ings one finds rapidly the value of s closing the diagram and thus corresponding to the real margin of safety.

EXAMPLE.

Figures 8 a and b give an example of all the computations and drawings necessary for the determination of the factor of safety for a determined sliding-circle. We find s = 0,8.

The same examination has be done with the method of slices, also giving s = 0,8.

The method of slices asked 2 hours, for one trial. The complete determination by three trials asked 5 hours.

The proposed method gives the real value of the coefficient of safety within 1¼ hour.

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I e 6

THE Ø = 0 ANALYSIS OF STABILITY AND ITS THEORETICAL BASIS.

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I. INTRODUCTION.

It is known that saturated clays when tested under conditions of no water content change behave, with respect to the applied stresses at failure, as purely cohesive materials with an angle of shearing resistance Ø equal to zero. This experimental observation is the basis for a method of stability analysis which is now becoming widely used and which is known as the Ø = 0 analysis of stability.

Experience is showing that the analysis is reliable in practice; leading to correct estimates of bearing capacity of clay soils and of earth pressure, and giving satisfactory results for the factor of safety of clay slopes. Yet there are important limitations to the range of application of the Ø = 0 analysis and to the soils in which it can be applied. These limitations must be fully appreciated in the application of the method to practical problems.

II. EXPERIMENTAL EVIDENCE OF Ø = 0.

In 1915 Langtry Bell carried out a series of shear box tests on various cohesive soils in which a restricted opportunity for water content change under the normal pressure was allowed. These tests showed that, at least for the softer clays, the angle of shearing resistance Ø was small.

Bell also presented equations for active and passive earth pressure in cohesive soils and for their bearing capacity. He did not, however, conclude that Ø should be taken as zero for clay soils in the analysed of these problems.

So far as the author is aware this assumption was first made explicitly by Fellenius in 1922 in connection with the stability of clay slopes, when he put forward the equation shown in Fig. 5 (a). Later, in 1927, Fellenius dealt with the problem more fully and, in particular, he derived the concept of the stability number $c/\gamma H$ which of great significance in earth pressure and slope stability in clays.

But at this time little was known of the shear properties of clays and there was insufficient evidence for the acceptance of an analysis based on the Ø = 0 assumption. The problem was, in fact, not placed on a firm experimental basis until 1932 when Terzaghi published the tests results shown in Fig. 1. These demonstrated that when a saturated clay is tested in the triaxial apparatus, under conditions of no water content change (the so-called "immediate" of "quick" triaxial test), the angle of shearing resistance is zero; although the angle of shearing resistance obtained in a test where the clay is allowed to consolidate under the applied stress is very considerably greater than zero.

Subsequent work has confirmed this result (see, for example, Jurgenson 1934a, Golder and Skempton 1948), and it follows that in saturated clays, tested under conditions of no water content change, the criterion of failure may be expressed in the form

$$\sigma_1 - \sigma_3 = 2c \qquad (1)$$

where σ_1 and σ_3 are the major and minor applied principal stresses at failure and c is the "cohesion" or shear strength of the clay at the particular water content of the test.

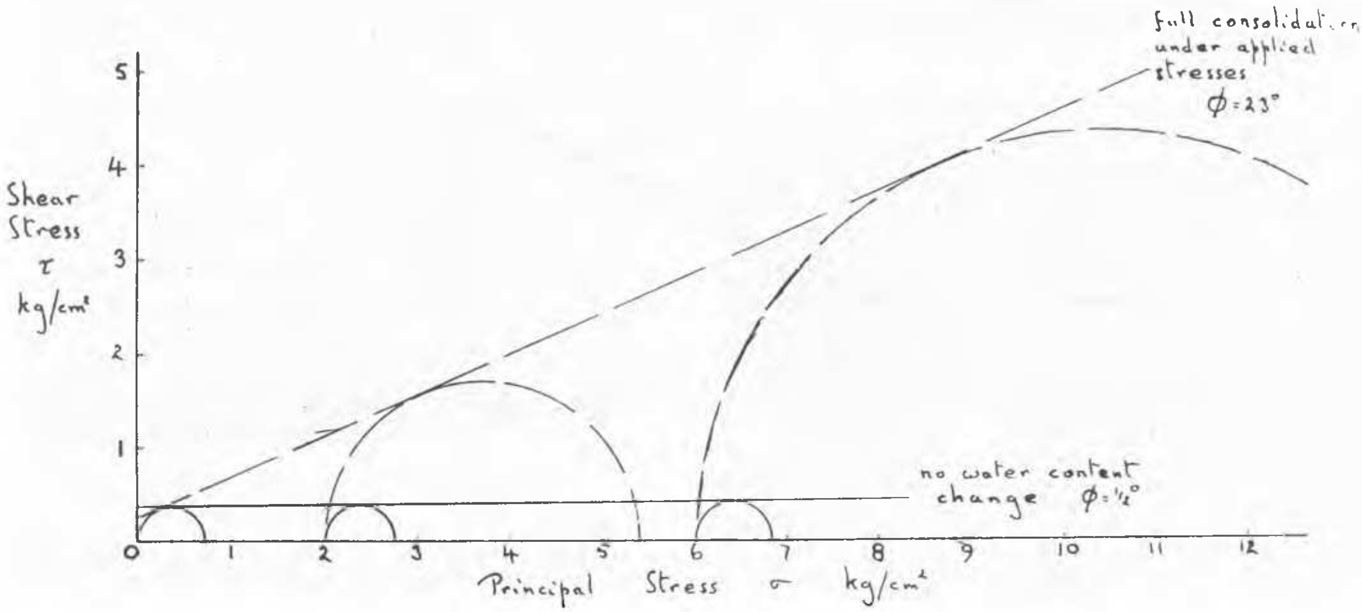
An analysis of stability can therefore be based upon equation (1) and will lead to correct results with respect to the applied stresses at failure, provided the basic conditions implicit in this equation are complied with.

