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CALCULATION OF SHEET PILE WALLS IN COHESIVE SOIL

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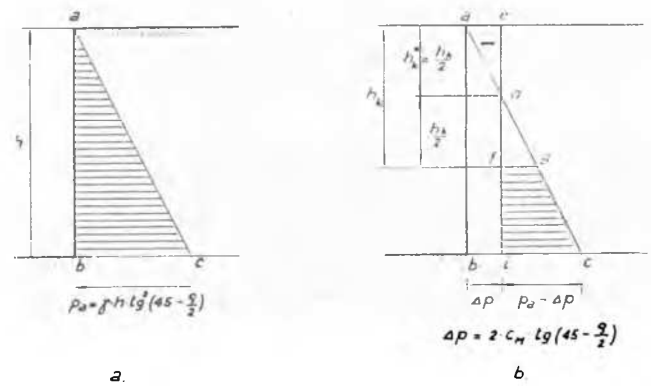
I. INTRODUCTION.

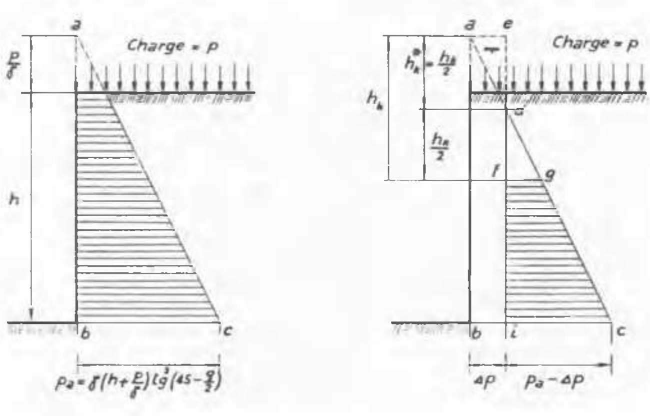
In practice the sheet pile walls are mostly calculated by the method of Krey 1) (linear distribution of stresses) or by the parabolic-method built up by O. Mohr on basis of tests executed by Engels. 2)

For the calculation of sheet pile walls in cohesive soil the cohesion is considered according to Jacoby 3) in such a way that the surface of the earth (fig. 1) for the active



Distribution of the active earth pressure without and with cohesion.

FIG.1 ab



Distribution of the active earth pressure with charge p without and with cohesion.

FIG.1 cd

earth pressure is reduced by $h_k^x = h_k/2$

With the average value of the cohesion

$$C_M = \frac{\gamma h_k}{4}\, tg\left(45^\circ - \frac{\rho}{2}\right) \tag{1}$$

we receive

$$h_k = \frac{4 C_M}{\gamma}\, tg\left(45^\circ + \frac{\rho}{2}\right) \tag{2}$$

Hereby is the meaning of:

- γ : unit weight of the soil
- ρ : angle of internal friction

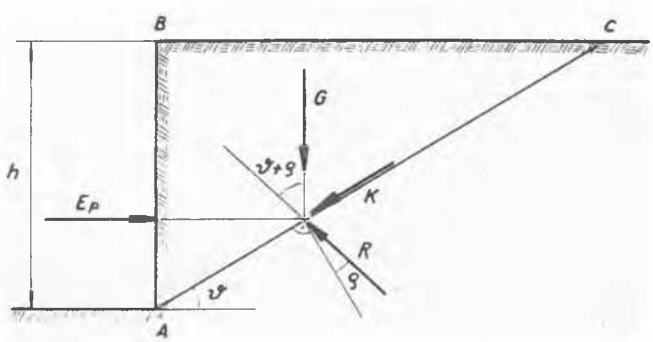
Deducting from the classical triangle of earth pressure of fig. 1 a the earth pressure reduced by the cohesion (rectangle with the width Δp and the height h), whereby

$$\Delta p = 2 C_M\, tg\left(45^\circ - \frac{\rho}{2}\right) \tag{3}$$

we receive fig. 1 b. In the triangle ade of fig. 1 b a negative earth pressure is produced (tension), which can only be taken up as long as there is a possibility of adherence between the soil and the sheet piles. As this is not the case with steel sheet piles the cleft will enlarge itself continually until the entire superior earth pressure will be equal to Zero, i.e. that the soil is standing free to the depth h_k. If we reckon on the basis of the earth pressure triangle dci instead of the trapezoid of earth pressure fgci we have to take into account a bigger active earth pressure. By reducing the surface of the earth by h_k^x according to fig. 1 we obtain therefore an additional security.

