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A SIMPLIFIED METHOD FOR COMPUTING THE BEARING CAPACITY OF THE SOIL SUPPORTING FOOTINGS OR PIERS

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INTRODUCTION.

In engineering practice, it is often useful to dispose of a simplified method for estimating rapidly the bearing capacity of the soil beneath footings or piers.

By this estimation two main conditions are to be taken in consideration: firstly the foundation load will be limited in such a way that shear failure and loss of stability of the soil cannot occur, and secondly the foundation must be designed to distribute the load so that dangerous settlements of the superstructure are prevented.

The ultimate value of the bearing capacity of the soil and the magnitude of the settlements depend not only on the mechanical properties of the soil, but also on the size of the loaded area, its shape, and its location with reference to the surface of the soil. It is evident that settlements are dependent on the allowed unit load.

In order to reduce the time required to solve both problems of stability and deformation of the soil, for some practical cases of loading conditions, diagrams are presented, which greatly simplify the use of formulas.

BASIC FORMULAS.

1) The bearing capacity of the soil.

In accordance with the investigations of Prandtl and Buisman, M. De Beer obtained a general formula for the ultimate bearing capacity of the soil supporting a continuous and centric loaded footing with a width b. In that formula, it is assumed that the shearing resistance s of the soil is determined by the equations:

$$s = c + p_b . tg\,\varphi + (\sigma - p_b)\, tg\,\varphi', \text{ when } \sigma > p_b, \quad (1)$$

$$s = c + \sigma\ .tg \qquad , \text{ when } \sigma > p_b, \quad (2)$$

wherein:
- c = the cohesion
- φ = the angle of internal friction
- φ' = the "apparent" angle of friction
- σ = the effective normal stress
- p_b = the effective normal stress beside the base of the footing.

For cohesionless soils (c = 0), M. De Beer obtains for the ultimate bearing capacity d_r, the following formula:

$$d_r = V''_b \cdot p_b + V'_g \cdot \gamma_k \cdot b \quad (3)$$

in which:
- b = the width of the footing.
- γ_k = the unit weight of the earth below the footing, in correlation with effective normal stress.

V''_b and V'_g = factors which depend on φ and φ'

$$V''_b = \left[e^{\pi\, tg\,\varphi'} \cdot tg^2\left(\frac{\pi}{4} + \frac{\varphi'}{2}\right) - 1 \right] \frac{tg\,\varphi}{tg\,\varphi'} - 1 \quad (4)$$

An analytic expression of the factor V'_c, whose values depend only on φ', has been elaborated by Prof. Raes. Some values of V'_g, obtained by means of this expression, are contained in table I. Corresponding values of V''_b, given by (4) for the case $\varphi = 30°$, are also indicated.

TABLE I.

φ'	V''_b	V'_g	φ'	V''_b	V'_g
5°	4,7	0,2476	20°	9,6	3,4522
10°	5,8	0,7233	25°	13,0	7,1634
15°	7,3	1,6413	30°	18,2	15,190

2) The settlements.

The final settlement Z of a rigid footing may be computed from the Terzaghi formula:

$$Z = \int_{\infty}^{0} \frac{dh}{C} \cdot 2{,}3 \cdot \log \frac{\sigma_0 + i \cdot \Delta p}{\sigma_0} \quad (5)$$

wherein:
- C = coefficient of compressibility
- dh = thickness of an elementary layer
- σ_0 = original effective stress upon the layer dh, located at a depth z below the base of the footing.
- Δp = increase of the effective stress at the level of the base.
- i = influence value of the surcharge Δp, on the vertical normal stress at a depth z.
- $\sigma_0 + i \cdot \Delta p$ = final effective vertical stress upon the layer dh (depth z). At the level of the base (z = 0), where the influence value i = 1, and the original effective

vertical stress $\sigma_o = p_b$, acts a unit load:

$$p = (\sigma_o + i.\Delta p)_o = p_b + \Delta p.$$

INFLUENCE VALUE i DUE TO A SURFACE LOAD Δp.

The vertical normal stress increase $d\sigma_z$ at an arbitrary point A, located at a depth z below an elementary surface dS of a loaded area S, which carries a load Δp, can be obtained by the Boussinesq-Buisman formula:

$$d\sigma_z = \frac{2\,dS}{\pi z^2} \cdot \cos^6 \theta \cdot \Delta p, \qquad (6)$$

wherein θ is the angle between the vertical axis through the point A and the vector (A,dS).

The influence value i for the point A is obtained by integration of the expression (6) over the whole loaded area S:

$$\sigma_z = \iint_S d\sigma_z = \iint_S \frac{2 \cdot dS}{\pi z^2} \cdot \cos^6 \theta \cdot \Delta p = i \cdot \Delta p \quad (7)$$

It has been proved (Voitus van Hamme, Delft) that for some particular points of a continuous, a rectangular, a square or a circular footing, the settlements computed in using the formula (3), are practically independent of the law of distribution of the contact pressure over the base of the footing. In these particular points the settlements may then be computed in the assumption that the distribution of the contact pressure is uniform. Further by absolute rigidity of the footing, the settlements will be uniform and equal to these calculated for the particular points. The location of these points is indicated in figure 1. (points P). The following table (table II) contains the calculated influence values i, at different depths of the vertical axis passing through the point P of a continuous, a square and a circular footing. The depths are expressed by the ratio of the ordinate z to the width of the footing.

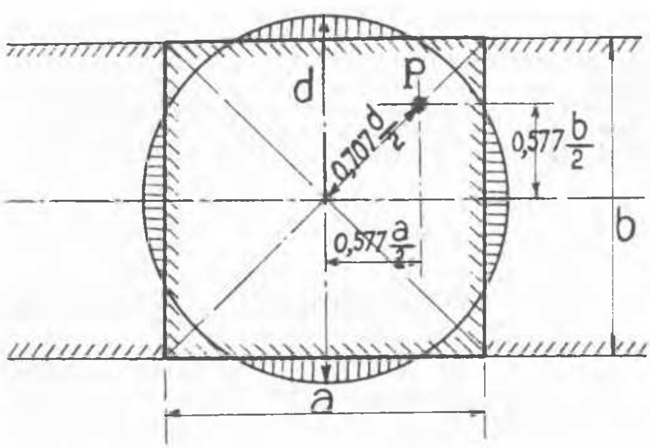


FIG.1

THE CHARACTERISTICS OF THE SOIL.

