a direct fundation, but one of a deep fundation (piles, fundation-pits). For these latter problems neither of the considered formulas is applicable, but direct information can be obtained from the results of deep penetration tests in situ.

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I e 5

IMPROVEMENT OF THE METHOD OF CALCULATION OF THE EQUILIBRIUM ALONG SLIDING CIRCLES.

H. Raedschelders - Ghent (Belgium).

INTRODUCTION.

In controling the equilibrium of slopes, one has to consider two possible failures. The first is in relation to a failure of the base and can be examined by the theory of the bearing capacity of the soil. The second, called slope failure, will occur when the shearing resistance of the earth along a certain surface is not large enough to make an equilibrium with the own weight of the slope and the waterpressure acting upon it.

We will consider the possibility of a slope failure in case of a well-defined toe-circle and we will propose a method for the determination of the factor of safety of that given slope in case of an homogeneous soil mass.

The shearing resistance of the earth is determined by the equation

$$\tau = c + \sigma \operatorname{tg} \varphi .$$

Often the control of equilibrium along sliding circles is limited to the verification of the equation of the moments about the centre of rotation. The method becomes very simple in case of an angle of internal friction $\varphi = 0$. Then we have the equation (fig. 1)

$$W.\, l_w = c_o.l_o.R.$$

where:

W = weight of the body of earth in tons per unit of length of the slope.

l_w = lever arm of the weight W with reference to the centre O of the toe circle in meter.

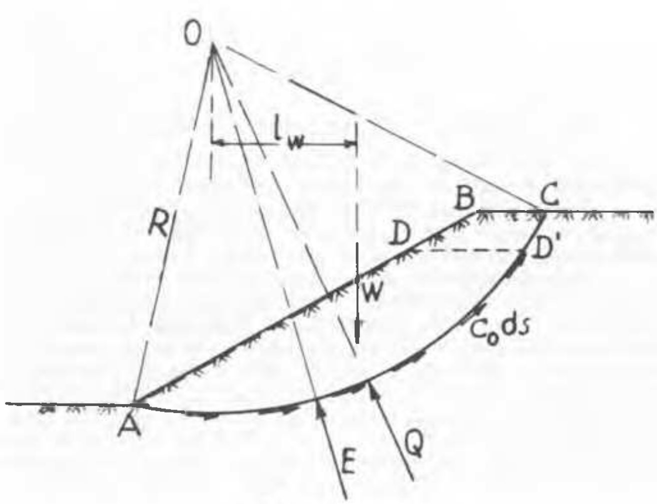


FIG. 1

c_o = cohesion in t/m^2 required to have an equilibrium.

l_o = length of the sliding surface in meter.

R = radius of the sliding circle in meter

On the fig. 1 are also shown the quantities Q and E.

Q = resultant of the normal effective stresses (in tons)

E = resultant of the waterpressures on the sliding circle.

The dotted line ADD' represents the hydrostatic pressureline above the sliding circle.

The factor of safety can then be found by the comparison of the required cohesion with the existing cohesion c_e of the soil. The latter has to be determined by means of laboratory-tests on undisterbed samples.

The factor of safety can be written :

$$s = \frac{c_e}{c_o}$$

The control of the equilibrium of rotation is insufficient, because each state of equilibrium is controled by three conditions: the equilibrium of translation along two mutual perpendicular directions (for instance a vertical and an horizontal direction) and the equilibrium of rotation.

In the different methods of controling the stability of slopes, one or two of the 3 conditions of equilibrium are often overlooked. For instance in the method consisting of cutting the sliding mass into slices by means of pseudo-sliding surface, often only the polygone of forces is drawn, thus taking only into account the conditions of translation. To take notice of the condition of rotation, it is necessary to draw also the pressure line, thus taking into account the value, the direction and the point of application of all forces involved. Fig. 2 shows an example of this method.

