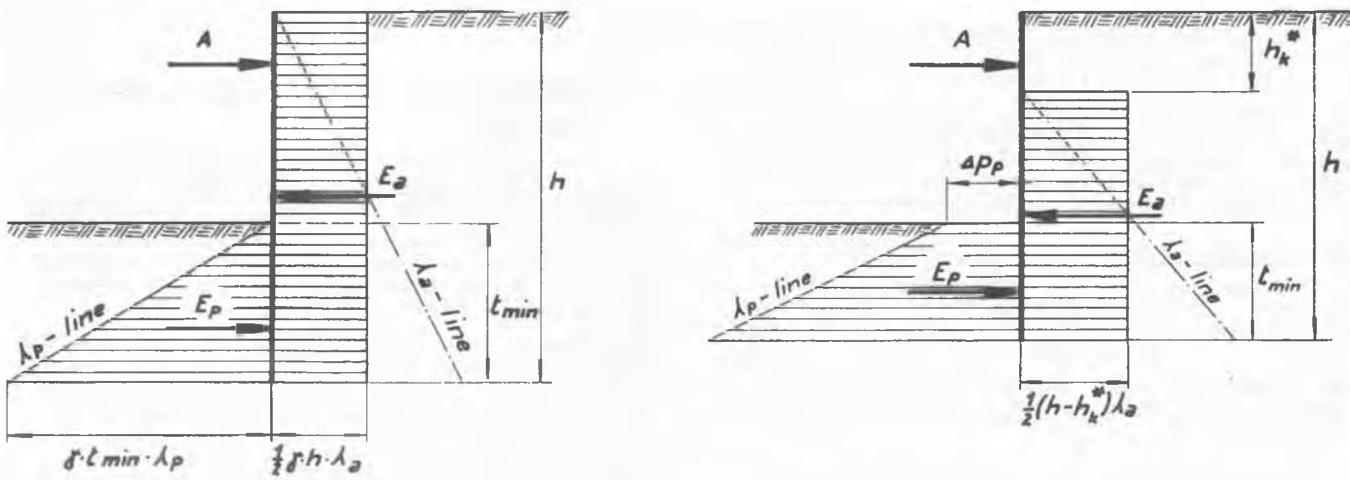

a. Without cohesion

b. with cohesion

FIG.10

2) H. Engels: Zur berechnung der Bohlwerke. Zentralblatt der Bauverwaltung, 1903, Nr. 44, S. 273, Nr. 103, S.649.
3) E. Jacoby: Grundsätzliches über die Berechnung von Spundwänden. Bautechnik, 1941, Nr. 8, S. 88. (Berichtigung S. 256).
4) C.F. Kollbrunner: Fundation und Konsolidation, Band II. Schweizer Druck- und Verlagshaus, Zürich, 1948.
5) F. Kötter: Die Entwicklung der Lehre vom Erddruck. Jahresbericht der deutschen mathem. Ver. 1891-92, S.128.
6) H. Reissner: Theorie des Erddruckes. Enzyklopädie der mathematischen Wissenschaften, Bd. IV 2, II, S.387. Teubner, Leipzig, 1907-1914.
7) K.v. Terzaghi: Distribution of the lateral pressure of sand on the timbering of cuts. Proceedings of the Intern. Conf.on Soil Mechanics and Found. Eng.Harvard University, Cambridge, Mass. Vol. I, p. 211, 1936.
8) A. Spiliker: Mitteilung über die Messung der Kräfte in einer Baugrubenaussteifung. Bautechnik, 1937, Heft 1, S. 16.
9) J. Ohde: Zur Theorie des Erddruckes unter besonderer Berücksichtigung der Erddruckverteilung. Bautechnik, 1938, Heft 10/11, S. 150.
10) C. Klenner: Versuche über die Verteilung des Erddruckes über die Wände ausgesteifter Baugruben. Bautechnik, 1941, Heft 29, S.316.
11) H. Lehmann: Die Verteilung des Erdangriffes an einer oben drehbar gelagerten Wand. Bautechnik, 1942, Heft 31/32, S. 273.
12) H. Dörr: Erddruck auf die Wände ausgesteifter Baugruben. Bautechnik, 1942, Heft 6,S. 54.

-o-o-o-o-o-o-

SUB-SECTION I g

STRESS DISTRIBUTION.

I g 1 APPROXIMATIVE CALCULATIONS OF THE STRESS-DISTRIBUTIONS DUE TO CONCENTRATED VERTICAL LOADS

IR. G.A. OOSTERHOLT
Technical University, Delft, Holland

In the Netherlands, Prof.dr.ir. F.K.Th. van Iterson was the first author, who tried to study this problem by means of approximative calculations 1). He computed the stress distribution, arising in a homogeneous isotropic material, if a smooth semi-sphere is pressed into it (fig. 1). If i denotes the vertical settlement of the sphere, the radial displacement of a point, located on the sphere in the direction θ, will amount to $i.\cos\theta$. Supposing that the radial stress ρ_θ spreads linear in the material, the radial displacement of the point considered will also be proportional to ρ_θ. In consequence of this, the stress distribution can be written,

$$\rho_\theta = \rho_{max} \cos\theta \qquad (1)$$

which is the same as Bousinesq's equation. Further

$$\rho_{max} = \frac{3}{2}\frac{P}{\pi r_0^2}\cos\theta \qquad (2)$$

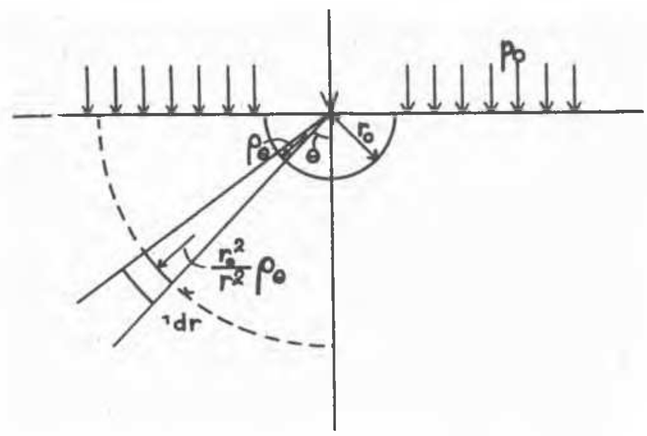

FIG. 1

A few years later on, Prof.ir.A.S. Keverling Buisman studied the same problem, for materials with a modulus of elasticity, which increases in linear proportion to the depth. If γ denotes the unit weight of the soil and C the constant from Terzaghi's law, the modulus of elasticity may be expressed by $\gamma . r . \cos\theta . C$ provided that the increase of the stresses due to the vertical load is very small. The total compression in the direction θ then amounts to

$$\int_{r_o}^{\infty} \rho_\theta \left(\frac{r_o}{r}\right)^2 \frac{dr}{\gamma r \cos\theta} = \frac{\rho_\theta}{2\gamma C \cos\theta}$$

As this is equal again to $i \cos\theta$, he found

$$\rho_\theta = 2.\gamma.C.i.\cos^2\theta \qquad (3)$$

$$\rho_{max} = 2 \cdot \frac{P}{\pi r_o^2} \qquad (4)$$

Comparing these new formulae with (1) and (2), prof. Buisman stated, that the latter might apply to some special cases, for instance if the capillary tensions are so high, that the increase of the effective normal stresses due to the weight of the soil, as well as to the load, may be neglected. On the contrary (3) and (4) should apply to a mass of soil without any capillary tensions, provided that the loads are very small.

The author has now tried to compute the stress distribution, if the loads are greater. The same method of calculation has been followed, but the increase of the modulus of elasticity when the sphere is pressed into the earth, has been taken into account. The computations are based again on the assumption, that the stress trajectories are straight lines through the centre of the sphere 3). The stress ρ_θ, acting in the direction θ (fig. 1) at the contact surface between the sphere and the soil, then expands in such a manner, that the stress amounts to $\frac{r_o^2}{r^2}\rho_\theta$ at a distance r from the centre of the sphere. There the original vertical stress was $p_o + \gamma r \cos\theta$ if p_o denotes the load per unit of area and γ the unit weight of the soil. Assuming, that Terzaghi's law may be applied at an inclined direction, the compression of the element plotted in fig. 1 amounts to

$$\frac{dr}{C} \cdot \ln \frac{p_o + \gamma r \cos\theta + \rho_\theta r_o^2/r^2}{p + \gamma r \cos\theta}$$

and the total compression of all elements in the same direction can be written

$$\int_{r_o}^{\infty} \frac{dr}{C} \cdot \ln\left\{1 + \frac{\rho_\theta r_o^2}{r^2(p_o + \gamma r \cos\theta)}\right\}$$

If i denotes again the settlement of the sphere, the intregal will be equal to $i \cos\theta$. Thus the general equation for the problem is found to be

$$C i \cos\theta = \int_{r_o}^{\infty} \ln\left\{1 + \rho_\theta \frac{r_o^2}{r^2(p_o + \gamma r \cos\theta)}\right\} dr \qquad (5)$$

The integration will be worked out for two different cases. In the first place p_o is assumed to be zero, which is true for a load at the surface of the earth. If we substitute

$$a^3 = \frac{r_o^2 \rho_\theta}{\gamma \cos\theta}$$

the integral can be reduced to

$$C.i.\cos\theta = r \ln\left(1 + \frac{a^3}{r^3}\right)\Big|_{r_o}^{\infty} + \int_{r_o}^{\infty} \frac{3a^3 dr}{r^3 + a^3}$$

By splitting into fractions we then obtain

$$C.i.\cos\theta = r \ln\left(1 + \frac{a^3}{r^3}\right) - \frac{a}{2} \ln \frac{r^2 - ar + a^2}{(r+a)^2} + a\sqrt{3}\, \mathrm{bg\,cotg} \frac{-2r + a}{a\sqrt{3}} \Big|_{r_o}^{\infty}$$

Substituting $\frac{r_o}{a} = \psi$ the formula can be written

$$\frac{C i}{r_o} \cos\theta = \frac{\pi\sqrt{3}}{\psi} - \ln\left(1 + \frac{1}{\psi^3}\right) + \frac{1}{2\psi} \ln \frac{\psi^2 - \psi + 1}{(\psi + 1)^2} - \frac{\sqrt{3}}{\psi}\, \mathrm{bg\,cotg} \frac{1 - 2\psi}{\sqrt{3}} = f(\psi) \qquad (6)$$

while $$\psi^3 = \frac{r_o \gamma \cos\theta}{\rho_\theta} \qquad (7)$$

The function $f(\psi)$ has been plotted in fig.2. If r_o, γ, and C are known and a value of i has been chosen, the relation between θ and ρ_θ can be computed.

For point loads P the formula becomes more simple. Multiplying (6) with ψ

$$\frac{C i \psi}{r_o} \cdot \cos\theta = \psi f(\psi)$$

and substituting $\psi \to 0$ we find:

$$\frac{C i \gamma^{1/3}}{\rho_\theta^{1/3} r_o^{2/3}} \cos^{4/3}\theta = \lim_{\psi = 0} \psi f(\psi) = \frac{2\pi}{\sqrt{3}}$$

Writing in the well known way

$$P = \int \rho_\theta \cos\theta \, dF$$

we find the relation between i and ρ_θ and P for point loads at the surface of the earth:

$$i = \frac{3.57}{C} \sqrt[3]{\frac{P}{\gamma}} \qquad (8)$$

$$\rho_\theta = \frac{3P}{\pi r^2} \cos^4\theta \qquad (9)$$

In the second place we assume p_o to be much greater than $\gamma r \cos\theta$ in the vicinity of the sphere; generally this will approximately be true for deep footings, like pilepoints. Equation (5) then is reduced to:

$$C.i.\cos\theta = \int_{r_o}^{\infty} \left(1 + \frac{\rho_\theta \cdot r_o^2}{r^2 p_o}\right) dr$$

In an analogous way as before, we can write the solution as follows:

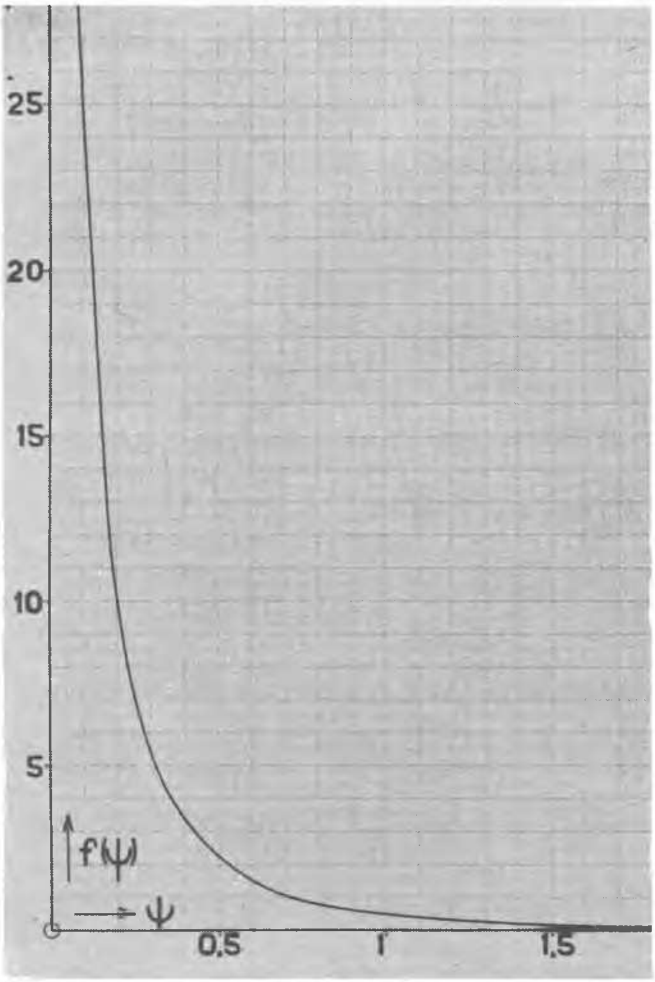

FIG. 2

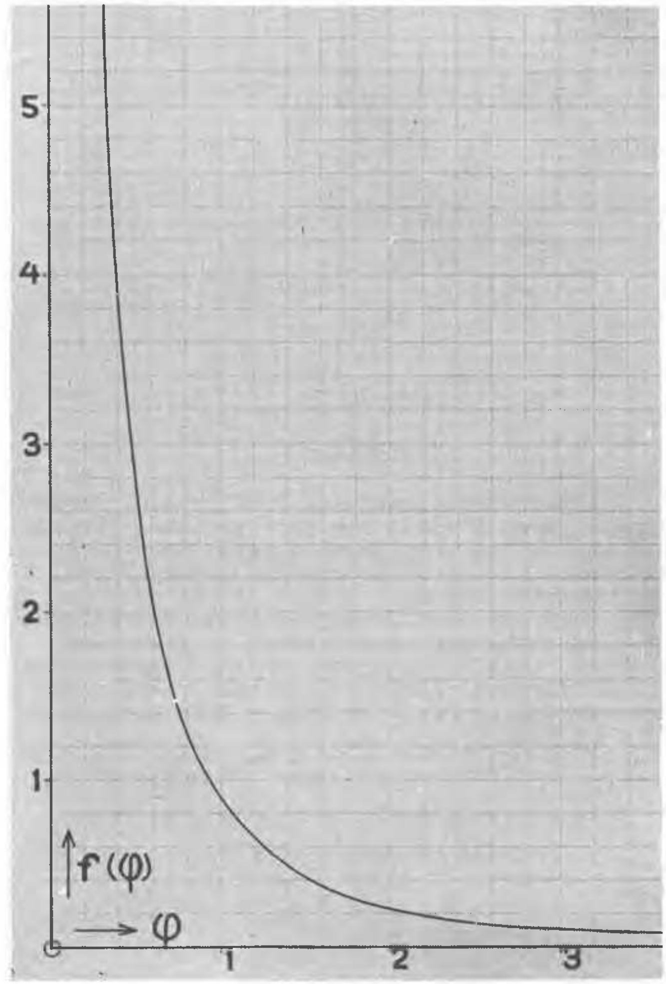

FIG. 3

$$\frac{Ci}{r_o}\cos\theta = \frac{\pi}{\varphi} - \ln\left(1+\frac{1}{\varphi^2}\right) - \frac{2}{\varphi}\,\mathrm{bgtg}\,\varphi = f(\varphi) \qquad (10)$$

when

$$\varphi = \sqrt{\frac{p_o}{\rho_\theta}} \qquad (11)$$

The function $f(\varphi)$ has been drawn in fig. 3. For point loads we now can derive the following equatuons:

$$\rho_\theta = \frac{2P}{\pi r^2}\cos^2\theta \qquad (12)$$

and

$$i = \frac{1}{C}\sqrt{\frac{2\pi P}{p_o}} \qquad (13)$$

If the new formulae for point loads are compared with those of the mentioned authors, it is striking, that the concentration factors are higher. For a constant modulus of elasticity (Boussinesq, Van Iterson) the concentration factor amounts to 3/2 ; in the corresponding case of a soil with a constant vertical stress p_o (form. 12) we now found 2. Calculating some stress distributions by the aid of (10) and (11), it turned out, that the concentration factor decreases with increasing values of φ and for $\varphi \rightarrow \infty$ the formula becomes identical with Boussinesq's equation $\varphi \rightarrow \infty$ means that $\rho_\theta \rightarrow 0$, so $E = C.p_o$ = a constant).

When the resistance against compression increases with the depth, the difference is yet more striking; Prof. Buisman found

$$\rho_\theta = \frac{2P}{\pi r^2}\cos^2\theta$$

while for point loads now was found

$$\rho_\theta = \frac{3P}{\pi r^2}\cos^4\theta$$

Calculating ρ_θ according to (6) and (7) it turns out, that the concentration factor decreases with increasing values of ψ and for $\psi \rightarrow \infty$ the formula is identical again with prof. Buisman's formula $\psi \rightarrow \infty$ corresponds also with

$\rho_\theta \rightarrow 0$ so $\quad E = C\gamma r\cos\theta$

It was well known from several experiments that the concentration factors could be considerably greater than indicated by Boussinesq's formula; different causes have been sought to explain this (Fröhlich, Buisman) . Results of all tests of loads at the surface of soil, made by several investigators, were controlled by Fröhlich 4); he found semi-empirically, that a formula, identical with (6), was in the best harmony with these tests. Obviously the main cause of the great stress concentration therefore seems to be the fact, that the modulus of elasticity increases, when the load is brought upon the soil; other factors however, as mentioned by Buisman and Fröhlich, will also have influence.

The author is well conscious, that the calculations are a simplified scheme of reality. As long as the laws, governing the deformations of soils, are not known better however, it is necessary to seek the best approximation of the problem. The object of the above, therefore, was only to complete a train of thought, that was ended half-way and to obtain by this a better harmony with reality.

REFERENCES.

1) De Ingenieur 1928 No.38/39.

2) De Ingenieur 1932 No.37; Grondmechanica p. 180.

3) Fröhlich, O.K. Druckverteilung im Baugrunde. 1934 9.

4) Fröhlich, op cit., p. 128.

-o-o-o-o-o-o-

Ig 2

ON THE DEPTH OF FOUNDATION

F. SZELĄGOWSKI
Polytechnical School, Warsaw

Among many conditions required of a new building the most important one is its stability against loading.

Stability of a building will not change, if the position of its foundation does not change.

It follows that the construction of a foundation should prove stable against any rotary movement, against any horizontal slipping, and finally against differential (varying) settlement, which might occur due to the upward pressure of the soil underneath the foundation.

The object of these lines is to set forth such a theory for depth of foundations which will make impossible the above mentioned movements.

In this connection it is necessary to say that an upward movement of the soil from underneath the foundation would be prevented when the pressure on the foundation bed of earth caused by the load of the foundation does not change the state of the extreme equilibrium of the soil determined by the equations:

$$t_{nt} = t_{nn}\,tg\,\varphi \qquad (1)$$

in the case of loose earths, and

$$t_{nt} = t_{nn}\,tg\,\varphi + C \qquad (2)$$

in the case of firm cohesive soil.

The first one is known as Rankine's formula of earth pressure, whereas the second one as Coulomb's formula.

Both of them when expressed in functions of stresses N_1, N_2 and T have the following form:

$$\sqrt{(N_1-N_2)^2+4T^2}-(N_1+N_2)\sin\varphi=0 \qquad (3)$$

in the case of loose earths, and

$$\sqrt{(N_1-N_2)^2+4T^2}-(N_1+N_2)\sin\varphi=2C\cos\varphi \qquad (4)$$

in the case of firm cohesive soil, where φ means the angle of friction, whereas C means the force of adhesion per suqare unit.

Rankine and Pauker were the first to initiate the study of the subject.

Rankine in the basis of the theory of equilibrium of cohesionless soil, and Pauker on the basis of equilibrium of retaining walls have both come independently from one another to the same results. In fact Rankine determined the smallest depth of a foundation by the formula:

$$h=H\left(\frac{1-\sin\varphi}{1+\sin\varphi}\right)^2$$

whereas Pauker: $h=H\,tg^4\left(45°-\frac{\varphi}{2}\right)$

where H means the height of a column of earth exerting pressure on the base corresponding to the pressure of the extreme equilibrium of loose earth.

It is known that

$$\frac{1-\sin\varphi}{1+\sin\varphi}=\frac{tg(45°-\varphi/2)}{tg(45°+\varphi/2)}=\frac{tg(45°-\varphi/2)}{ctg(45°-\varphi/2)}=tg^2\left(45°-\frac{\varphi}{2}\right)$$

hence

$$h=H\left(\frac{1-\sin\varphi}{1+\sin\varphi}\right)^2=H\,tg^4\left(45°-\frac{\varphi}{2}\right) \qquad (5)$$

where φ in both these formulae means the same angle of friction, commonly known as the angle of natural sliding of the soil.

Neither Rankine's nor Pauker's deductions which served as the starting point to formula (5) are correct from the theoretical point of view as both these authors assume in the vertical plane BC (fig. 1) a sudden turn at

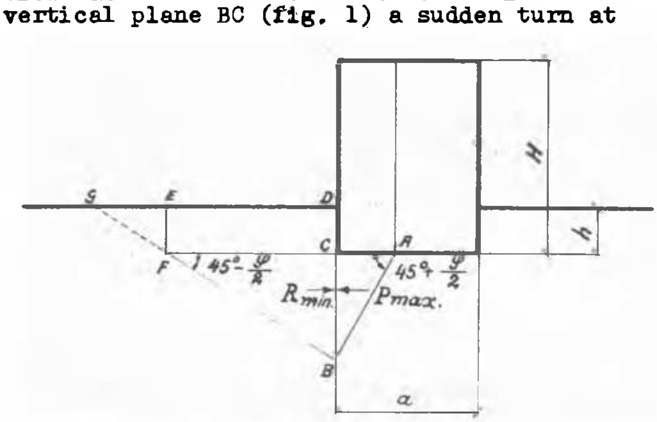

FIG. 1

an equal angle in the Lamé's ellipse of stresses. They assume as well the surfaces of sliding being planes which from an angle of $45° + \frac{1}{2}\varphi$ towards the level change their position to $45° - \frac{1}{2}\varphi$ in a non continuous way. Moreover the stresses of friction have not been taken into account.

In spite of these drawbacks formula (5) due to its simple form grew very popular in the technical literature and, as it always happens, all the drawbacks have been compensated by a factor of safety, the value of which has been lately reduced from 1,75 to 1,5. In