Location of the water level and unit weight of the soil.

The effective pressure p_b at the level of the base of the footing or the effective pressure σ_o at an arbitrary depth below the base depend on the location of the groundwater level and the unit weight of the soil.

In the following investigations it is arbitrary assumed that the water level corresponds with the foundation level. Further it is supposed that the pressure in the water is hydrostatic.

As unit weights of the soil, we admit:
- soil above the water level : $\gamma = 1,6\ t/m^3$
- soil below the water level : $\gamma_n = 2,0\ t/m^3$.

Taking in account the hydrostatic pressure in the pore-water, the unit weight γ_k to introduce in the formula (3) is:

$$\gamma_k = (2,0 - 1,0)\ t/m^3 \text{ or } 1\ t/m^3 \qquad (8)$$

Hence, if the surface of the ground is horizontal and does not carry any surcharge, we obtain:
- effective pressure p_b :

$$p_b = 1,6\ h \qquad (h \text{ in m., } p_b \text{ in } t/m^2) \quad (9)$$

wherein h is the depth of the base below the surface of the ground.
- effective pressure σ_o :

$$\sigma_o = 1,6h + 1z \qquad (h \text{ and } z \text{ in m, } \sigma_o \text{ in } t/m^2) \quad (10)$$

in which z is the depth below the level of the base.

THE SHEARING RESISTANCE OF THE SOIL.

It is assumed that the shearing resistance s of the soil located below the base of the footing can be expressed by the formulas (1) and (2). Assuming furthermore that the soil is cohesionless (c = 0), and has an angle of internal friction $\varphi = 30^o$ for a normal stress smaller than p_b, the shearing resistance s of the idealized soil we will introduce in our calculations, can be represented by figure 2. The shearing resistance of the soil located

TABLE II

Influence values i.

continuous footing width b		square footing side a		circular footing diameter d	
depth $\frac{z}{b}$	i	depth $\frac{z}{a}$	i	depth $\frac{z}{d}$	i
0	1,00	0	1,00	0	1,00
0,2	0,95	0,2	0,90	0,2	0,87
0,4	0,82	0,4	0,67	0,4	0,64
0,6	0,70	0,6	0,52	0,6	0,48
0,8	0,62	0,8	0,40	0,8	0,37
1,0	0,56	1,0	0,33	1,0	0,28
1,2	0,50	1,2	0,26	1,2	0,23
1,4	0,45	1,4	0,21	1,4	0,19
1,6	0,41	1,6	0,18	1,6	0,16
1,8	0,37	1,8	0,15	1,8	0,13
2,0	0,35	2,0	0,13	2,0	0,12

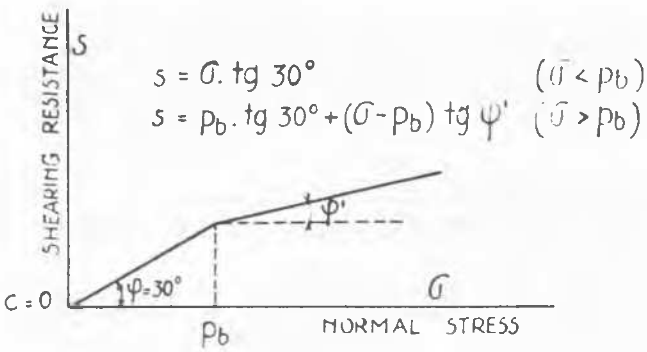


FIG. 2

above the level of the base of the footing is neglected. An approximate value of φ' can be obtained from triaxial compression tests or derived from deepsounding results. The following table contains some limits of φ' found for different soil types:

TABLE III.

Soil type	φ'
soft alluvium	5^{o} - 10^{o}
firm tertiary clay	10^{o} - 15^{o}
loam, or clayey sand	15^{o} - 25^{o}
loose sand	25^{o} - 30^{o}
compact sand	$>30^{o}$

THE COMPRESSIBILITY OF THE SOIL.

In computing the settlements of a soil stratum which is subjected to a foundation load, it is generally assumed that the soil is homogeneous, isotropic and perfectly elastic. Furthermore we select a foundation material whose deformation characteristics correspond to a given coefficient of compressibility, namely C = 100. The coefficient of compressibility of the real soil has to be determined by consolidation tests or can be derived from deepsounding results. Some values of C obtained for some soil types are given in the table IV.

TABLE IV.

soil type	C
peat	5 - 10
clay	10 - 20
sandy clay, loam	20 - 50
sand (according to the compacity)	5= - 500

BEARING CAPACITY DIAGRAMS.

Combining the equations (3), (8) and (9), we get:

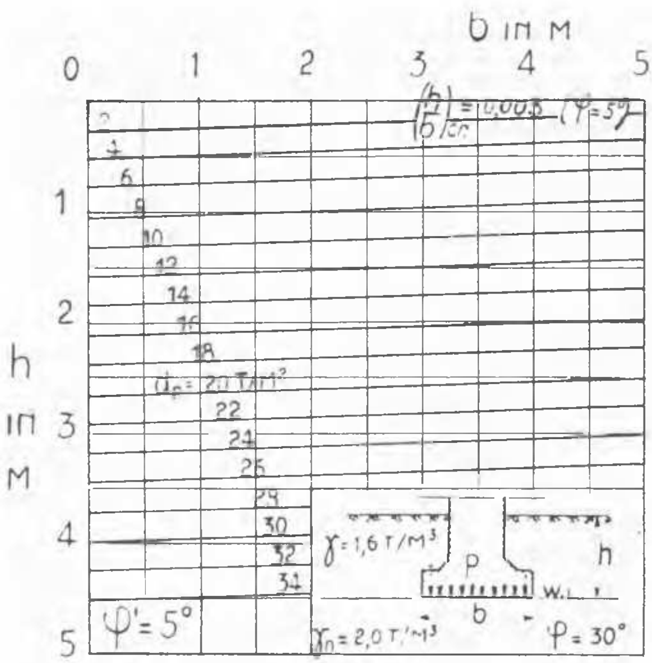
$$d_r = 1{,}6\ V''_b \cdot h + V'_g \cdot b \qquad (11)$$