The control by means of slices, even when complete, still presents a few inaccuracies :

1) one admits that the effective soil reactions K_1... on the pseudo-sliding surfaces between the different slices are parallel to the tangent on the sliding circle and that the points of application of these forces are located in the middle third of the height. Thus the point of application is not exactly known and a more or less arbitrary assumption can be made on its account.

2) The reaction Q is assumed to be a tangent of the R sin φ circle. This is only true for an elementary part of the surface but not for a certain length : this inaccuracy can be

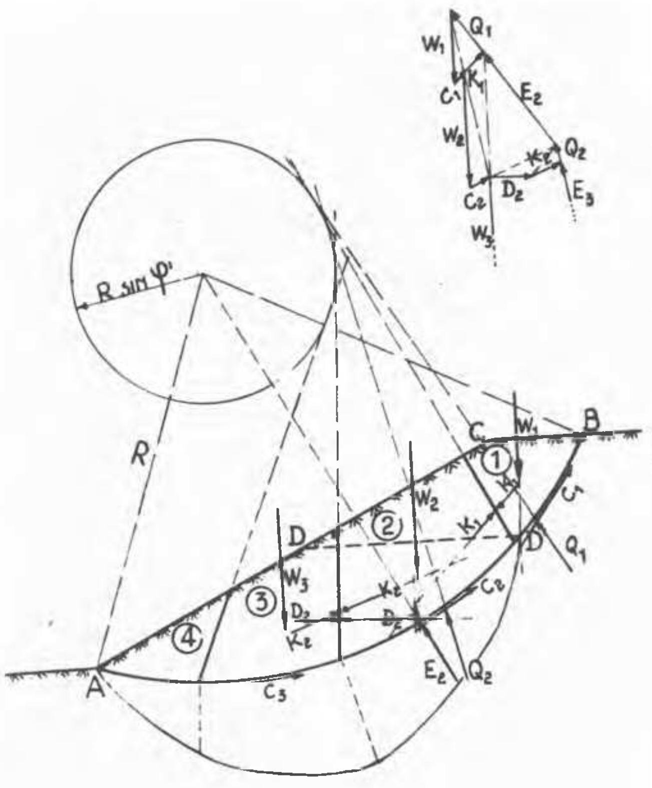


FIG. 2

neglected when a sufficient number of slices is considered.

3) The method takes much more time than the control of the moments.

4) The factor of safety s has to be taken on the two elements of the shearing resistance τ. For each considered sliding circle one has to repeat the total calculations and drawings for several values of s and this with new values of C and φ given by the relations $c' = \frac{c_e}{s}$ and $\varphi' = \text{arc.tg} \left[\frac{\text{tg}\ \varphi}{s}\right]$

One has to make at least two or three trials for each surface.

THEORY OF THE NEW METHOD.

In case of an homogeneous soil mass we will suggest a more rapid method, nevertheless taking into account the three conditions of equilibrium.

The simplified method consists in the consideration of only one slice.

The advantage of the method lies in the rapidity of the examination of the equilibrium along a sliding circle without neglecting some important points on theoretical view.

On the same time there are no longer difficulties with the reactions between the different slices, because they are no longer to be considered.

On the other hand the resultant of all the reactions Q is no longer a tangent of the R sinφ-circle, but it can be proved that the error is on the side of safety.

We will now consider all the forces acting on the sliding mass of earth :

1) The weight of the earth located above the arc is W tons per unit of length of the slope. This force is acting along a vertical line through the centre of gravity of the area of the slice.

