12 and 13 represent the diagrams h in function of b for the parameters d_g/γ. Apart from that these diagrams are indentical to those of the fig. 7, 8 and 9.

2) When the value of h/b becomes very large, then the problem is no longer one concerning

a direct fundation, but one of a deep fundation (piles, fundation-pits). For these latter problems neither of the considered formulas is applicable, but direct information can be obtained from the results of deep penetration tests in situ.

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I e 5

IMPROVEMENT OF THE METHOD OF CALCULATION OF THE EQUILIBRIUM ALONG SLIDING CIRCLES.

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INTRODUCTION.

In controling the equilibrium of slopes, one has to consider two possible failures. The first is in relation to a failure of the base and can be examined by the theory of the bearing capacity of the soil. The second, called slope failure, will occur when the shearing resistance of the earth along a certain surface is not large enough to make an equilibrium with the own weight of the slope and the waterpressure acting upon it.

We will consider the possibility of a slope failure in case of a well-defined toe-circle and we will propose a method for the determination of the factor of safety of that given slope in case of an homogeneous soil mass.

The shearing resistance of the earth is determined by the equation

$$\tau = c + \sigma \operatorname{tg} \varphi .$$

Often the control of equilibrium along sliding circles is limited to the verification of the equation of the moments about the centre of rotation. The method becomes very simple in case of an angle of internal friction $\varphi = 0$. Then we have the equation (fig. 1)

$$W.\ l_w = c_o.l_o.R.$$

where:

W = weight of the body of earth in tons per unit of length of the slope.

l_w= lever arm of the weight W with reference to the centre O of the toe circle in meter.

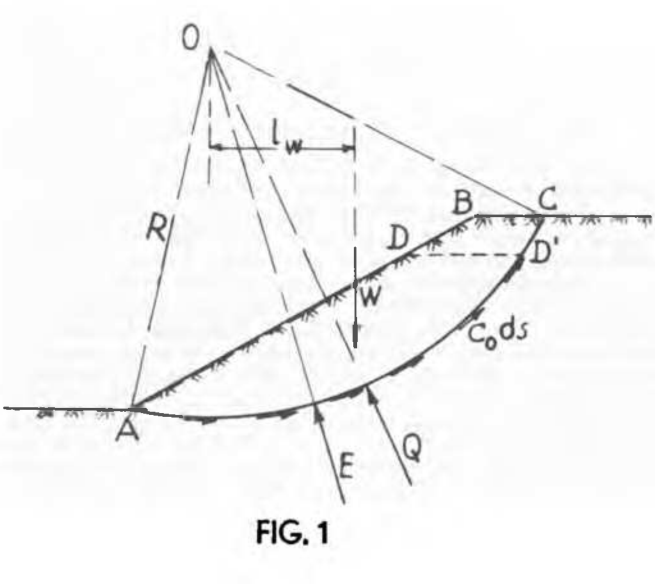

FIG. 1

c_o = cohesion in t/m^2 required to have an equilibrium.

l_o = length of the sliding surface in meter.

R = radius of the sliding circle in meter

On the fig. 1 are also shown the quantities Q and E.

Q = resultant of the normal effective stresses (in tons)

E = resultant of the waterpressures on the sliding circle.

The dotted line ADD' represents the hydrostatic pressureline above the sliding circle.

The factor of safety can then be found by the comparison of the required cohesion with the existing cohesion c_e of the soil. The latter has to be determined by means of laboratory-tests on undisterbed samples.

The factor of safety can be written :

$$s = \frac{c_e}{c_o}$$

The control of the equilibrium of rotation is insufficient, because each state of equilibrium is controled by three conditions: the equilibrium of translation along two mutual perpendicular directions (for instance a vertical and an horizontal direction) and the equilibrium of rotation.

In the different methods of controling the stability of slopes, one or two of the 3 conditions of equilibrium are often overlooked. For instance in the method consisting of cutting the sliding mass into slices by means of pseudo-sliding surface, often only the polygone of forces is drawn, thus taking only into account the conditions of translation.To take notice of the condition of rotation, it is necessary to draw also the pressure line, thus taking into account the value, the direction and the point of application of all forces involved. Fig. 2 shows an example of this method.

The control by means of slices, even when complete, still presents a few inaccuracies :

1) one admits that the effective soil reactions K_1... on the pseudo-sliding surfaces between the different slices are parallel to the tangent on the sliding circle and that the points of application of these forces are located in the middle third of the height. Thus the point of application is not exactly known and a more or less arbitrary assumption can be made on its account.

2) The reaction Q is assumed to be a tangent of the R sin φ circle. This is only true for an elementary part of the surface but not for a certain length : this inaccuracy can be