expressed by (16): $\tau_m = \mu\sigma_m$

from which we can derive the stresses and strains at breakage under some fundamental conditions of stresses or strains.

3. Under some fundamental conditions of stresses or strains, such as pure shear, simple normal stress and equal normal stresses etc. in two- and three-dimentional problems, the stress-strain relations were determined and the obtained results showed:
a. They can express quite well the tendencies of the well known experimental results.
b. Elasticity can be applied to the new theory as its special case when the changes of stresses in a soil mass remain in small amounts compared with the initially existed stress σ_0
c. The effects of the new constant μ or Poisson's ratio ν on the deformations under different loading conditions can be clearly explained.
d. The stresses and strains at breakage in the case of pure shear, simple normal stress or equal stresses can be determined. We can know the effects of a new constant μ on the stresses or strains at breakage, the tendencies of which coincide with those of the well known experimental results.

The relations between the breaking strengths under different loading conditions, such as simple compression or tension test, shear test and triaxial loading test can be explained.

4. Under such stress conditions as plane stresses, plane strains, and triaxial symmetrical stresses, the envelopes of Mohr's circles were determined. They are not always straight lines, but are ellipses, parabolas, or hyperbolas in accordance with the values of μ under the different stress conditions.

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I e 11

ON THE BEARING CAPACITY OF PILES

Prof. Dr. József. JAKY

The bearing capacity of piles consists of two parts: point resistance and skin friction. It is an old problem of Science of Foundations to compute both numerically. The point resistance is essentially the bearing capacity of a shallow pier founded at very great depth, the skin friction is produced by earth pressure acting on the surface of the pile. Several attempts have been made to compute theoretically the bearing capacity of piles 1), 2), but no one is used generally, because they give too low values in view of point resistance, considerably lower than gained by pile load tests.

The aim of this paper is to find a formula giving point resistance and skin friction value in accordance with the reality. It is intended to deal with the bearing capacity of individual piles only.

Sliding surfaces in cohesive soil at instant of failure beneath the base of footings in depth h are - according to theory of Prandtl-Caquot - composed of planes and logarithmic spirals (Fig. 1). If the base lies at depth h, sliding is on both sides hindered by load $p_1 = h\gamma$ and the pressure producing failure 3):

$$p_0 = \left(h\gamma + c\cot\phi\right) tg^2\left(45° + \frac{\phi}{2}\right) e^{\pi tg\phi} - c\cot\phi \quad (1)$$

Point resistance beneath the pile with cross section A:

$$P_1 = A\left[\left(h\gamma + c\cot\phi\right) tg^2\left(45° + \frac{\phi}{2}\right) e^{\pi tg\phi} - c\cot\phi\right] \quad 1a)$$

On the surface of the pile in state of rest earth pressure at rest is acting, its value for cohesive soil:

$$E_0 = \frac{h^2\gamma}{2} k_0 - hk_1 \quad (2)$$

wherein k_0 and k_1 are coefficients of earth pressure at rest 4).

Let the perimeter of the pile be equal to U, the coefficient of skin friction $tg\delta$, the bearing capacity produced by skin friction:

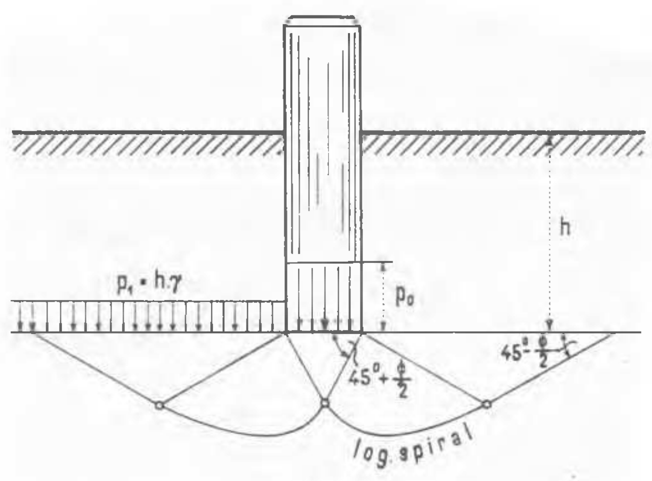

FIG.1

$$P_2 = U tg\delta\left(\frac{h^2\gamma}{2} k_0 - hk_1\right)$$

and the total bearing capacity:

$$P = A\left[\left(h\gamma + c\cot\phi\right) tg^2\left(45° + \frac{\phi}{2}\right) e^{\pi tg\phi} - c\cot\phi\right] + U tg\delta\left(\frac{h^2\gamma}{2} k_0 - hk_1\right) \quad (3)$$