2) The resultant of the cohesive forces can no longer be approached by multiplying the cohesion per unit of length of the sliding surface by the length of the surface of the sliding circle. In case of a constant cohesion on the whole surface of sliding, the fig. 3

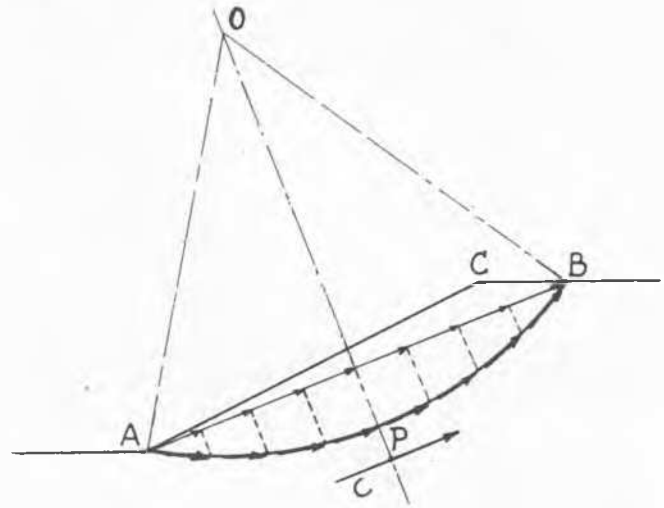


FIG. 3

shows that the resultant can be determined by the equation

$$C = c_o \cdot \text{chord AB}.$$

The direction is that of the chord AB and the point of application can easely be found by the method of moments about the centre O. By that way we find :

$$OP = R \cdot \frac{\text{arc AB}}{\text{chord AB}}$$

For the method of slices this correction had not to be made because for each slice of a limited length the factor $\frac{\text{arc AB}}{\text{chord AB}}$ is nearly equal to 1.

3) The waterpressure E can be computed by means of a flow net, which values make it possible to determine the waterpressure in each point of the sliding surface. This is an exact method for a permeable soil-mass. For cuts in a clay stratum it is safe to consider in each point of the sliding surface an hydrostatic waterpressure corresponding to the surface of the slope and the free water-table existing before the cut was digged, the adaptation of the groundwater to the new situation being very slow.

All these elementary forces are directed to the center of the sliding circle. E can be found graphically by composing all the elementary forces. By this way we know the resultant in value and direction. The point of application follows from the fact that E has to go through the centre of the circle.

4) The resultant Q can be found by means of the polygone of forces which must be closed with it.

Since the factor of safety will be applied on the two elements of the shearing resistance, one has to do a little "trial and error" work to find that factor. But now the trials are very quick and without any serious calculation.

By means of comparison it will be possible to estimate the importance of the error introduced by the fact that the resultant Q is not exactly to the R.sinφ-circle.

PRACTICAL APPLICATION OF THE NEW METHOD.

1) Computation of the weight W of the sliding earth mass. Since we consider a slope with a width of 1 meter, we must only determine the slice in m^2 and multiplicate it with the unit weight γ . In all cases the considered surface of the slice can be represented by a sum of a circle-segment and one or more triangles with a positive or negative value (fig. 4). The

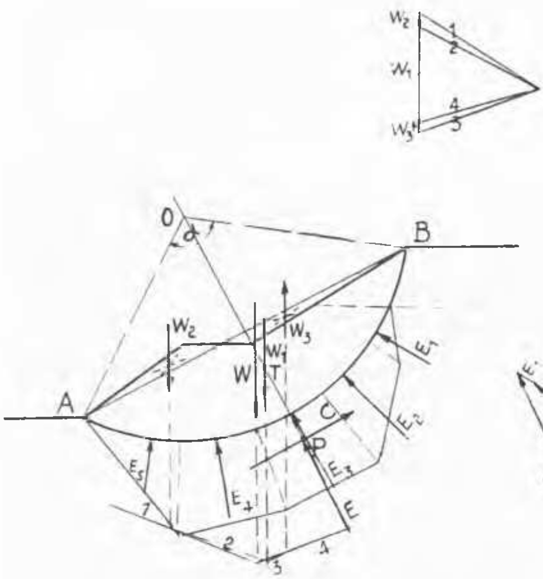


FIG. 4

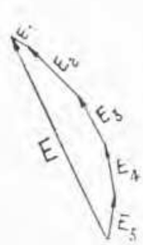


FIG. 4 b

formula for the surface of the segment, with the angle α and a radius R, being

$$S_s = \frac{\pi R^2 \alpha}{360} - \frac{R^2 \sin\alpha}{2}$$

and

$$OT = \frac{AB^3}{12\ S_s}$$

The resulting force W is the algebraic sum of the components. Its direction is vertical and its points of application is easily found by graphical integration, as indicated on fig. 4.