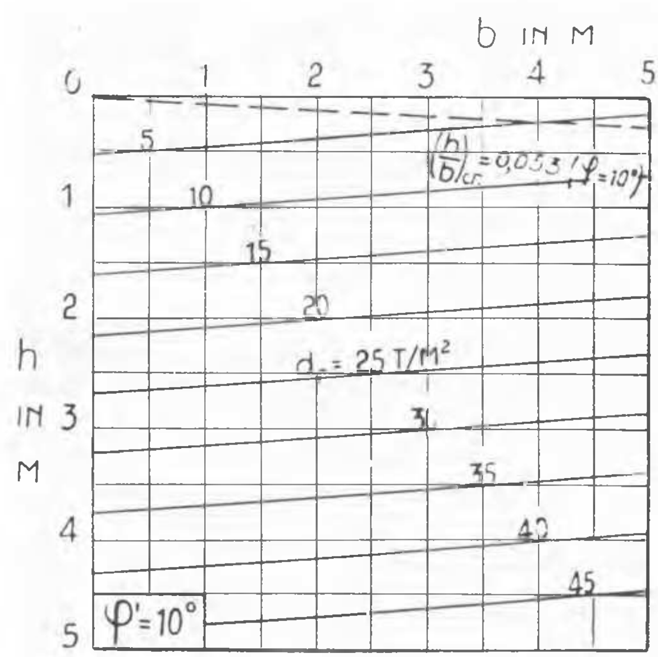
This equation shows that the bearing capacity d_r is a linear function of the depth h below surface and the width b of the footing.

In the figures 3 to 8, where b is plotted against h for various angles φ', the expression (11) is represented by a net of straight lines of equal bearing capacity.

SETTLEMENT GRAPHS.

In integrating graphically the equation (5), in which C = 100, settlements have been computed for various widths of the footing and various depths below surface. Different values of the unit load p were taken in consideration namely p = 5 - 10 - 15 - 20 t/m². These operations have been repeated for a continuous long strip, for a square and for a circular loaded area. The influence values i of the table II were used. In addition to this, it will be said that the integration of the expression





Lines of equal bearing capacity for continuous footings.

FIG. 3

FIG. 4

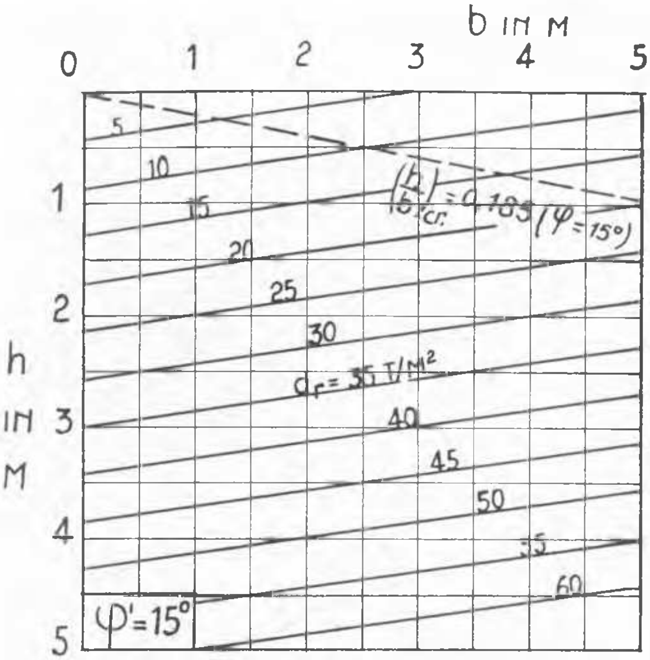


FIG. 5

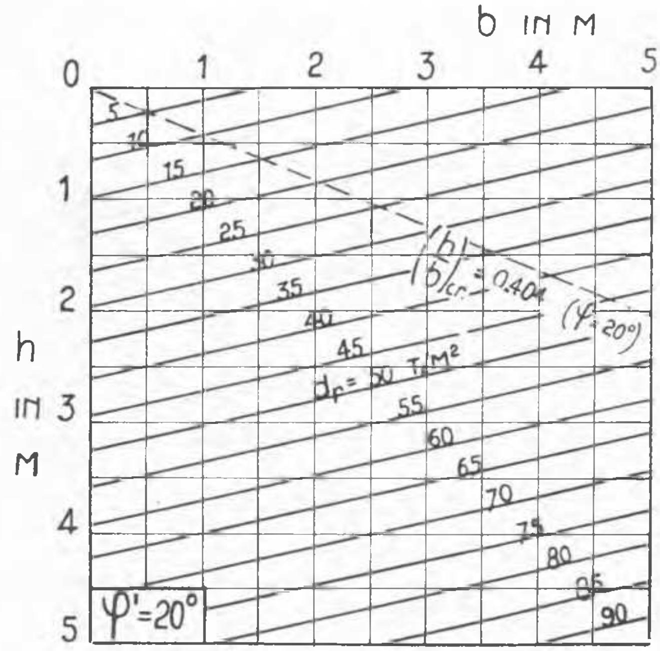
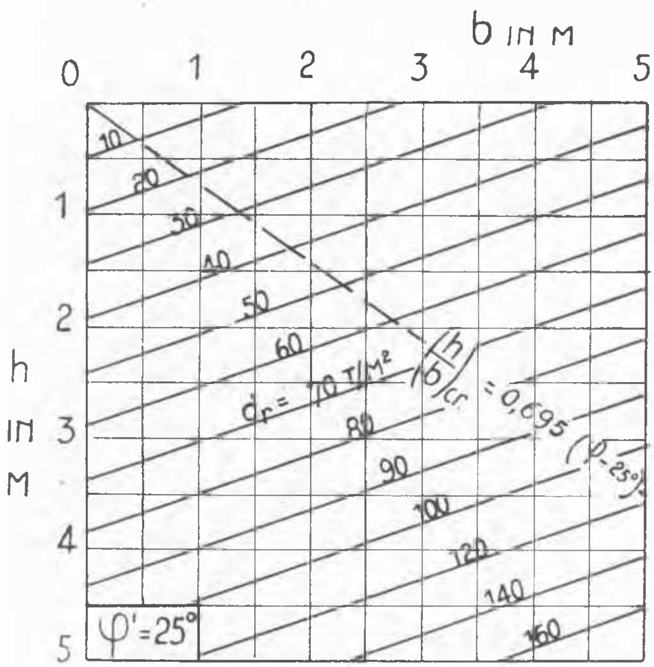
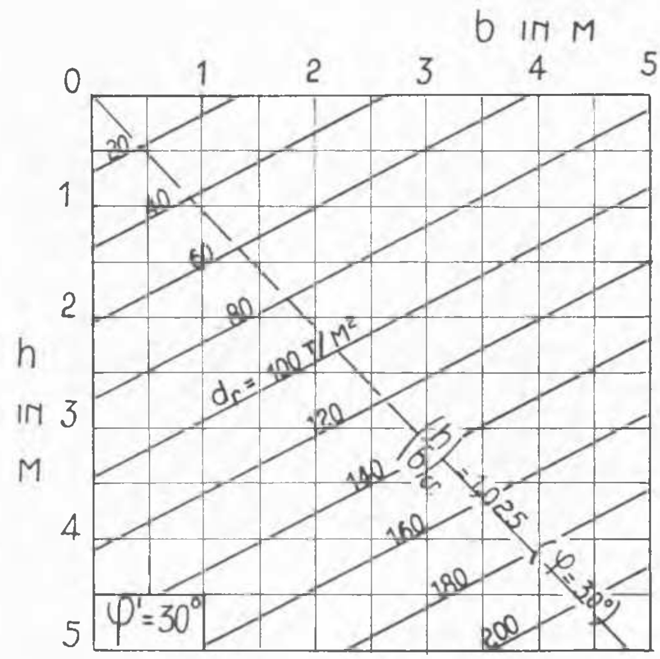