For the passive earth pressure Jacoby is increasing the earth surface by the same value h_k^x. This hypothesis seems to be quite convincing at the first look, but it must be pointed out that for bigger angles and exacter calculations it cannot be adopted. 4)

According to fig. 2 and fig. 3 we obtain:



Passive earth pressure E_p on vertical wall with smooth surface. Friction and cohesion.

FIG. 2

$$\Delta E_p = k \frac{\sin(90^\circ + \rho)}{\sin\{90^\circ - (\vartheta + \rho)\}} = k \frac{\cos \rho}{\cos(\vartheta + \rho)}$$

$$= \frac{C_M \cdot h}{\cos(90^\circ - \vartheta)} \cdot \frac{\cos \rho}{\cos(\vartheta + \rho)} \tag{4}$$

$$\Delta p_p = \frac{\Delta E_p}{h} = \frac{C_M}{\sin \vartheta} \cdot \frac{\cos \rho}{\cos(\vartheta + \rho)} \tag{5}$$

The angle for the most dangerous surface of sliding on passive earth pressure is:

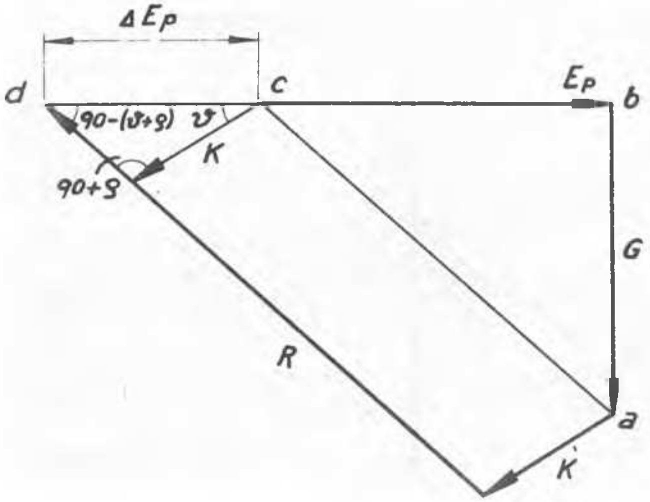
$$\vartheta = 45^\circ - \frac{\rho}{2} \tag{6}$$

Therefore equation (5) is converted into:

$$\Delta p_p = \frac{C_M}{\sin(45^\circ - \rho/2)} \cdot \frac{\cos \rho}{\cos(45^\circ + \rho/2)} \tag{7}$$

and after trigonometrical transformations

$$\Delta p_p = 2 C_M\, tg\left(45^\circ + \frac{\rho}{2}\right) \tag{8}$$



Force polygon $G - K - R - E_p$

cb = E_p (without cohesion)

db = E_p (with cohesion)

FIG. 3

Calculating $p_k = \Delta p_p$ according to Jacoby we get

$$p_k = \gamma h_k^x \cdot \lambda_p = \gamma h_k^x \, tg^2\left(45^\circ + \frac{\rho}{2}\right) \qquad (9)$$

with equation (1) follows:

$$p_k = 2 C_M \cdot tg^3\left(45^\circ + \frac{\rho}{2}\right) = \Delta p_p \, tg^2\left(45^\circ + \frac{\rho}{2}\right) \qquad (10)$$

i.e. the cohesion determined by Jacoby on the earth surface is the effective cohesion multiplied with $tg^2(45^\circ + \rho/2)$