2) Computation of the resultant of the cohesive forces C' (fig.4). We saw already that this resultant C is given by the equation

$$C = c_o \cdot \text{chord } AB$$

Its direction is parallel to the chord and the point of application P is given by the formula

$$OP = R. \frac{\text{arc. } AB}{\text{chord } AB}$$

3) The general method to find the resultant of the waterpressures on the sliding surface has already been outlined in the theory. This general method is a graphical one and is illustrated by the example of fig. 4b.
In some cases an algebraic method can be more rapid. When a diagram of the waterpressures is made by drawing the pressure in each point of the sliding surface upwards on the vertical through that point, it often happens that the so obtained diagram of waterpressures consists on one or more straight lines. For instance in fig. 5 it is assumed that in a few pervious layers the original waterlevel DD' is not altered and that in each point the waterpressure is given by the vertical distance

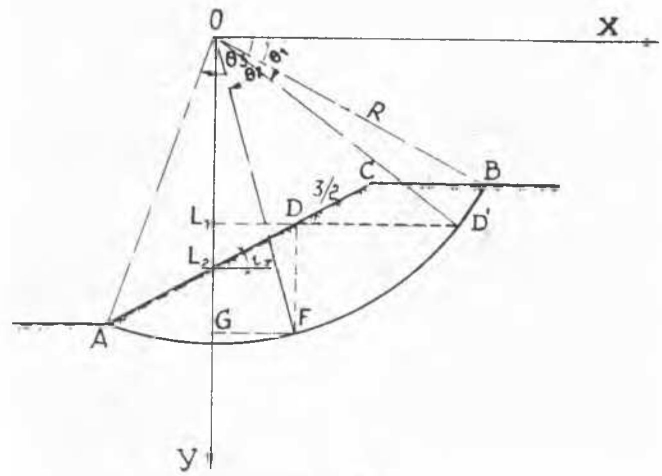


FIG. 5

of the considered point to the soil surface. In that case the diagram of waterpressures consists in the two straight lines AD and DD'. Considering the vertical through the centre O of the sliding surface, we define on this vertical the points of intersection L_1 and L_2 with the two straight lines DD' and AD of the waterpressure diagram. The lengths $b_1 = OL_1$, $b_2 = OL_2$, and the angles $\vartheta_1, \vartheta_2, \vartheta_3$ as indicated on fig. 5 are considered. The factor tg i is often known or can be measured, i being the angle of the considered straight line of the waterpressure diagram with the horizon. In case of fig. 5, tg i_1 = 0 and tg i_2 = 2/3 =0,666

The vertical and horizontal components V and H of the resultant of the waterpressure E are then given by the formulas:

$$H = \sum \left\{ bR\left[\sin\vartheta_2 - \sin\vartheta_1\right] + \frac{R^2}{4}\left[\cos 2\vartheta_2 - \cos 2\vartheta_1\right] + \right.$$
$$\left. - \frac{R^2 \operatorname{tg} i}{2}\left[\vartheta_2 - \vartheta_1 + \frac{\sin 2\vartheta_2 - \sin 2\vartheta_1}{2}\right] \right\} \qquad (1)$$

$$V = \sum \left\{ -bR\left[\cos\vartheta_2 - \cos\vartheta_1\right] - \frac{R^2}{2}\left[\vartheta_2 - \vartheta_1 - \frac{\sin 2\vartheta_2 - \sin 2\vartheta_1}{2}\right] + \right.$$
$$\left. + \frac{R^2 \operatorname{tg} i}{4}\left[\cos 2\vartheta_2 - \cos 2\vartheta_1\right] \right\} \qquad (2)$$