FIG. 6





Lines of equal bearing capacity for continuous footings.

FIG. 7

FIG. 8

(5) was limited to a depth equal to two times the smallest plan dimension of the loaded area. The results of these computations (about 200 in number) have permitted to draw the graphs of figures 9 to 20, in which the width b is plotted against the depth h, and curves of equal settlement are represented.

It may be seen in figures 9, 13 and 17, that the curves of equal settlement approach an horizontal asymptote with an ordinate $h_c = \frac{5}{1,6} = 3,125$ m. At this critical depth, the weight of the soil balances the unit load $p = 5\ t/m^2$ of the footing. For values of h greater than h_c, the underlying strata will have a tendency to swell.

Continuous footing.

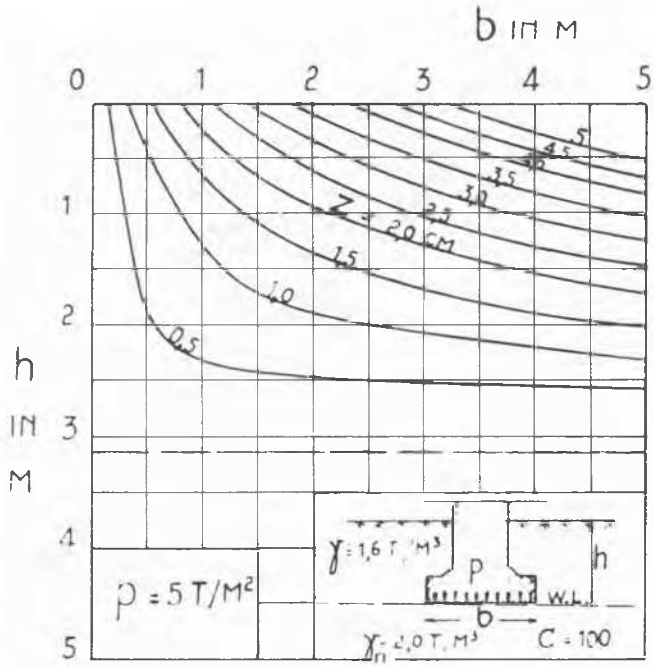


FIG. 9

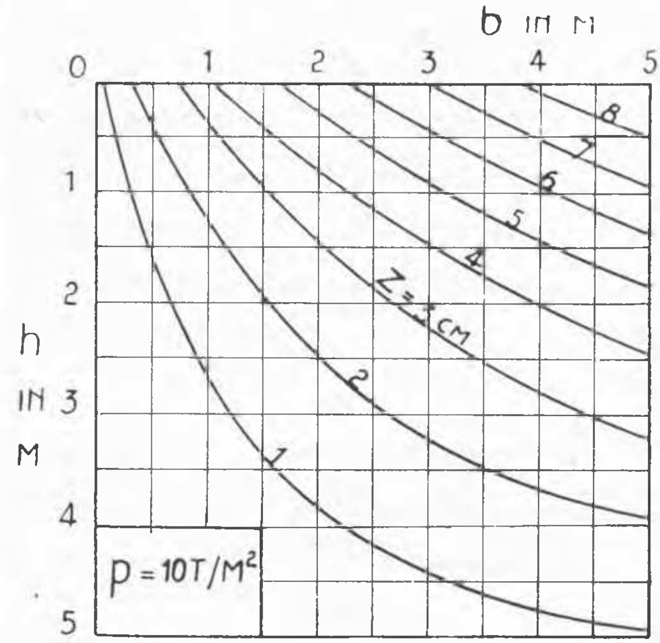
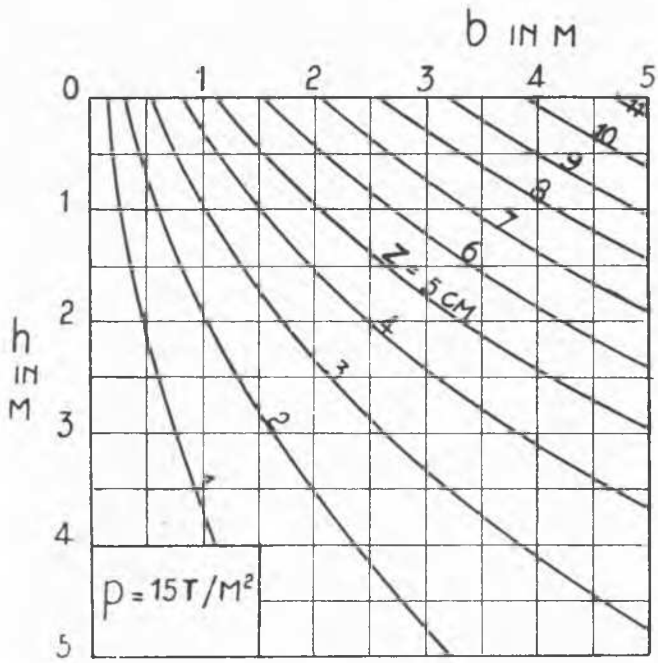
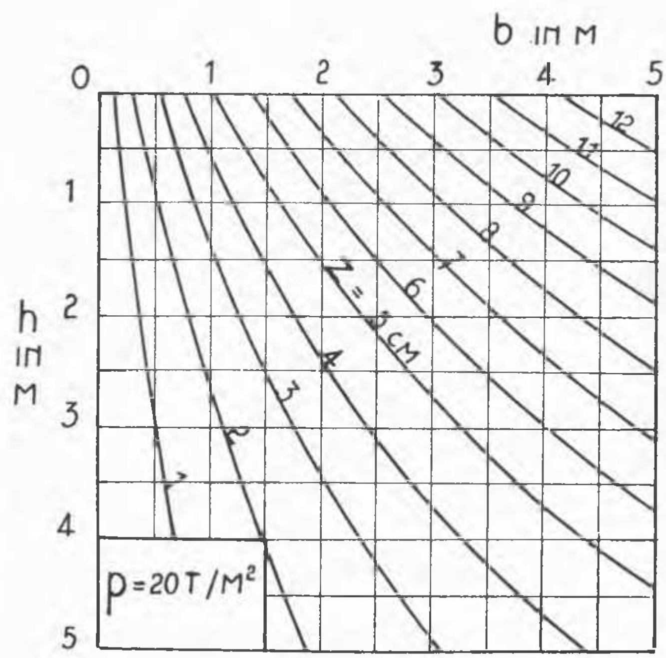


FIG. 10





Curves of equal settlement for different values of unit load.

FIG. 11 FIG. 12

SIMPLIFIED METHODS FOR ESTIMATING BEARING CAPACITY AND SETTLEMENTS.

1) Where the soil characteristics and the loading conditions correspond to the assumptions introduced by elaborating our diagrams, the use of them, for example for design of footings, presents no more difficulties. Even interpolation can be easily effected for intermediary values of φ' or p.

2) Equation (11) and figures 3 to 8 refer to continuous footings. Practically the equation (11) and the corresponding graphs are also applicable to rectangular footings whose

Square footing.

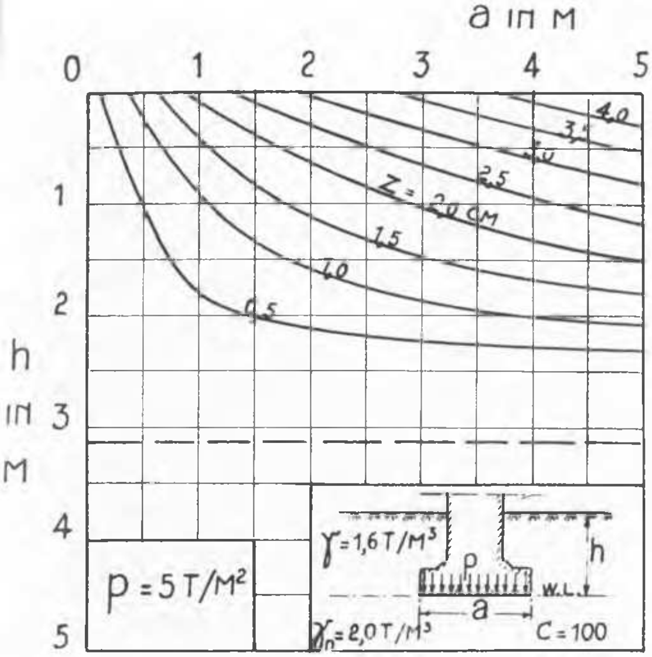


FIG.13

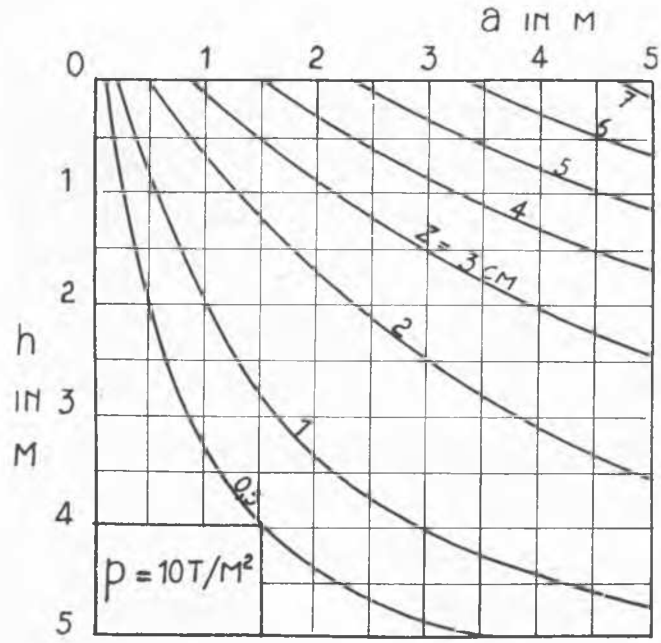
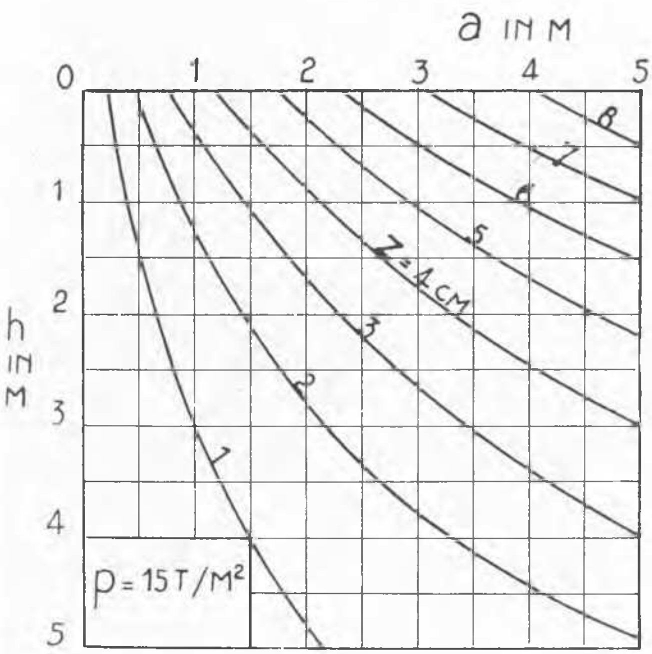
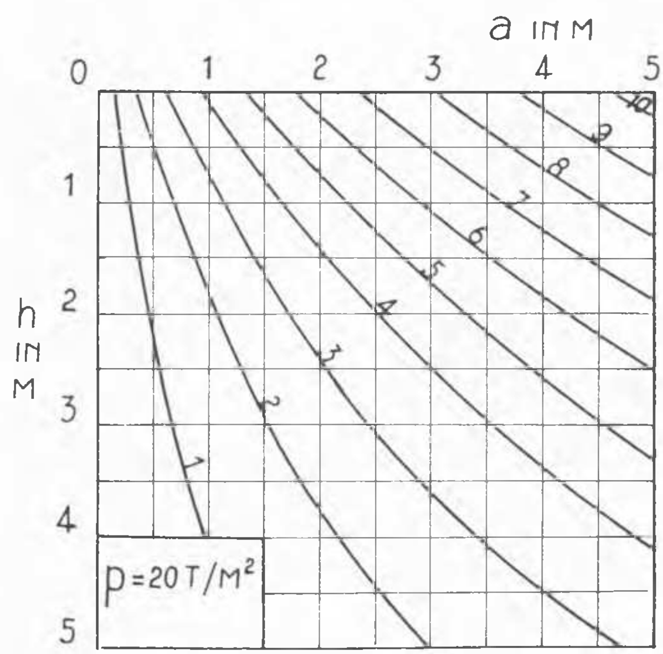


FIG.14





Curves of equal settlement for different values of unit load.

FIG.15

FIG.16

lengths are large compared to their widths. Laboratory experiments made in Delft have shown that, for a loaded area whose width is equal to its length, such as a square or a circular area, the bearing pressure is about 30 per cent greater than for a strip load. On the basis of this experience, the diagrams may be used for a square or a circular footing: it is only necessary to multiply the indicated values of d_r by the coefficient 1,3.

3) The formula (11) has been obtained for a centric loaded strip. By eccentric loading, and if the eccentricity is small, the use of the formula,(11) may give sufficient results,

Circular footing.

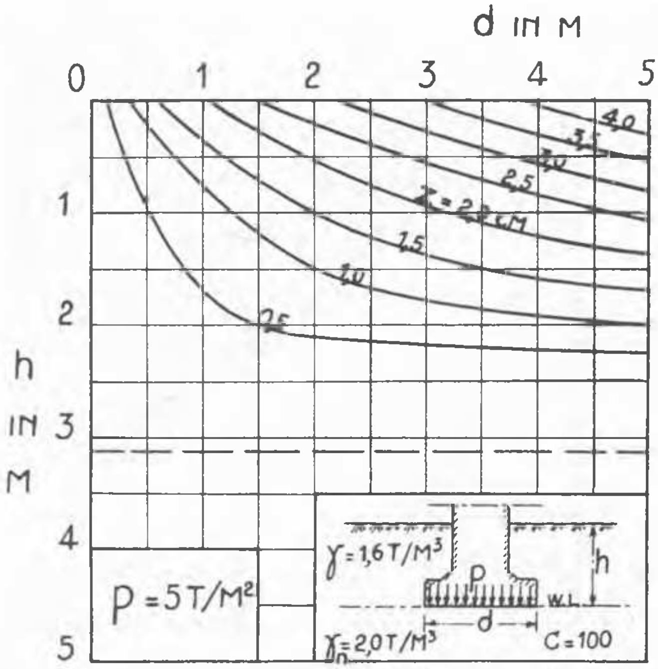


FIG.17

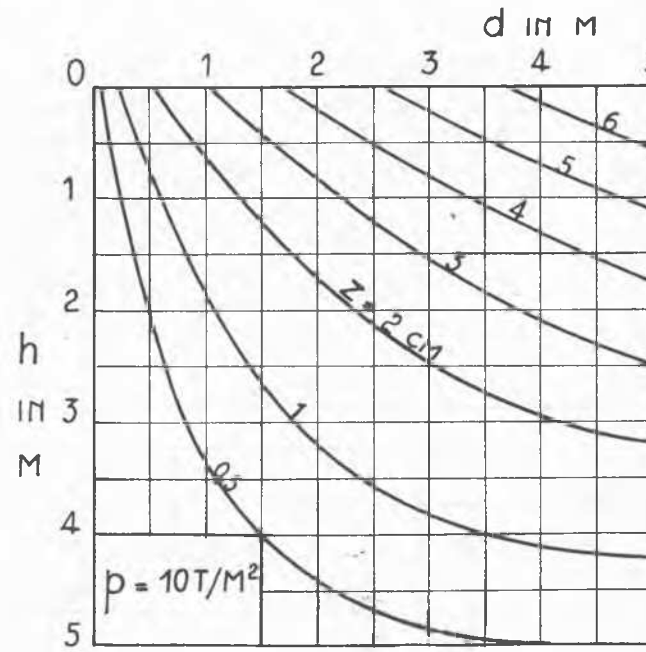
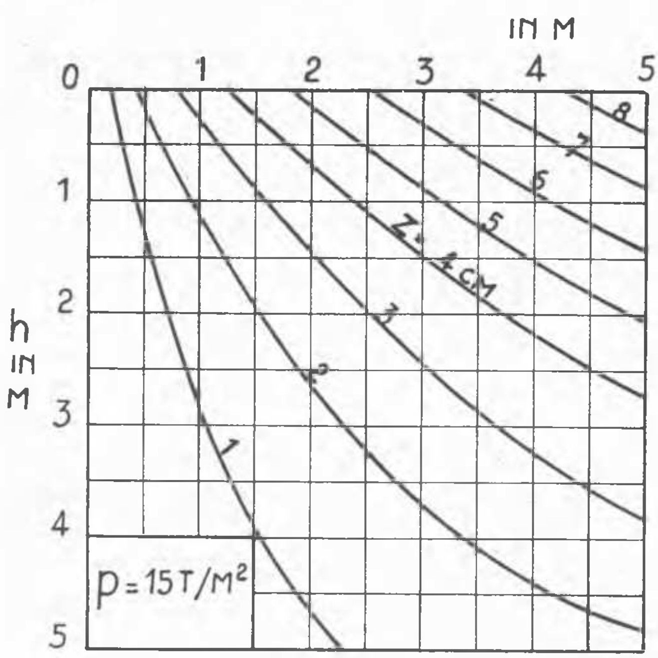
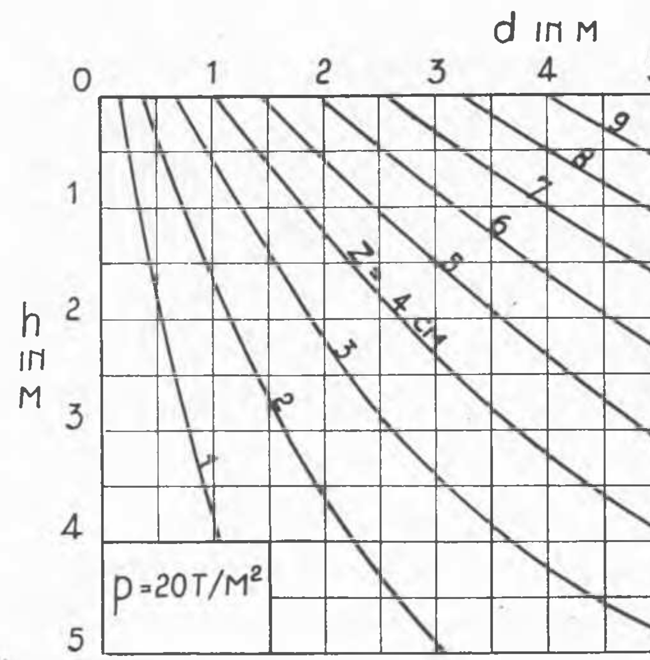


FIG.18





Curves of equal settlement for different values of unit load.

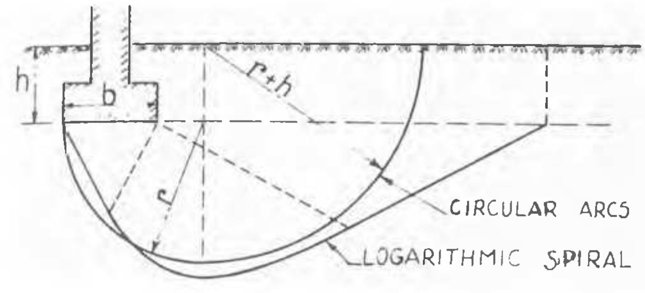
FIG.19

FIG.20

when the width b is substituted by two times the complemental eccentricity 2u.

4) The developed formula of the bearing capacity assumes that the surface of failure consists of a logarithmic spiral tangent to two straight lines (Prandtl's theory).

M. Andersen (Minnesota) has developed formulas in which it was assumed that the surface of rupture can be approximated by circular arcs, as shown in figure 18. In comparing the two methods, for a cohesionless soil in which $\varphi = \varphi'$, Mr. De Beer has found that the method of circular arcs gives smaller values than the Prandtl method, when the width of the



Prandtl and Andersen Methods.

FIG. 21

footing is great compared to the depth of foundation (shallow foundation). Fig 21. On the contrary, the Prandtl-method leads to more safe results when dealing with deep foundations. Mr. De Beer has shown elsewhere that the applicability of both methods can be limited by a critical value of the ratio of the depth to the width of the footing. Since this value depends only on the angle of internal friction, the zones of applicability of both methods may be separated by the straight lines shown in figures 3 to 8 (dotted lines). Above these critical lines, the curves of equal bearing capacity may give exaggerated values of d_r.

On the other hand, in the developed formula, the shearing resistance of the soil located above the level of the base of the footings has been disregarded. However on the safe side, when dealing with deep foundations, such simplification is not justified, because the resulting error may be excessive.

Now the diagrams developed for the bearing capacities, give ultimate values, at which rupture will occur. If they are applied to design of footings, regardless the problem of settlement, suitable factors of safety should be used. When dealing with deep footings or piers, a factor of safety, s = 2 or less, may be allowed. When the load of the construction is transmitted by means of shallow foundations, a factor of safety of s = 4 or more may be necessary.

5) The bearing capacity diagrams have been computed in the assumption that the surface of the ground beside the foundation is horizontal and not surcharged. When the surface of the ground carries a uniformly distributed surcharge q per unit of area, the graphs can be used by introducing an equivalent depth of foundation h_f, by means of the following equations:

$$p_b = q + 1,6\ h = 1,6\ h_f$$

$$h_f = h + \frac{q}{1,6} \qquad (12)$$

6) A similar method can be used when the real unit weights of the soil are not the same as those introduced in the elaboration of the diagrams.

Let: γ' = the real unit weight of the soil above the water level.

γ'_k = the real unit weight of the soil below the base of the foundation, reduced by the unit weight of water.

q = a uniform surcharge.

h_f, b_f = equivalent depth and width of foundation which lead to an equal bearing capacity in an unsurcharged soil with characteristics:

$\gamma = 1,6\ t/m^3$, $\gamma_k = 1\ t/m^3$.

The ultimate bearing value may then be computed, by replacing h and b, with h_f and b_f:

$$p_b = q + \gamma' .\ h = 1,6\ h_f \qquad (13)$$

$$\gamma'_k\ .\ b = (2,0 - 1,0)\ b_f \qquad (14)$$

7) If the groundwater level does not correspond with the level of the base, but is located only at a depth $h_n < h$ below the surface of the ground, we can introduce the equivalent depth h_f:

$$p_b = 1,6\ h_n + (2,0 - 1,0)\ (h - h_n) = 1,6\ h_f \qquad (15)$$

If the water level is far below the level of the base of the footing, an equivalent width b_f is given by:

$$1,6\ b = (2,0 - 1,0)\ b_f \qquad (16)$$

8) The use of the formula (11) is justified when in the proximity of the foundations, the surface of the ground has not to be modified by excavation or cutting works. In construction of buildings, it is often necessary to found the footing near a deep cut needed to provide basement room, for example for cellars. If h represents the depth of the foundation with reference to an horizontal section through the bottom of the cut, the method based on the equation (11) could be used. Nevertheless for small values of φ' and small values of h, this method can lead to very unfavorable results, because the preconsolidation under the original natural pressure was disregarded. In order to take in account the effect of the preconsolidation, Mr. de Beer presents in this case the following bearing capacity formula:

$$d_r = p_{b,o}\left[X(\varphi,\varphi')\, tg\,\varphi \left(\frac{p_b}{p_{b,o}}\right)^{\frac{tg\,\varphi'}{tg\,\varphi}} - \frac{tg\,\varphi}{tg\,\varphi'} + 1\right] + V'_q \cdot \gamma_k \cdot b \qquad (17)$$

wherein:

$p_{b,o}$ = the original effective vertical pressure at the level of the foundation, before the cut was opened, and which corresponds to the weight of the earth below the surface of the ground (height h_o).

p_b = the actual effective pressure beside the base of the foundation, and which is exerted by the earth located below the bottom of the cut (height h).

$X(\varphi,\varphi')$ = a function of φ and φ'. Some values of $X(\varphi,\varphi')$, when $\varphi = 30^0$, are given in table V.

TABLE V

φ'	$X(\varphi,\varphi')$	φ'	$X(\varphi,\varphi')$
5^0	17,9	20^0	17,6
10^0	14,0	25^0	22,9
15^0	14,6	30^0	31,8

In order to obtain a convenient solution of the equation (15), we introduce an equivalent depth h_f of foundation, by means of the following equations:

$$V_b'' \cdot 1{,}6\, h_f = \left[X(\varphi, \varphi')\, tg\, \varphi \left(\frac{p_b}{p_{b,o}}\right)^{\frac{tg\varphi'}{tg\varphi}} - \frac{tg\,\varphi}{tg\,\varphi'} + 1 \right] \cdot p_{b,o} \quad (18)$$

$$K = \frac{X(\varphi, \varphi')\, tg\, \varphi \left(\frac{p_b}{p_{b,o}}\right)^{\frac{tg\varphi}{tg\varphi'}} - \frac{tg\,\varphi}{tg\,\varphi'} + 1}{V''_b} \quad (19)$$

$$1{,}6\, h_f = K . p_{b,o} = K \, . \, 1{,}6\, h_o .$$

$$h_f = K . \, h_o . \quad (20)$$

The factor K has been computed for different values of the ratio $\frac{p_b}{p_{b,o}}$ or $\frac{h}{h_o}$, and different values of φ'. Values of K for $\varphi = 30°$ are given in table VI.

the value of the final settlement Z, depends on the opposite value of the coefficient of compressibility C, the probable settlement for the real soil will be obtained by multiplying the values of the graphs by the ratio $\frac{100}{C}$, in which C is the real coefficient of compressibility.

If the load conditions are not the same as those introduced in the basic graphs, a graphical interpolation can lead to a sufficient accurate solution of the problem.

If the soil characteristics, as unit weight and water level, differ from the assumed ones, in the most cases the graphs give an upper limit for the real settlement. In introducing equivalent values for the width or the depth

TABLE VI

h/h_o	φ'					
	5°	10°	15°	20°	25°	30°
1,0	1,000	1,000	1,000	1,000	1,000	1,000
0,8	0,928	0,905	0,875	0,850	0,825	0,800
0,6	0,838	0,798	0,756	0,707	0,656	0,600
0,4	0,718	0,662	0,598	0,533	0,469	0,400
0,2	0,528	0,462	0,390	0,322	0,259	0,200

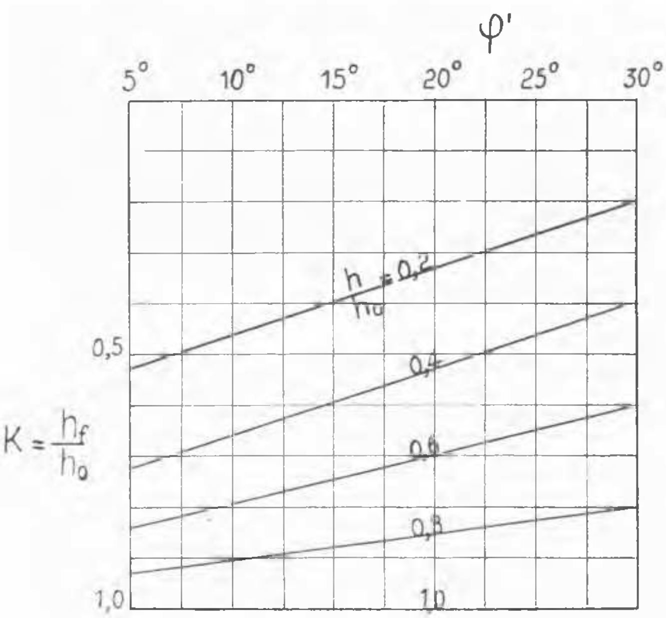


FIG. 22

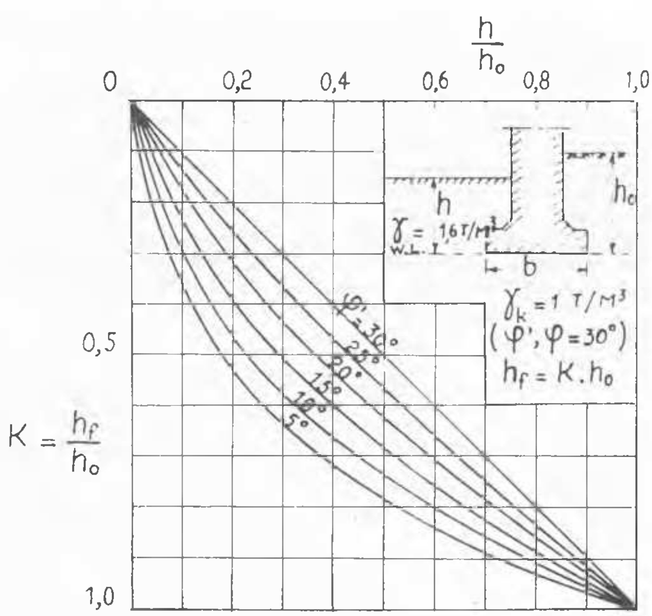


FIG. 23

For rapid interpolation, the values of the table VI may be plotted as in figures 22 and 23.

To obtain directly by the usual procedure, (diagrams 3 to 8) the bearing capacity d_r of the footing near a cut, it is now only necessary to select the tabular or the graphical value of the ratio K, and to calculate the equivalent depth of foundation h_f, by means of the equation (20).

9) The settlement graphs represented by figures 9 to 20, are constructed for a given degree of compressibility of the soil, namely C = 100.

Since, in accordance to the basic formula,(5), of the footings, as was done for the use of the bearing capacity diagrams, useful and sufficient approximate information on the settlement problem may rapidly be obtained.

10) Even, when the compressibility of the soil has not been previously determined, the diagrams may give useful indication in order to avoid differential settlement. For instance in the case of an approximately homogeneous soil, it will be possible to choice judiciously the size and the depth of the different footings of the construction, in such a way that the final settlements of the various parts of the construction will be as uniform as possible.

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