Eq. (3) is plotted on Fig. 2. taking angle of friction $\phi = 35°$ for sand, $\phi = 10°$ for clay, pile lengths up to h = 10 m. In the same figure are plotted results obtained from Dörr's formula:

$$P = A h\gamma tg^2\left(45° + \frac{\phi}{2}\right) + U\frac{h^2\gamma}{2} tg\delta\left(1 + tg^2\phi\right) \quad (4)$$

From this comparison it may be stated that Dörr's formula

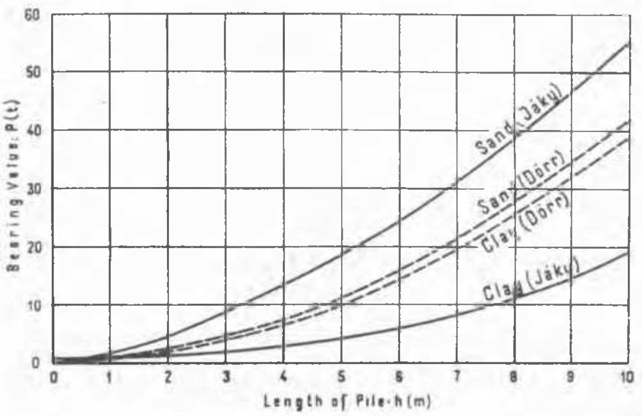

FIG. 2

1) generally gives too low bearing capacities,
2) bearing capacities in sand or clay scarcely differ but this fact contradicts practical observation. The shortcoming of Dörr's formula originates from computing erroneously the point resistance (first term in Eq. (4)) and from determining the pressure of the soil on the skin by means of the theory of the passive earth pressure (second term) altough the smaller earth pressure at rest would have to be considered.

Altough results given by Eq. 3. are closer to reality, they give still lower values than observed by load tests. This fact may be explained as follows:

The point of the pile is on all sides surrounded with soil, therefore, there is no obstacle whatever to the sliding surface being developed right up the surface of the pile as shown on Fig. 3.

This state of stress has been investigated by the author as early as in 1945 - generalizing Prandtl's theory and neglecting the weight of soil : $\gamma = 0$. He named this combined state of stress as "cleft-resistance" and deduced its value to be

$$p_z = c\cot\phi\left[tg^2\left(45°+\frac{\phi}{2}\right)e^{2\pi tg\phi}-1\right] \quad (5)$$

This value exceeds importantly Prandtl's resistance to punching ("Schneidefestigkeit") characterized by Eq. (1) and gives for large angles of internal friction many times as much as Eq.(1). p_z values for different angles of internal friction are shown on Table I.

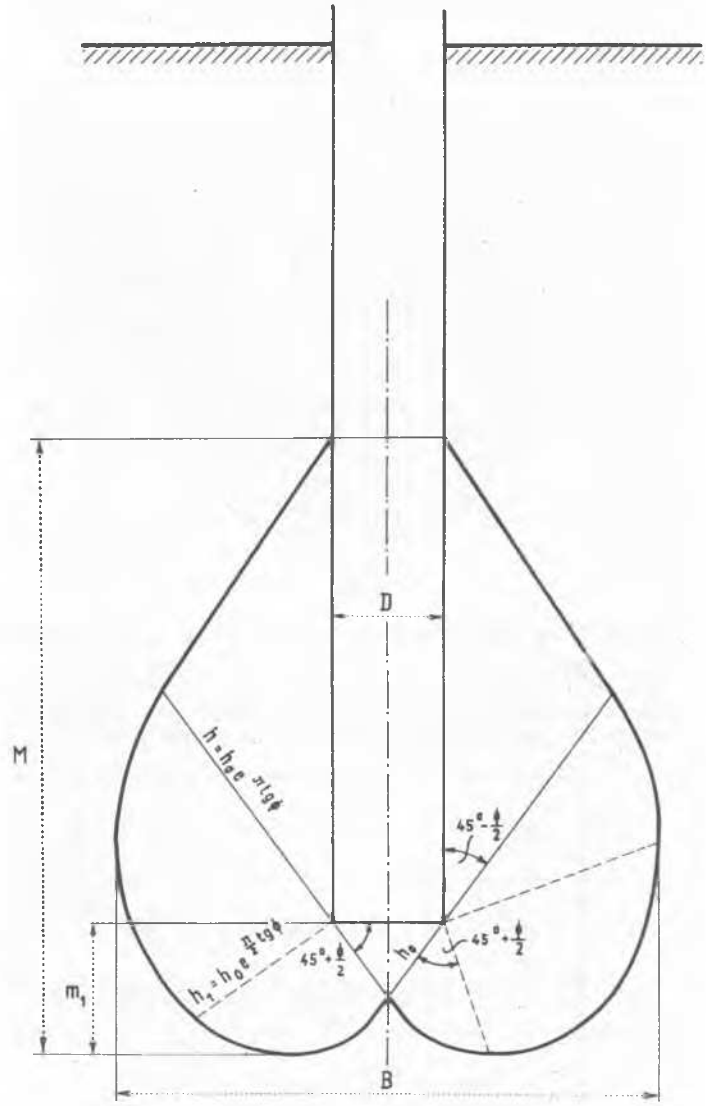

FIG. 3

TABLE I

Angle of internal friction	0°	10°	20°	25°	30°	35°	40°	45°
Coefficient k	0	18.6	52.6	97.0	194.0	423.0	1063	3122

The value of point resistance for cross-section A:

$$P_z = A\,c\cot\phi\left[tg^2\left(45°+\frac{\phi}{2}\right)e^{2\pi tg\phi}-1\right] \quad (5a)$$

The dimensions of the bearing bulb developed around the point of the pile may be easily determined, the most important of them are:

$$m_1 = \frac{D}{2}\,\frac{\cos\phi}{\cos\left(45°+\frac{\phi}{2}\right)}\,e^{\left(\frac{\pi}{4}+\frac{\phi}{2}\right)tg\phi} \quad (6)$$

$$M = D\left[tg\left(45°+\frac{\phi}{2}\right)e^{\pi tg\phi}+\frac{1}{2}\,\frac{\cos\phi}{\cos\left(45°+\frac{\phi}{2}\right)}\,e^{\left(\frac{\pi}{4}+\frac{\phi}{2}\right)tg\phi}\right] \quad (7)$$

$$B = D\left[1+\frac{\cos\phi}{\cos\left(45°+\frac{\phi}{2}\right)}\,e^{\left(\frac{3}{4}\pi+\frac{\phi}{2}\right)tg\phi}\right] \quad (8)$$

The quite different shape of bulbs developed in sand ($\phi = 35°$) and clay ($\phi = 10°$) is shown on Fig. 4. There is a mass of bulb in sand ∽ 25 times as large as in clay and that is roughly the relation between the bearing capacities, too.

The depth to which the pile has to be driven in order to develop a complete bearing bulb is:

$$h_1 \geqq M - m_1 = D\,tg\left(45°+\frac{\phi}{2}\right)e^{\pi tg\phi} \quad (9)$$

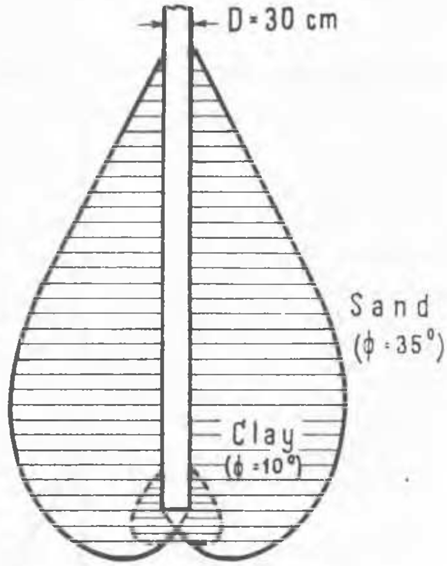

FIG. 4

condition satisfied by short piles too, because, for instance:
$\phi = 35^o$ and D = 30 cm $D\,tg\left(45^\circ + \frac{\phi}{2}\right) e^{\pi tg\phi} = 5.20\text{ m}$
that is the bearing bulb is developed in the case of individual piles by lengths: $h > 5\text{ m}$

which is valid up to the moment when $P_2 > P_1$ i.e.:

$$A\,c\cot\phi\left[tg^2\left(45^\circ + \frac{\phi}{2}\right)e^{2\pi tg\phi} - 1\right] \geq$$

$$\geq A\left[\left(h\gamma + c\cot\phi\right)tg^2\left(45^\circ + \frac{\phi}{2}\right)e^{\pi tg\phi} - c\cot\phi\right]$$

whence the other boundary depth:

$$h_2 \leq \frac{c}{\gamma}\,ctg\phi\left(e^{\pi tg\phi} - 1\right) \qquad (10)$$

For instance: $\phi = 35^o$ and $c = 2\ t/m^2$, $\gamma = 2\ t/m^3$

$h_2 = 11.50$ m.

Result: by pile length between limits characterized by Eq. (9) and (10) the point resistance does not depend on the depth and is a constant given by Eq. (5 a), if $h > h_2$ point resistance increases with increasing depth, its value is given by Eq. (1 a). So piling is economical beyond $h > h_1$ only, on the other hand, it is superfluous to drive piles deeper than $h = h_1$, supposing homogeneous soil strata.

Numerical example.

a) Physical properties of soft clay : $\phi = 10^o$, $c = 3\ t/m^2$, $\gamma = 2\ t/m^3$; diameter of pile: D = 30 cm.

The limit depths: h = 0.62 m, h = 6.30 m.
Point resistance of a pile h = 5.0 m long : $P_2 = 4.0$ t

b) Dense sand soil: $\phi = 35^o$. Dense sand has a certain cohesion. according to test results:

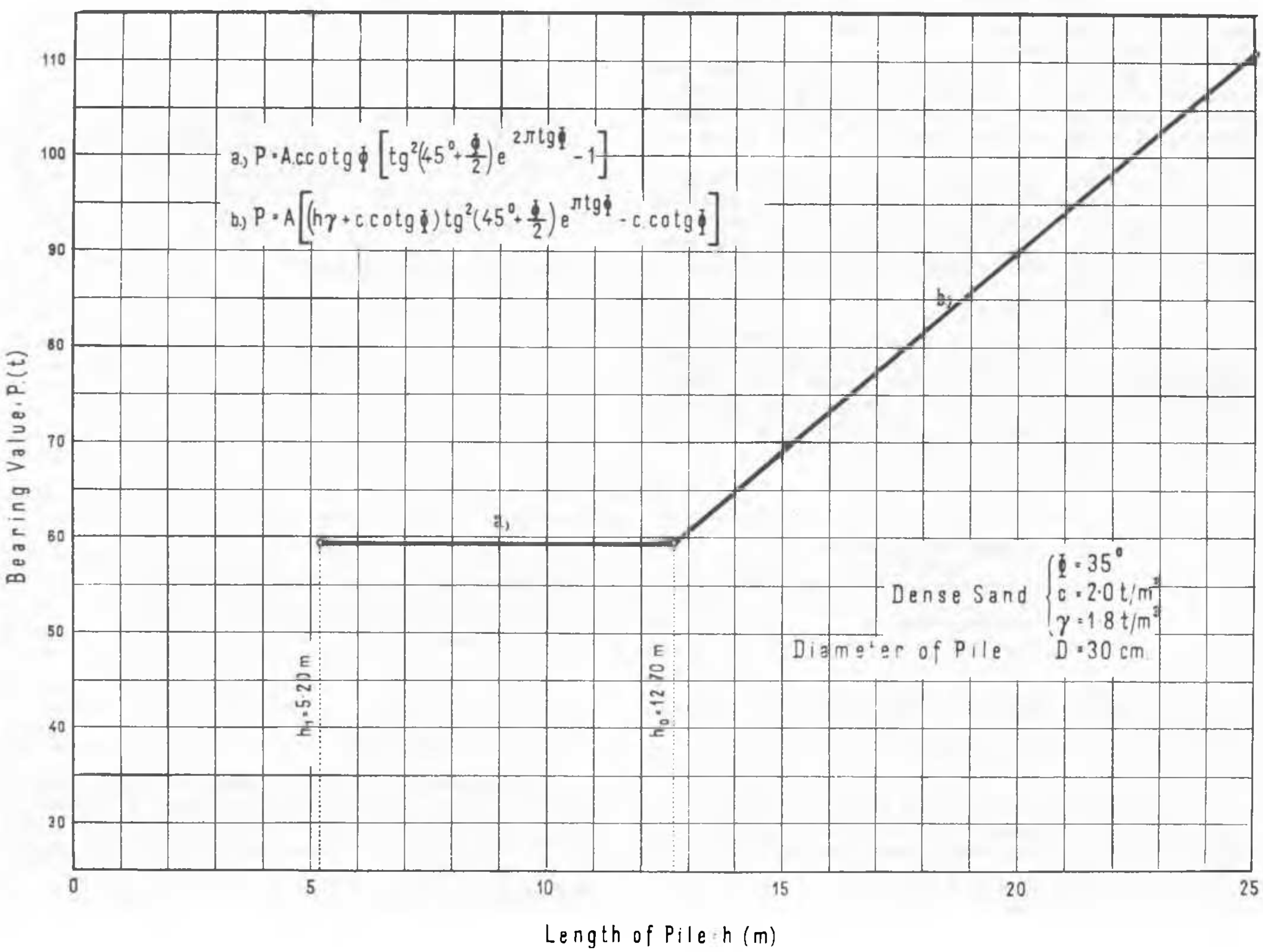

FIG. 5

$c = 0.1 - 0.2$ kg/cm^2
Taking $c = 2$ t/m^2 $\gamma = 1.8$ t/m^3, the depths h_1 and h_2 (D = 30 cm) : $h_1 = 5.20$ m, $h_2 = 12.70$ m. If h = 10.0 m, from Eq. (5 a) : $P_2 = 62$ t.

Taking a safety factor n = 3, the allowable point resistance: $P_1^m \cong 20$ t which remains constant up to $h_2 = 12.70$ m, beyond that the point resistance increases linearly as shown on Fig.5.

The width of the bulb : B = 3.72 m, thus, the distance of the piles must be B = 3.72 m, otherwise the sliding surfaces intersect each other and the surfaces of shear would be insufficient.

In the case of pile group the angle between the surfaces of sliding joined symmetrically is to be examinated and - based on that - the new state of stress revealed. Without dealing with this question, it may be stated that the bearing capacity of the pile placed in a group where the distance between them: $d < B$ will be considerably smaller and the question may be justified whether the piling with small distances be economical.

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SUB-SECTION I f

EARTH PRESSURE

I f 1

PRESSURE IN SILOS

Prof. Dr. J. JAKY

The so-called classical theory 1) to compute pressures exerted by granular material stored in silos is erroneous from the point of view of Statics, because it supposes the angle of friction between the wall and the granular material to be a constant. Since the conditions of equilibrium are expressed for an elementary slice having infinitely small height but finite width, stresses σ_y, σ_x and τ acting in an arbitrary point of the mass and their distribution cannot be determined, from which it follows that the obtained formulae are not in accordance with the dimensions of the silo. The aim of this paper is to give an approximate solution which is correct from point of view of Statics and takes the influence of dimensions -diameter of silo- on pressure correctly into account. The variation of pressures during storing time is not dealt with, nor are pressure problems of emptying and filling treated. 2)

I. BASIC ASSUMPTIONS.

The problem of pressure in silos is essentially that of earth pressure of cohesionless materials, which might be solved by means of the classical earth pressure theory if we could make a plausible assumption on the shape of the surface of sliding. For want of it, the solution is obtained by making use of empirical curves gained by measurements in silos. 3)

Considering a silo of 2b width, bounded by parallel vertical walls it is certain that a wall of AC = y_0 hight represents that limit down to which the classical earth pressure is acting, (Fig. 1) since the surfaces of slid-

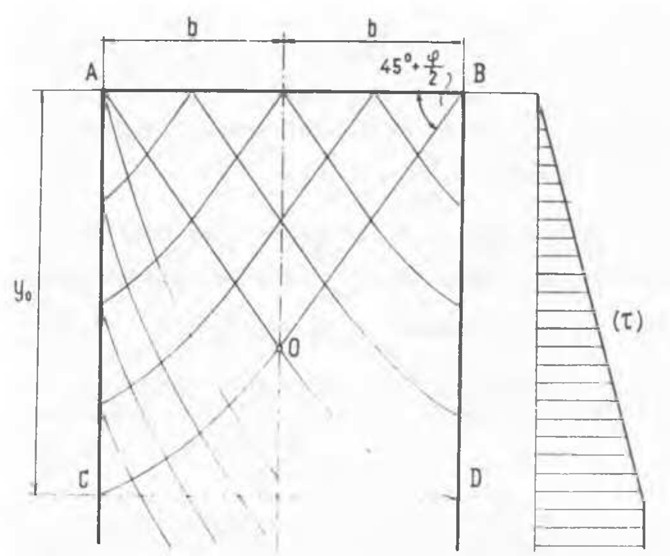

FIG. 1

ing (planes + curves) still run out to superficies AB free from external load, the so-called silo effect is produced solely in depths $y > y_0$.

Assuming approximately the surfaces of sliding joining to AO as logarithmic spirals, the depth y_0 may be computed as