The two components H and V with their value and direction give, by composing, the resultant of the waterpressures on the sliding

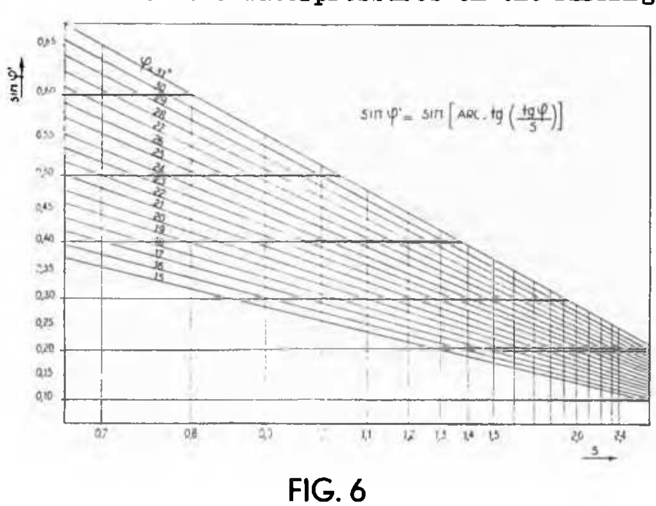


FIG. 6

surface also in value and direction. The point of application follows from the fact that this resultant has to go through the centre O of the sliding surface.

4) Now we make a graphical composition of all the previous determined forces :

We draw first W known in value, direction and point of application as indicated under 1). We

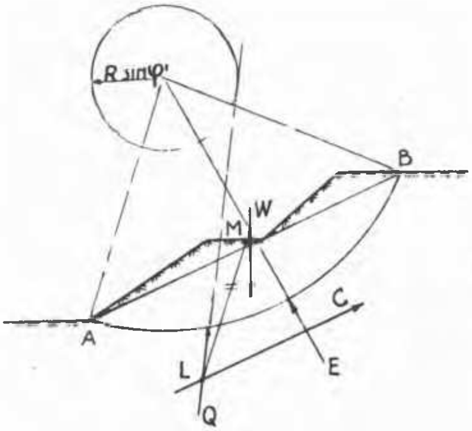


FIG. 7

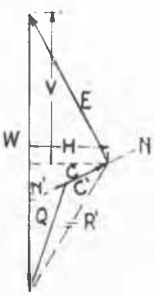


FIG. 7 b

compute the value, direction and point of application of E as explained under 3). We know the direction of C, and also its point of application, because these two quantities are independent of the unknown required cohesion c_o.

On the other hand composing the two known forces W and E, as done in fig. 7b we get the force R'. The working line of R' goes through the point of intersection M (fig. 7) of the forces W and E. This line cuts the working line of C in the point L. The unknown force Q must go through this point L. Farther this same force is assumed to be tangent to the circle R sin φ'