It should be noted that $(\sigma_1 - \sigma_3)$ is the compression strength of the clay and therefore $c = \frac{1}{2}$ compression strength (2)

This result provides a ready means of determining the cohesion of a clay sample, since the compression strength can easily be measured.

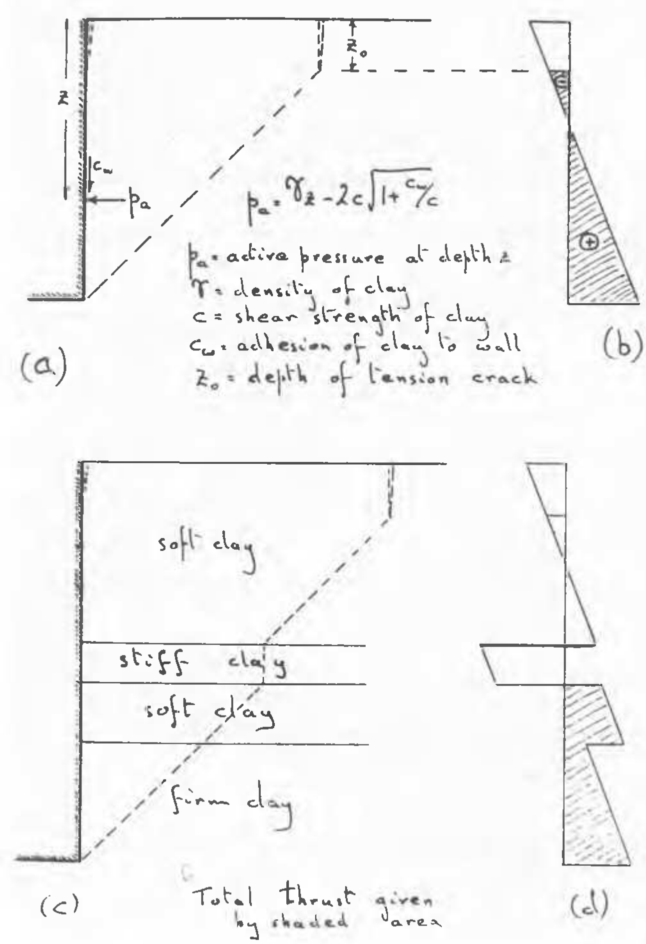
III. THE Ø = 0 ANALYSIS OF STABILITY.

Based upon equation (1) the Ø = 0 analysis has been developed for the calculation of active and passive pressure and bearing capacity of clays and the calculation of factors of safety in clay slopes. The methods are summarised in the following paragraphs:



Triaxial Tests on Clay, showing $\phi = 0$ (approx.) with no Water Content Change. (Terzaghi 1932)

FIG. 1



Application of $\phi = 0$ Analysis to Calculation of Active Earth Pressure in Clays.

FIG. 2

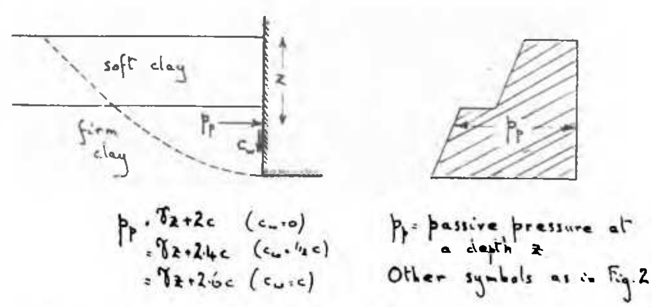
1) Active earth pressure (Fig. 2). For a vertical wall with horizontal backing

$$p_a = \gamma z - 2c\sqrt{1 + c_w/c} \qquad (3)$$

This gives a pressure distribution with depth as shown in Fig. 2(b). With a tension crack of depth z_o the total thrust on the wall is equal to the algebraic sum of the shaded area. With variable strata a diagram such as that shown in Fig. 2(d) is obtained.
The total thrust is again equal to the algebraic sum of the shaded areas.

The distribution of the total thrust depends on the mode of yield of the wall.

2) Passive earth pressure (Fig. 3)



Application of $\phi = 0$ Analysis to Calculation of Passive Pressure in Clays.

FIG. 3

$$p_p = \gamma z + 2c \quad (c_w = c) \qquad (4)$$

$$= \gamma z + 2.4c \quad (c_w = \tfrac{1}{2}c)$$

$$= \gamma z + 2.6c \quad (c_w = c)$$

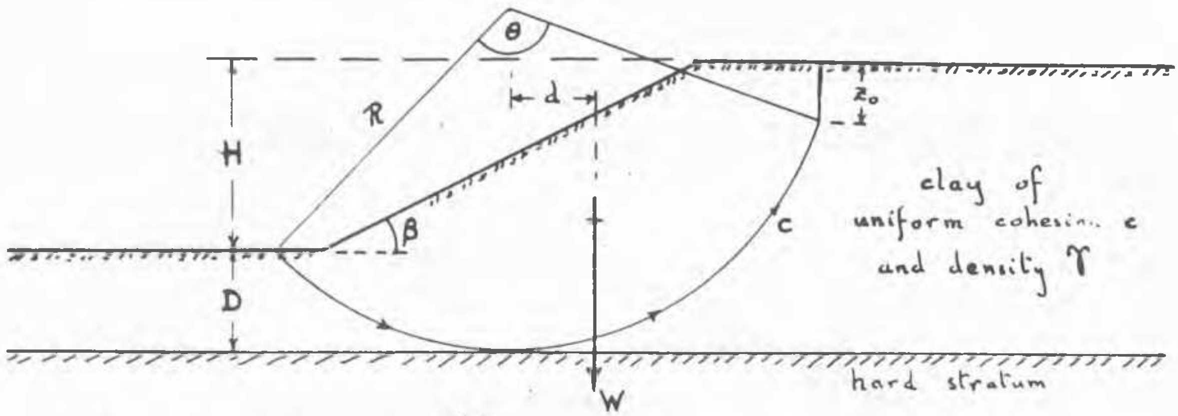
the coefficients of c have been derived by Packshaw (1946) from the modified wedge theory given by Terzaghi (1943).

3) Bearing capacity of shallow foundations (Fig. 4)

$$q_u = \gamma z + (2 + \pi)\,c \quad \text{strip footings} \qquad (5)$$

$$q_u = \gamma z + 6.7c \quad \text{square footings} \qquad (6)$$

The strip footings equation was introduced by

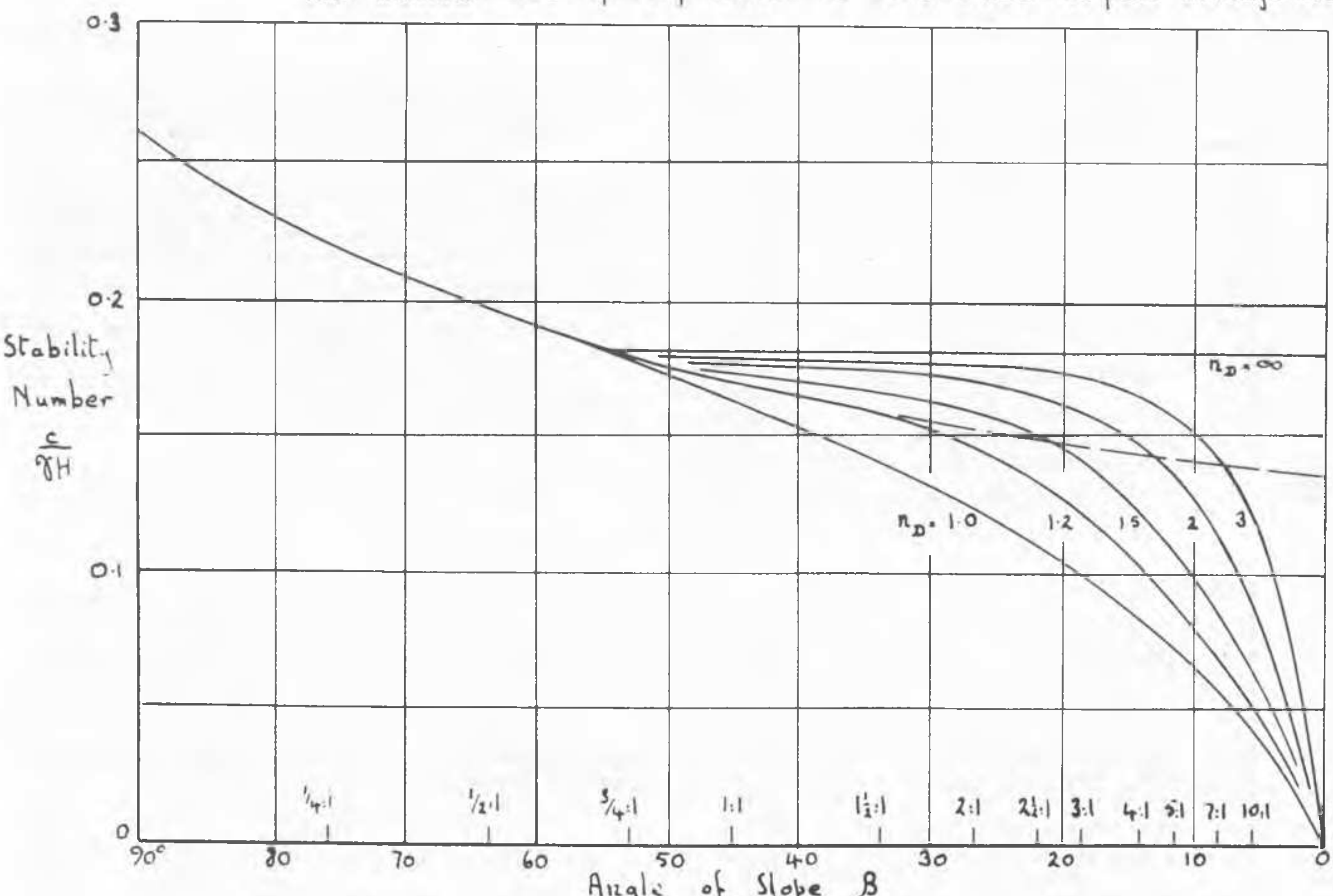


Factor of safety $= \dfrac{cR^2\theta}{Wd}$ Depth factor $n_D = \dfrac{H+D}{H}$

(b) Stability Number Chart

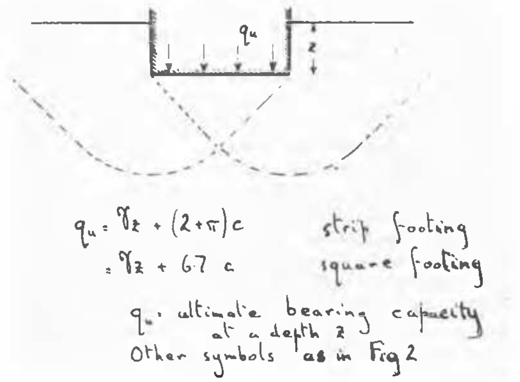
Allo.. for tension crack by appropiate reduction in c

Use dotted line if slip circle is constrained to pass through toe



Ø = 0 Analysis in Slopes of Uniform Clay.
(Fellenius 1927 and Taylor 1937)

FIG. 5



Application of $\emptyset$ = 0 Analysis to Calculation of Ultimate Bearing Capacity in Clays.

FIG. 4

Jurgenson (1934 b) from a theoretical analysis by Prandtl (1920). The square footing equation is based upon experimental work by Golder (1941) and Terzaghi (1942) and a field investigation by Skempton (1942). With variable strata an average value of c in a depth equal to the width of the footing can generally be taken.

An alternative approach is to calculate the shear stresses $(\sigma_1 - \sigma_3)/2$ in the ground beneath the foundation and ensure that at no point is the stress in excess of the cohesion of the clay (Cooling 1942).

4) Stability of slopes (Figs. 5 and 6)

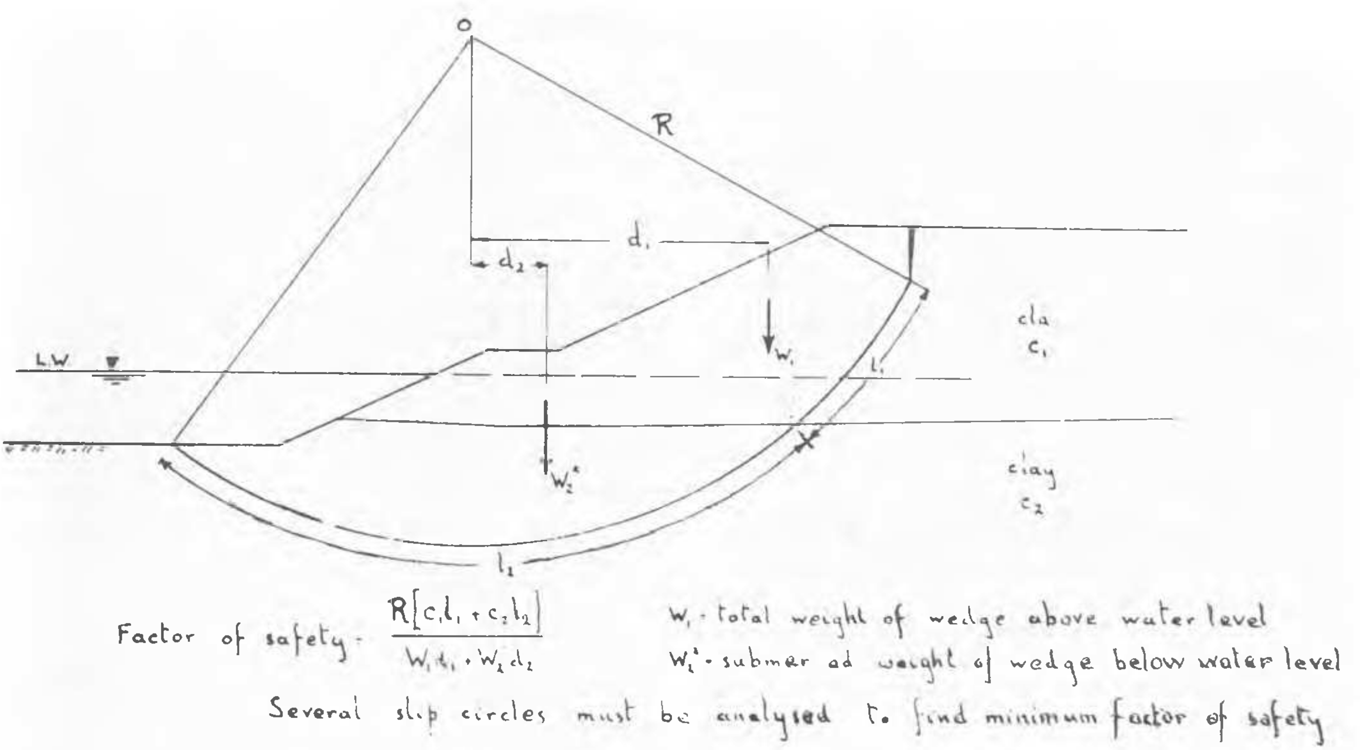
For homogeneous clays the problem has been solved by Taylor (1937), Fig. 5, following the original work of Fellenius. For variable strata the procedure is to analyse a chosen slip circle, as shown in Fig. 6, and then repeat with other circles until a minimum factor of safety has been established.

IV. LIMITATIONS OF THE $\emptyset$ = 0 ANALYSIS.

The foregoing methods of analysis have been applied with success in practical problems.(see Skempton and Golder 1948 for a summary of ten field examples). It must,however, be realised (a) that the analyses are based on the assumption that the soil behaves as if $\emptyset$ = 0 and (b) that there are three important limitations to this assumption. These limitations are:

1) For fully saturated clays the angle of shearing resistance is zero only when there is no water content change under the applied stresses. The rate of consolidation of clays is so small that, in most cases, the change in water content during construction is negligible. Therefore the $\emptyset$ = 0 analysis applies, from this point of view, almost exactly to the conditions obtaining immediately after construction. Thus the bearing capacity of a foundation when the structure is completed, the stability of a dock wall after dredging in front of the wall has been carried out, or the stability of a cutting after excavation is completed, are all given correctly by the $\emptyset$ = 0 analysis.

But with time water content changes will take place under the changed stress-conditions,and the shear strength of the clay will progressively alter from that used in the analysis. In the case of a foundation the change will be due to consolidation and the $\emptyset$ = 0 analysis is therefore conservative in that it gives the lower limit of bearing capacity. In some problems it is justifiable to allow for the increase in strength during construction (see, for example, Bishop 1948). In the case of dock walls and cuttings, however, the



$\emptyset$ = 0 Analysis of Clay Slope with two Strata.

FIG. 6

general tendency may sometimes be a reduction in strength, and this possibility must be carefully considered in design.

The special case of stiff-fissured clays, where the changes in strength with time are great, is considered in another paper by the author (Skempton 1948 b).

2) Even in fully saturated clays, tested under conditions of no water content change, the true angle of internal friction $\emptyset_f$ of the clay is not zero. Although the clay, at any particular water content, behaves with respect to the applied stresses at failure as if it were a purely cohesive material (when tested under these conditions) its behaviour is at all times controlled by the true cohesion and true angle of internal friction and the effective stresses. As shown by Terzaghi (1936) the presence of the true internal friction is apparent from the observation that the shear planes in a compression test specimen are inclined to the horizontal at angles greater than 45°, (45° corresponding to the case of true non-frictional materials). The inclination α is in fact theoretically equal to $(45^\circ + \frac{1}{2}\emptyset_f)$ for all tests including those where no water content change take place and the angle of shearing resistance is zero.

In tests on individual undisturbed samples it is often difficult to obtain consistent values of α owing to lack of homogeneity. The author has, however, examined a number of cases where reasonably consistent results have been obtained and it appears that a significant correlation exists between $\emptyset_f$ and porosity (Skempton 1948a). In Fig. 7 the results for saturated clays are given x) These soils without exception, were found to have a zero angle of shearing resistance although as will be seen, the shear planes were in all cases inclined at angles greater than 45°.

It therefore follows that whereas the applied stresses at failure are given correctly by the $\emptyset = 0$ analysis, according to equation (1), the shear surface is controlled by the true angle of friction $\emptyset_f$: an angle which is greater than zero. Thus the $\emptyset = 0$ analysis will not, in general, lead to a correct prediction of the actual shear surface, nor will the analysis, theoretically, give a correct factor of safety if the values of c are applied to an observed shear surface. In practice the errors arising from this latter point may often, though not always, be small.

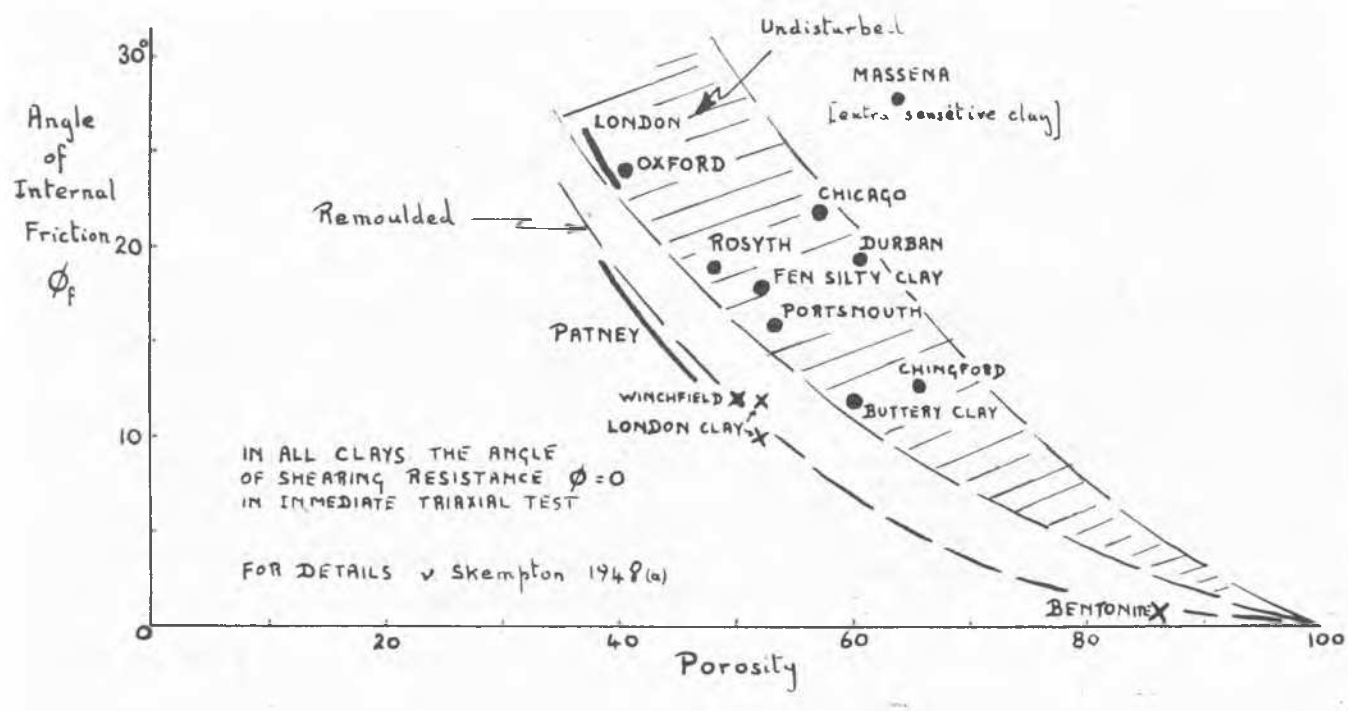
3) Clays which are not fully saturated do not give an angle of shearing resistance equal to zero when tested under conditions of no overall water content change. Thus in many problems associated with earth dams, embankments and compacted bases and sub-grades the $\emptyset = 0$ analysis will not apply. In addition it appears that some silts, even when fully saturated, show an angle of shearing resistance in the immediate triaxial test greater than zero (Golder and Skempton 1948). For these soils also the $\emptyset = 0$ analysis does not apply.

V. PRACTICAL EXAMPLES.

Three practical examples will be taken as illustrative of the above points.

1) Eau Brink Cut. (Skempton 1945). Here the actual shear surface was determined with reasonable accuracy, and the shear strengths of the strata were measured as one-half the unconfined compression strength. Using the $\emptyset = 0$ analysis the factor of safety was found to be 1.0, which is correct. But the slip surface corresponding to this factor of safety lays definitely behind the actual slip surface. This is analogous to the fact that the 45° plane in a compression test is flatter than the actual shear plane. Moreover the factor of safety as calculated on the actual slip surface was 1.30

x.) Lack of homogeneity is largely eliminated in remoulded clays and their $\emptyset_f$ porosity relation is therefore also given in Fig. 7. It is clear that a definite correlation exists for these samples.



Angle of Internal Friction in Saturated Clays.

FIG. 7

In triaxial compression tests carried out with no water content change ϕ was zero. Yet the shear planes were inclined at angles of more than 45^o without exception.

2) Chicago Subway. (Peck 1942). Here the earth pressure as calculated on the $\phi = 0$ analysis agreed well with the observed loads in the struts. The conclusion drawn by Peck was that the clay had an "angle of internal friction" equal to zero. In the author's opinion the correct conclusion is that during the few weeks of observation no appreciable water content change could take place and the essential basis for the $\phi = 0$ assumptions in saturated clays was therefore complied with. Nevertheless the clay would almost certainly have a true angle of friction ϕ_f in excess of zero. The strut load observations could not possibly supply any evidence on this point. But the author would be surprised if the shear planes in the compression tests were inclined at 45^o, or if any more elaborate analysis such as that developed by Hvorslev (1937) would show that $\phi_f = 0$ for this clay.

3) Wynne-Edwards' Critical Height Tests. In these tests, which are described by Skempton and Golder (1948), it was found that the critical height of a vertical bank of remoulded London Clay was 8 ft. This result is in reasonable agreement with the height calculated on the $\phi = 0$ analysis using the measured values of c and γ for the clay and the observed depth of tension crack. Yet the slip plane, which was well defined in each of the three tests, was inclined at 51^o, and not at 45^o. Moreover the value of ϕ_f, deduced from the relation $\alpha = 45^o + \frac{1}{2}\phi_f$ is 12^o and this is in close agreement with the values of ϕ_f found from the inclination of the shear planes in compression specimens of remoulded clay at the particular porosity (52 percent), of the clay used in the critical height tests, see Fig. 7.

VI. CONCLUSIONS.

1) When tested with no water content change a saturated clay behaves with respect to the applied stresses at failure as if it were a purely cohesive material with an angle of shearing resistance ϕ equal to zero.

Consequently an analysis of stability based on this result will give correct results for earth pressure, bearing capacity, or the factor of safety of a clay slope for the period when water content changes are negligible. This will often be the case for the conditions at the completion of construction.

2) With increasing time after construction water content changes may occur which lead to different strengths in the clay from those assumed in the $\phi = 0$ analysis. These should be allowed for as far as possible in the design. The special case of stiff-fissured clays is an extreme example of this effect.

3) The fact that the $\phi = 0$ analysis leads to a correct evaluation of stability under conditions of no water content change does not in any way prove that the clay has a true angle of internal friction ϕ_f equal to zero. On the contrary there is evidence that most clays exhibit an angle of internal friction definitely in excess of zero. This leads to the conclusion that, in so far as the position of the shear surface is controlled in part by the angle of internal friction (friction being a directional porosity and not merely a coefficient representing increase in shear strength under increasing pressure), the $\phi = 0$ analysis will lead to an incorrect placing of the shear surface. It is also concluded that a $\phi = 0$ analysis based on the actual shear surface will, at least theoretically, lead to an incorrect estimate of the forces and of stability.

The correct estimate of stability will be obtained from a $\phi = 0$ analysis using the shear surface compatible with the $\phi = 0$ assumption, even though this surface is itself not coincident with the true slip surface.

4) The $\phi = 0$ analysis cannot be applied to partially saturated clays, nor to those silts which show an angle of shearing resistance greater than zero in the immediate triaxial test.

5) It may be possible to evolve an analysis which overcomes the difficulties expressed in conclusions (3) above. Meanwhile, provided its limitations are appreciated, the $\phi = 0$ analysis is a method of great value in civil engineering design.

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I e 7

A SIMPLIFIED METHOD FOR COMPUTING THE BEARING CAPACITY OF THE SOIL SUPPORTING FOOTINGS OR PIERS

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INTRODUCTION.

In engineering practice, it is often useful to dispose of a simplified method for estimating rapidly the bearing capacity of the soil beneath footings or piers.

By this estimation two main conditions are to be taken in consideration: firstly the foundation load will be limited in such a way that shear failure and loss of stability of the soil cannot occur, and secondly the foundation must be designed to distribute the load so that dangerous settlements of the superstructure are prevented.

The ultimate value of the bearing capacity of the soil and the magnitude of the settlements depend not only on the mechanical properties of the soil, but also on the size of the loaded area, its shape, and its location with reference to the surface of the soil. It is evident that settlements are dependent on the allowed unit load.

In order to reduce the time required to solve both problems of stability and deformation of the soil, for some practical cases of loading conditions, diagrams are presented, which greatly simplify the use of formulas.

BASIC FORMULAS.

1) The bearing capacity of the soil.

In accordance with the investigations of Prandtl and Buisman, M. De Beer obtained a general formula for the ultimate bearing capacity of the soil supporting a continuous and centric loaded footing with a width b. In that formula, it is assumed that the shearing resistance s of the soil is determined by the equations:

$$s = c + p_b . tg\,\varphi + (\sigma - p_b)\, tg\varphi' , \text{ when } \sigma > p_b, \quad (1)$$

$$s = c + \sigma\ .tg \qquad , \text{ when } \sigma > p_b, \quad (2)$$

wherein:

- c = the cohesion
- φ = the angle of internal friction
- φ' = the "apparent" angle of friction
- σ = the effective normal stress
- p_b = the effective normal stress beside the base of the footing.

For cohesionless soils (c = 0), M. De Beer obtains for the ultimate bearing capacity d_r, the following formula:

$$d_r = V''_b \cdot p_b + V'_g \cdot \gamma_k \cdot b \qquad (3)$$

in which:

- b = the width of the footing.
- γ_k = the unit weight of the earth below the footing, in correlation with effective normal stress.

V''_b and V'_g = factors which depend on φ and φ'

$$V''_b = \left[e^{\pi\, tg\varphi'} \cdot tg^2\left(\frac{\pi}{4} + \frac{\varphi'}{2}\right) - 1 \right] \frac{tg\,\varphi}{tg\,\varphi'} - 1 \qquad (4)$$

An analytic expression of the factor V'_c, whose values depend only on φ', has been elaborated by Prof. Raes. Some values of V'_g, obtained by means of this expression, are contained in table I. Corresponding values of V''_b, given by (4) for the case $\varphi = 30^\circ$, are also indicated.

TABLE I.

φ'	V''_b	V'_g	φ'	V''_b	V'_g
5°	4,7	0,2476	20°	9,6	3,4522
10°	5,8	0,7233	25°	13,0	7,1634
15°	7,3	1,6413	30°	18,2	15,190

2) The settlements.

The final settlement Z of a rigid footing may be computed from the Terzaghi formula:

$$Z = \int_{\infty}^{0} \frac{dh}{C} \cdot 2,3 \cdot \log \frac{\sigma_0 + i \cdot \Delta p}{\sigma_0} \qquad (5)$$

wherein:

- C = coefficient of compressibility
- dh = thickness of an elementary layer
- σ_0 = original effective stress upon the layer dh, located at a depth z below the base of the footing.
- Δp = increase of the effective stress at the level of the base.
- i = influence value of the surcharge Δp, on the vertical normal stress at a depth z.
- $\sigma_0 + i \cdot \Delta p$ = final effective vertical stress upon the layer dh (depth z). At the level of the base (z = 0), where the influence value i = 1, and the original effective