II. LINEAR STRESS DISTRIBUTION.

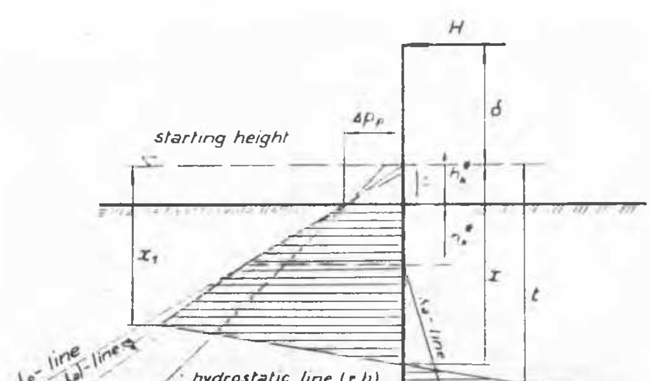
a. Self-supporting sheet pile wall.

The earth resistance on the earth surface is starting according to fig. 4 with the ordinate Δp_p (equation (8)) in front of the sheet piles.

With

$$\Delta p_p = \gamma s(\lambda_p - \lambda_a) \qquad (11)$$

we receive the starting height s above the effective earth surface in front of the sheet



Self supporting sheet pile wall in cohesive soil, charged by a singular force H.

FIG. 4

pile wall

$$s = \frac{2C_M \, tg(45^\circ + \rho/2)}{\gamma(\lambda_p - \lambda_a)} \qquad (12)$$

or, after introduction of the value $h_k^x = h_k/2$ and equation (1)

$$s = \frac{h_k^x}{\lambda_p - \lambda_a} \qquad (13)$$

The values x_1, x and σ of fig. 4 result from the equilibrium conditions to:

$$x_1 = \frac{k(4t + 6\delta) + 2s^2(t-s)}{t^2 - 2k - s^2} \qquad (14)$$

$$x = \frac{k(4t + 6\delta) + 2s^2(t-s)}{2tx_1 - x_1^2 - 2k - s^2} \qquad (15)$$

$$\sigma = \gamma(\lambda_p - \lambda_a)\left[\frac{x_1 x - 2k - s^2}{t - x}\right] \qquad (16)$$

Hereby are:

$$k = \frac{H}{\gamma(\lambda_p - \lambda_a) b}$$

$$\lambda_p = tg^2\left(45^\circ + \frac{\rho}{2}\right)$$

$$\lambda_a = tg^2\left(45^\circ - \frac{\rho}{2}\right)$$

b = width of the sheet pile wall

Δp_p can be found graphically in such a way that we draw through the point of intersection of the sheet pile wall and the earth surface which we consider increased by h_k^x the hydrostatical pressure line γh (pressure line with $\lambda = 1$, i.e. $\rho = 0$), and bring this line with the real earth surface to intersection. According to fig. 4 we receive:

$$\Delta p_p = \gamma h_k^x \qquad (17)$$

as well as: $\Delta p_p = \gamma s(\lambda_p - \lambda_a)$

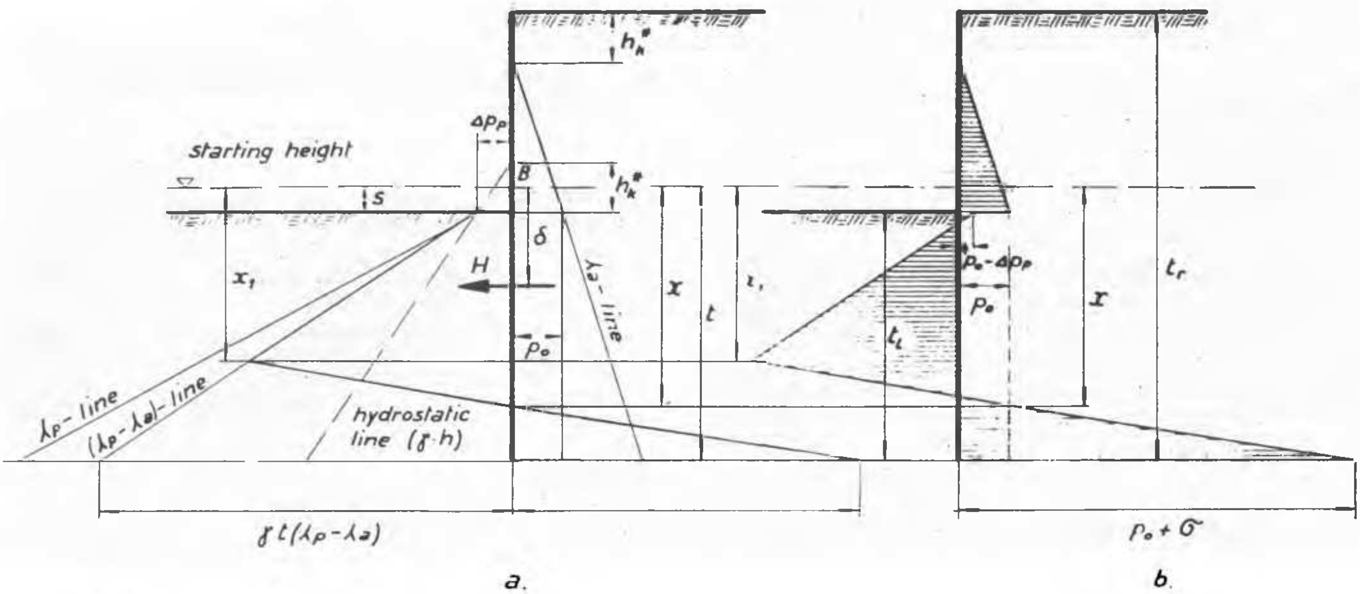
or

$$s = \frac{h_k^x}{\lambda_p - \lambda_a} \quad \text{i.e. equation (13)}$$

The bounds of the pressure figure are starting with the ordinate Δp_p according to equation (8) in the height of the effective earth surface (fig. 4) firstly according to the line of the biggest possible passive earth pressure (λ_p - line) to the depth h_k^x beneath the earth surface. Due to the fact that from this point the active earth pressure is reigning on the right hand side of the wall, the line underneath h_k^x will be steeper, i.e. parallel with the ($\lambda_p - \lambda_a$) - line up to depth x_1 beneath the starting height s .
The further development is the same as for cohesionless soil and is the result of considerations due to equilibrium conditions.

If the sheet pile wall is charged with active earth pressure, the calculation follows according to fig. 5. The starting height s must be introduced according to equation (12).

In that case the above mentioned graphical determination of Δp_p is only valid when c_M at the left and right hand side of the sheet pile wall will be considered as equal.
Of course, c_M beneath the bottom of the foundation trench (left hand side of the sheet pile wall, fig. 5) is often considerably bigger than c_M at the right hand side of the wall (reckoned from the original earth surface). When the average value of the cohesion from the original earth surface to the foot



a Active and passive earth pressure. b Summation of the pressures.

Self supporting sheet pile wall in cohesive soil, charged by earth pressure.

FIG. 5

of the sheet pile wall is introduced, we receive an additional security, because Δp_p is taken into account smaller than in reality.

According to fig. 5 b there must be:

$$\sigma + p_o \leq \gamma t_l \lambda_p + \Delta p_p - \gamma (t_r - h_k^*) \lambda_a \quad (18)$$

b. Simple proped sheet pile wall.

The cohesion is considered in the same way as mentioned above, i.e. for the active earth pressure we decrease the earth surface by h_k^* and introduce the cohesion value Δp_p for the passive earth pressure according to equation (8), or construct Δp_p graphically as shown above.

The sheet pile wall being charged with active earth pressure, we receive according to fig. 6a the minimal ramming depth t_{min} from the condition

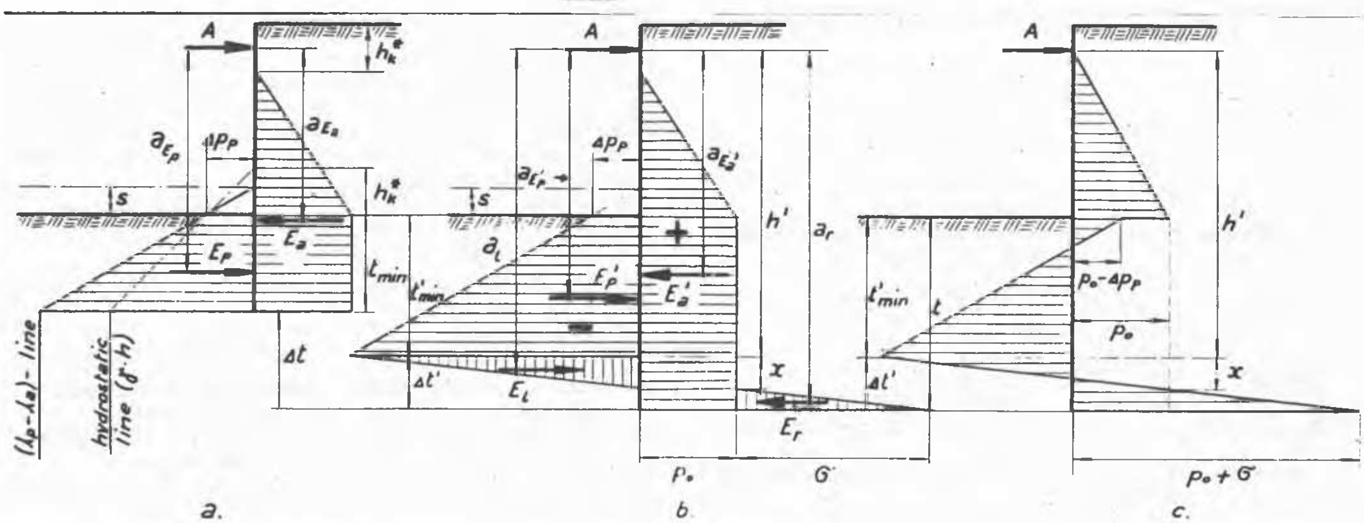
$$E a_{E_a} = E_p a_{E_p} \quad (19)$$

Increasing the ramming depth by Δt, E_a is growing to E'_a and the minimal ramming depth t_{min} up to t'_{min}. Up to this depth the stress distribution is considered as to be increasing in linear line. The position of point zero (fig. 6b) will be found from the relation:

$$E_l a_l = E_r a_r \quad (20)$$

The summation of the pressure figures is shown in fig. 6 c. The condition of safety for a sheet pile wall being rammed down deeper than the supposed minimal ramming depth t_{min} is always fulfilled.

The influence of the cohesion results in the following:



$\Delta t' = \Delta t - (t'_{min} - t_{min})$

FIG. 6

1) The minimal ramming depth will be smaller.
2) Graphically can be proved that, when the sheet pile wall is rammed deeper than t_{min}, the force in the prop will be reduced far more than without consideration of the cohesion. Therefore with cohesive soil the influence of ramming in deeper layers is actually bigger than with cohesionless soil.

III. PARABOLICAL STRESS DISTRIBUTION.

Supposing that the resistance W is increasing linearly with the depth the following equation will be valid according to fig.7:

$$W = W_o + \alpha h = \alpha h_\alpha \qquad (21)$$

The sheet pile wall charged with a linear force H, according to fig. 7 is resisting as

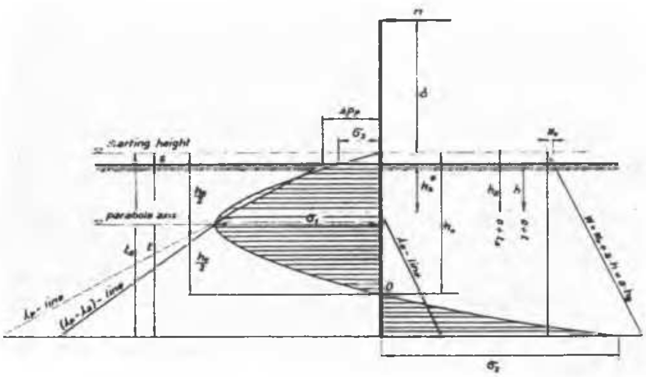


FIG.7

long as the following conditions will be fulfilled:

$$\sigma_1 \leqq \Delta p_p + \gamma h_k^x \lambda_p + \gamma\left(\frac{h_o}{2} - s - h_k^x\right)(\lambda_p - \lambda_a) \qquad (22)$$

$$\sigma_2 \leqq \Delta p_p + \gamma h_k^x \lambda_p + \gamma(t - h_k^x)(\lambda_p - \lambda_a) \qquad (23)$$

$$\sigma_3 \leqq \Delta p_p \qquad (24)$$

In the case of the self-supporting sheet pile wall being charged with active earth pressure, the distribution of pressure is shown in fig. 8. (Exceptions are possible with lower sheet piles and a very high h_k^x). The sheet pile wall is resisting as long as the following conditions are fulfilled:

$$\sigma_1 \leqq \Delta p_p + \gamma\left(\frac{h_o}{2} - s\right)(\lambda_p - \lambda_a) \qquad (25)$$

$$p_o + \sigma_2 \leqq \Delta p_p + \gamma t_l \lambda_p - \gamma(t - h_k^x)\lambda_a \qquad (26)$$

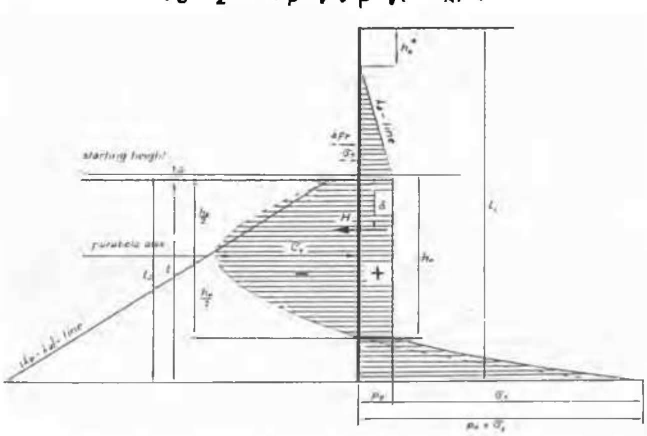


FIG. 8

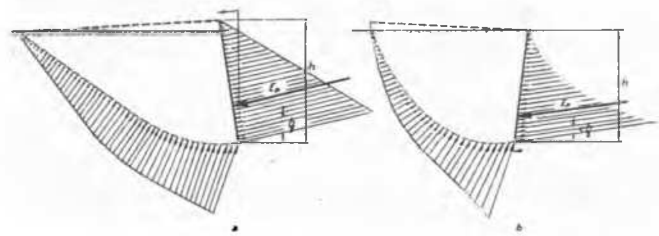
$$\sigma_3 \leqq \Delta p_p \qquad (27)$$

IV. REMARKS TO THE DISTRIBUTION OF EARTH PRESSURE.

As it is known from publications of Kötter 5), Reissner 6), Terzaghi 7), Spilliker 8), Ohde 9), Klenner 10), Lehmann 11), Dörr 12), a.s.o. the distribution of earth pressure is considerably more complicated as is supposed according to the classical theory of earth pressure.

From the publication of Ohde 9) we learn that self-supporting rigid sheet piles, turning round a lower point, can be determined with an earth pressure distributed triangularly over the height of the sheet pile wall. The classical theory of Coulomb, in spite of their simplifying suppositions, is therefore leading to quite satisfactory results. However, in the case of proped sheet piles the distribution of the active earth pressure cannot be supposed to be triangular.

With passive earth pressure the coherence between movements of the sheet piles, surface of sliding and distribution of pressure is similar to that with active earth pressure. The passive distribution of earth pressure for the two most important basic forms is according to Ohde shown in fig. 9.



Passive earth pressure according to Ohde.

FIG. 9

Until there will be available further fundamental and systematical tests, self-supporting sheet piles can be determined by means of the above mentioned equations. For proped sheet piles the calculations must be based on the active earth pressure distribution according to Klenner (rectangular distribution of earth pressure with the same contents of the rectangle than for the classical triangle) The passive distribution of earth pressure can be supposed to be triangular.
For the minimal ramming depth t_{min} the active and passive earth pressure are drawn in fig. 10. For the enlargement of the minimal ramming depth by Δt we draw your attention to the above mentioned remarks.

V . SUMMARY.

The theory built up by Jacoby with regard to the calculation of sheet pile walls in cohesive soil has been corrected. With the new theory it will be possible to take into account the cohesion mathematically or graphically

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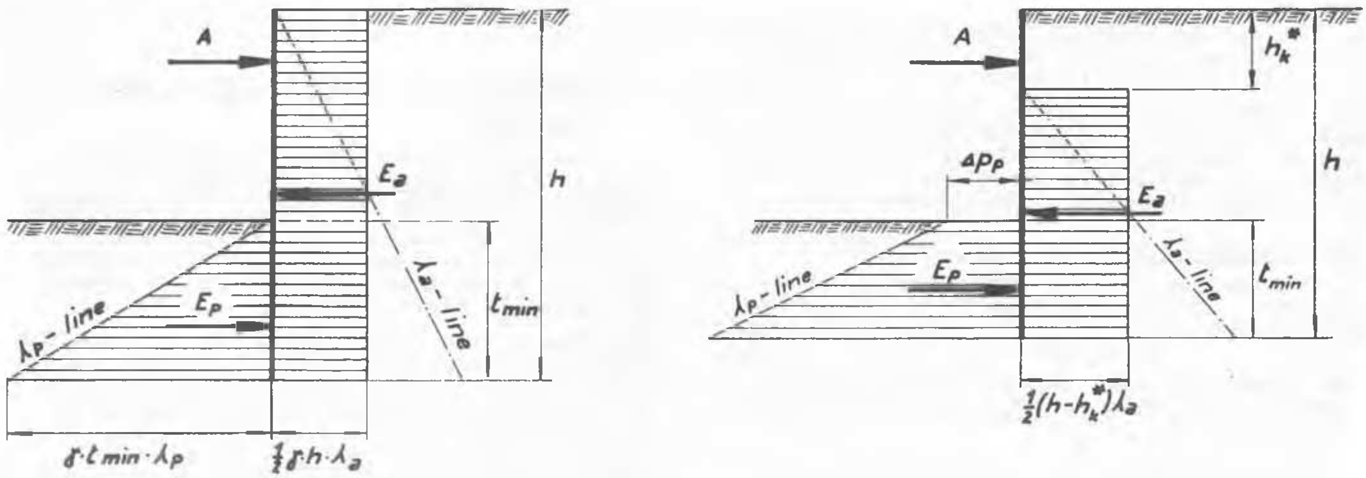

a. Without cohesion

b. with cohesion

FIG.10

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SUB-SECTION I g

STRESS DISTRIBUTION.

I g 1 APPROXIMATIVE CALCULATIONS OF THE STRESS-DISTRIBUTIONS DUE TO CONCENTRATED VERTICAL LOADS

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In the Netherlands, Prof.dr.ir. F.K.Th. van Iterson was the first author , who tried to study this problem by means of approximative calculations 1). He computed the stress distribution, arising in a homogeneous isotropic material, if a smooth semi-sphere is pressed into it (fig. 1). If i denotes the vertical settlement of the sphere, the radial displacement of a point, located on the sphere in the direction θ , will amount to $i.\cos\theta$. Supposing that the radial stress ρ_θ spreads linear in the material, the radial displacement of the point considered will also be proportional to ρ_θ .In consequence of this, the stress distribution can be written,

$$\rho_\theta = \rho_{max} \cos\theta \qquad (1)$$

which is the same as Bousinesq's equation. Further

$$\rho_{max} = \frac{3}{2} \frac{P}{\pi r_o^2} \cos\theta \qquad (2)$$