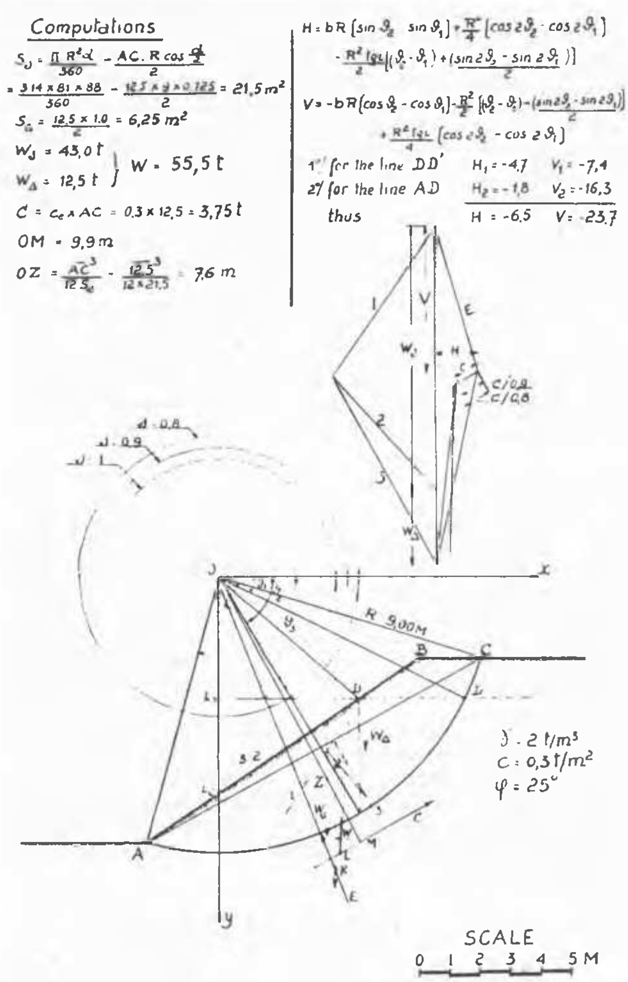
Now we adopt an arbitrary value for the safety coefficient s, and obtain then easily

$$c_o = \frac{c_e}{s},$$

From that, one gets $C' = c_o$. chord AB, which is put on the line NN' of fig. 7b. Now we try to close the polygone with a direction of Q given by a tangent on the circle R sin φ' through the point C (fig. 7), the angle φ' being given by the equation

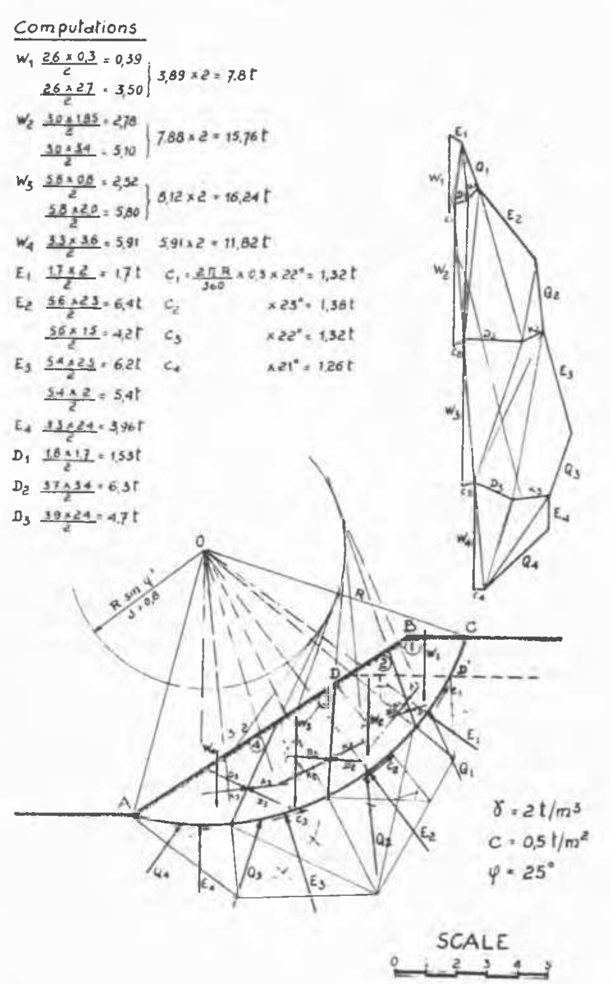
$$\varphi' = \text{arc.tg}\left[\frac{\text{tg}\,\varphi}{s}\right]$$

Sin φ' can be found immediately by the graphs of fig. 6. Generally the diagram of fig. 7b will not close, because the value of s has been arbitrarely adopted. But with a few try-



Example of proposed method

FIG. 8 a



Example of method of slices

FIG. 8 b

ings one finds rapidly the value of s closing the diagram and thus corresponding to the real margin of safety.

EXAMPLE.

Figures 8 a and b give an example of all the computations and drawings necessary for the determination of the factor of safety for a determined sliding-circle. We find s = 0,8.

The same examination has be done with the method of slices, also giving s = 0,8.

The method of slices asked 2 hours, for one trial. The complete determination by three trials asked 5 hours.

The proposed method gives the real value of the coefficient of safety within 1¼ hour.

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I e 6

THE Ø = 0 ANALYSIS OF STABILITY AND ITS THEORETICAL BASIS.

A. W. Skempton M.Sc., A.M.I.C.E., F.G.S.

University Reader in Soil Mechanics and Assistent Professor at Imperial College, University of London.

I. INTRODUCTION.

It is known that saturated clays when tested under conditions of no water content change behave, with respect to the applied stresses at failure, as purely cohesive materials with an angle of shearing resistance Ø equal to zero. This experimental observation is the basis for a method of stability analysis which is now becoming widely used and which is known as the Ø = 0 analysis of stability.

Experience is showing that the analysis is reliable in practice; leading to correct estimates of bearing capacity of clay soils and of earth pressure, and giving satisfactory results for the factor of safety of clay slopes. Yet there are important limitations to the range of application of the Ø = 0 analysis and to the soils in which it can be applied. These limitations must be fully appreciated in the application of the method to practical problems.

II. EXPERIMENTAL EVIDENCE OF Ø = 0.

In 1915 Langtry Bell carried out a series of shear box tests on various cohesive soils in which a restricted opportunity for water content change under the normal pressure was allowed. These tests showed that, at least for the softer clays, the angle of shearing resistance Ø was small.

Bell also presented equations for active and passive earth pressure in cohesive soils and for their bearing capacity. He did not, however, conclude that Ø should be taken as zero for clay soils in the analysed of these problems.

So far as the author is aware this assumption was first made explicitly by Fellenius in 1922 in connection with the stability of clay slopes, when he put forward the equation shown in Fig. 5 (a). Later, in 1927, Fellenius dealt with the problem more fully and, in particular, he derived the concept of the stability number $c/\gamma H$ which of great significance in earth pressure and slope stability in clays.

But at this time little was known of the shear properties of clays and there was insufficient evidence for the acceptance of an analysis based on the Ø = 0 assumption. The problem was, in fact, not placed on a firm experimental basis until 1932 when Terzaghi published the tests results shown in Fig. 1. These demonstrated that when a saturated clay is tested in the triaxial apparatus, under conditions of no water content change (the so-called "immediate" of "quick" triaxial test), the angle of shearing resistance is zero; although the angle of shearing resistance obtained in a test where the clay is allowed to consolidate under the applied stress is very considerably greater than zero.

Subsequent work has confirmed this result (see, for example, Jurgenson 1934a, Golder and Skempton 1948), and it follows that in saturated clays, tested under conditions of no water content change, the criterion of failure may be expressed in the form

$$\sigma_1 - \sigma_3 = 2c \qquad (1)$$

where σ_1 and σ_3 are the major and minor applied principal stresses at failure and c is the "cohesion" or shear strength of the clay at the particular water content of the test.

An analysis of stability can therefore be based upon equation (1) and will lead to correct results with respect to the applied stresses at failure, provided the basic conditions implicit in this equation are complied with.

It should be noted that $(\sigma_1 - \sigma_3)$ is the compression strength of the clay and therefore $c = \frac{1}{2}$ compression strength (2)

This result provides a ready means of determining the cohesion of a clay sample, since the compression strength can easily be measured.

III. THE Ø = 0 ANALYSIS OF STABILITY.

Based upon equation (1) the Ø = 0 analysis has been developed for the calculation of active and passive pressure and bearing capacity of clays and the calculation of factors of safety in clay slopes. The methods are summarised in the following paragraphs: