that was ended half-way and to obtain by this a better harmony with reality.

REFERENCES.

1) De Ingenieur 1928 No.38/39.

2) De Ingenieur 1932 No.37; Grondmechanica p. 180.

3) Fröhlich, O.K. Druckverteilung im Baugrunde. 1934 9.

4) Fröhlich, op cit., p. 128.

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Ig 2

ON THE DEPTH OF FOUNDATION

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Among many conditions required of a new building the most important one is its stability against loading.

Stability of a building will not change, if the position of its foundation does not change.

It follows that the construction of a foundation should prove stable against any rotary movement, against any horizontal slipping, and finally against differential (varying) settlement, which might occur due to the upward pressure of the soil underneath the foundation.

The object of these lines is to set forth such a theory for depth of foundations which will make impossible the above mentioned movements.

In this connection it is necessary to say that an upward movement of the soil from underneath the foundation would be prevented when the pressure on the foundation bed of earth caused by the load of the foundation does not change the state of the extreme equilibrium of the soil determined by the equations:

$$t_{nt} = t_{nn} \operatorname{tg} \varphi \qquad (1)$$

in the case of loose earths, and

$$t_{nt} = t_{nn} \operatorname{tg} \varphi + C \qquad (2)$$

in the case of firm cohesive soil.

The first one is known as Rankine's formula of earth pressure, whereas the second one as Coulomb's formula.

Both of them when expressed in functions of stresses N_1, N_2 and T have the following form:

$$\sqrt{(N_1-N_2)^2+4T^2} - (N_1+N_2)\sin\varphi = 0 \qquad (3)$$

in the case of loose earths, and

$$\sqrt{(N_1-N_2)^2+4T^2} - (N_1+N_2)\sin\varphi = 2C\cos\varphi \qquad (4)$$

in the case of firm cohesive soil, where φ means the angle of friction, whereas C means the force of adhesion per suqare unit.

Rankine and Pauker were the first to initiate the study of the subject.

Rankine in the basis of the theory of equilibrium of cohesionless soil, and Pauker on the basis of equilibrium of retaining walls have both come independently from one another to the same results. In fact Rankine determined the smallest depth of a foundation by the formula:

$$h = H\left(\frac{1-\sin\varphi}{1+\sin\varphi}\right)^2$$

whereas Pauker: $h = H \operatorname{tg}^4\left(45° - \frac{\varphi}{2}\right)$

where H means the height of a column of earth exerting pressure on the base corresponding to the pressure of the extreme equilibrium of loose earth.

It is known that

$$\frac{1-\sin\varphi}{1+\sin\varphi} = \frac{\operatorname{tg}(45°-\varphi/2)}{\operatorname{tg}(45°+\varphi/2)} = \frac{\operatorname{tg}(45°-\varphi/2)}{\operatorname{ctg}(45°-\varphi/2)} = \operatorname{tg}^2\left(45°-\frac{\varphi}{2}\right)$$

hence

$$h = H\left(\frac{1-\sin\varphi}{1+\sin\varphi}\right)^2 = H\operatorname{tg}^4\left(45°-\frac{\varphi}{2}\right) \qquad (5)$$

where φ in both these formulae means the same angle of friction, commonly known as the angle of natural sliding of the soil.

Neither Rankine's nor Pauker's deductions which served as the starting point to formula (5) are correct from the theoretical point of view as both these authors assume in the vertical plane BC (fig. 1) a sudden turn at

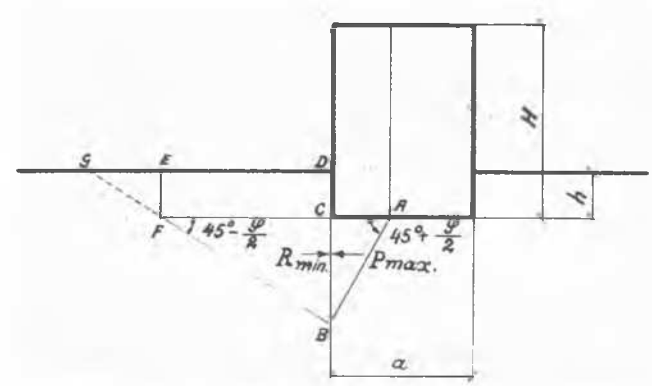

FIG. 1

an equal angle in the Lamé's ellipse of stresses. They assume as well the surfaces of sliding being planes which from an angle of $45° + \frac{1}{2}\varphi$ towards the level change their position to $45° - \frac{1}{2}\varphi$ in a non continuous way. Moreover the stresses of friction have not been taken into account.

In spite of these drawbacks formula (5) due to its simple form grew very popular in the technical literature and, as it always happens, all the drawbacks have been compensated by a factor of safety, the value of which has been lately reduced from 1,75 to 1,5. In order to improve the matter Jankowski derived from the above the following formula:

$$h=\frac{H}{2}\left(\frac{tg\frac{45^\circ-\varphi}{2}}{tg\frac{45^\circ+\varphi}{2}}\right)^2 \qquad (6)$$

Although the results of this formula were confirmed by the experiments of Kurdiumow, nevertheless the assumption of the surfaces of sliding of the upward pushed soil as planes still did not correspond to the real facts. From the experiments of Kurdiumow it was evident that the movement of the particles of the upward pushed soil procedes on continuous curves beginning right at the bottom of the foundation (fig. 2).

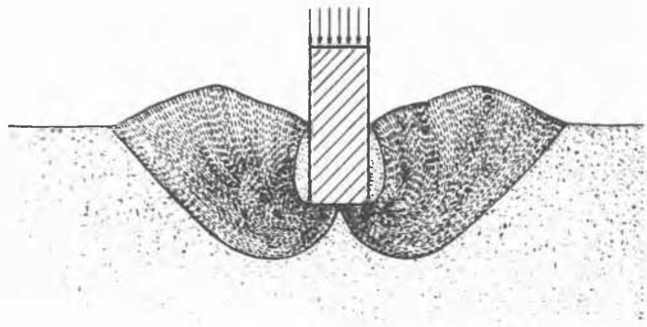

FIG. 2

Betzecki's formula 1) has a similar drawback. He has determined the necessary depth of a foundation (for loose earth) by the following formula:

$$h=H tg^4\left(45^\circ-\frac{\varphi}{2}\right)-\frac{a}{2tg(45^\circ-\varphi/2)}\left[1-tg^4\left(45^\circ-\frac{\varphi}{2}\right)\right] \qquad (7)$$

where a means half the width of a foundation (fig. 3).

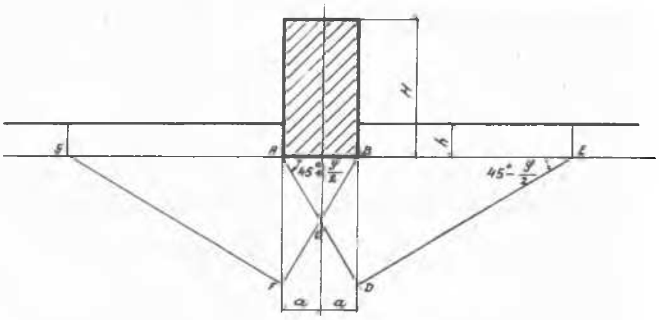

FIG. 3

In connection with the above it must be mentioned that in his reasoning Betzecki assumed the possibility of a formation of a wedge causing the upward push of clods of soil with quadrangularlike frofiles BCDE and ACFG. Apart from this it should be noted that Betzecki when giving his formula (7) has taken in account only the state of loading on the surfaces of sliding AC and BC, appearing in the extreme equilibrium of soil.

However, in the above reasoning the possibility of there being a wedge ABC of $90^\circ-\varphi$ at the top, which should force its way into the soil together with the foundation according to the experimental observations of Kurdiumow, has not been taken into account at all. The sliding of the soil, as already mentioned, takes place right at the middle of the foundation footing while the soil is being pushed off both sides.

The first attempt of approaching the real state of things was done by Caquot 2) who has given an equation in the following form:

$$h=\frac{H}{tg^2(45^\circ+\varphi/2)e^{\pi tg\varphi}} \qquad (8)$$

in the case of loose earths, and

$$h=\frac{H-\frac{C}{\gamma tg\varphi}\left[tg^2\left(45^\circ+\frac{\varphi}{2}\right)e^{\pi tg\varphi}-1\right]}{tg^2\left(45^\circ+\frac{\varphi}{2}\right)e^{\pi tg\varphi}} \qquad (9)$$

in the case of firm cohesive soil, where γ means the specific weight of earth.

Caquot in his reasoning in accordance with the previous work of Boussinesq and Résal considered a continuous joining of the surfaces of sliding inclined at an angle of $45^\circ+\frac{1}{2}\varphi$ towards the level with the surfaces of sliding inclined at an angle of $45^\circ-\frac{1}{2}\varphi$ by means of a logarithmical spiral (fig. 4).

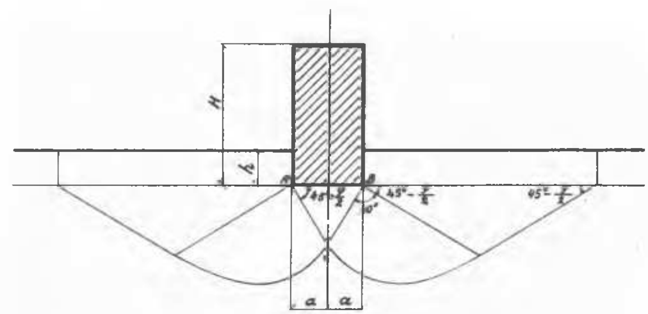

FIG. 4

In connection with the above assumption there arises a question whether the surfaces of sliding of the pushed off soil will have taken that configuration at all.

The answer to this question, judging by the results of experiments, is a negative one, as the experimental observations show that the surfaces of sliding change their configuration with the depth. Moreover the wedge of the top angle $90^\circ-\varphi$, which should force its way into the soil does not exist either.

Finally the last one who tried to determine the necessary depth of a foundation was Ritter whose formula based on the pushing off soil particles from underneath the foundation was given in the following form: 3)

$$h=\frac{2H-a\left[tg^5(45^\circ+\varphi/2)-tg(45^\circ+\varphi/2)\right]}{2tg^4(45^\circ+\varphi/2)} \qquad (10)$$

in the case of loose earth, and

$$h=\frac{2H-a\left[tg^5\left(45^\circ+\frac{\varphi}{2}\right)-tg\left(45^\circ+\frac{\varphi}{2}\right)\right]-\frac{2C}{\gamma tg\varphi}\left[tg^4\left(45^\circ+\frac{\varphi}{2}\right)-1\right]}{2tg^4\left(45^\circ+\frac{\varphi}{2}\right)} \qquad (11)$$

in the case of firm cohesive soil.

In his considerations Ritter assumed, as Betzecki and Caquot did before, the possibility of a formation of a wedge (fig. 5) causing the pushing off clods of soil having certain curved surfaces the shape of which was not exactly determined by him. This was due to the fact to the deduction of formulae (10) and (11). Ritter's starting point was

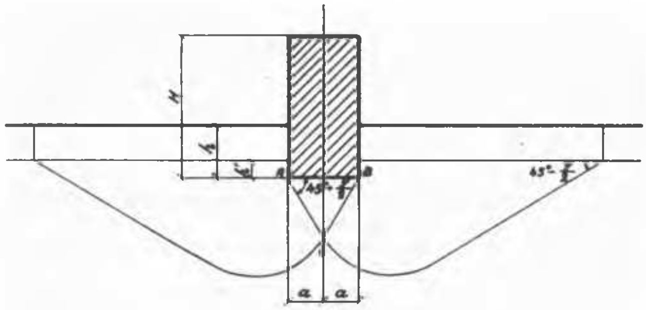

FIG. 5

the relation determining the values of principal stresses in the form independant of the sliding surfaces of the pushed off soil.

Analysing the above formulas (7) and (10), and taking into consideration the existance of the below quoted connection:

$$tg\left(45^\circ+\frac{\varphi}{2}\right)=\frac{1}{tg(45^\circ-\varphi/2)}$$

it must be stated that Ritter's formula (10) is identical with that of Betzecki (7). So here again both these authors, Betzecki and Ritter, starting from different points independently from one another arrive, as it can be seen, to identical formulas.

The above quoted formules, as it can be observed, are to a certain extent, based on assumptions incompatible with experiments which did clearly explain a complex process of the phenomenon of the thrust of soil from underneath the foundation.

Moreover on the basis of these experiments it was possible to come to a conclusion that just before the formation of the surfaces of sliding, i.e. before the thrust of soil from underneath the foundation there took place a strong condensation of the soil beneath the foundation. It could also be deduced that the parts of soil directly adhering to the side surfaces of the foundation did not come into the area of sliding, i.e. they were not included in the bulk of clods of soil pushed off (fig. 2).

Seeing the non defined configuration of the surfaces of sliding of the soil pushed off and the sheer impossibility of basing thereupon any exact calculation of the necessary depth of a foundation, it will be to the point to make the above mentioned on the basis of the pertaining results of the elasticity theory taking simultaneously into account Rankine's or Coulomb's condition.

This way was adopted by Pigeaud, 4) who using Boussinesq's definition of the stress distribution under the influence of intense pressure, did obtain an equation the whole of the question, however, without final deduction of respecting formulas concerning the calculation of the necessary depth of a foundation in the case of loose or firm earths.

That's why in this article having reference to Michell's formulas defining the stresses in continuous sphere under uniform load spread over a sector 2a long, the respective formulas are arrived at in quite a simple way.

Michell's formulas for the load shown on fig. 6 have the following form:

$$N_1=\frac{p}{\pi}\left(arctg\frac{a-x}{y}+arctg\frac{a+x}{y}\right)+\frac{2pya(x^2-y^2-a^2)}{\left[(y^2+x^2-a^2)^2+4a^2y^2\right]\pi}$$

$$N_2=\frac{p}{\pi}\left(arctg\frac{a-x}{y}+arctg\frac{a+x}{y}\right)-\frac{2pya(x^2-y^2-a^2)}{\left[(y^2+x^2-a^2)^2+4a^2y^2\right]\pi}$$

$$T=\frac{4ay^2xp}{\pi\left[(y^2+x^2-a^2)^2+4a^2y^2\right]}$$

Taking into account the above values in formulas

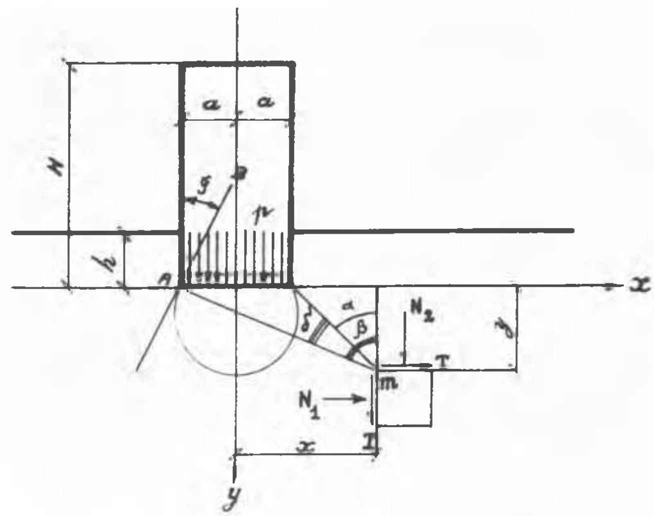

FIG. 6

(3) and (4) and then to simplify the calculation introducing the angles α, β, and $\delta=\beta-\alpha$, we shall obtain as the result the Rankine's and Coulomb's conditions in very simple forms:

$$\frac{2p}{\pi}\left(\sin\delta-\delta\sin\varphi\right)=0 \qquad (12)$$

$$\frac{p}{\pi\cos\varphi}\left(\sin\delta-\delta\sin\varphi\right)=C \qquad (13)$$

On the other hand it must be noted that Rankine's and Coulomb's conditions represent the maximum shear stresses.

Thus the extreme value of angle δ can be determined from the equation which will result of differentiating of equation (12) and (13). Thus when differentiating we arrive at:

$$\cos\delta-\sin\varphi=0$$

The value of angle

$$\delta=\frac{\pi}{2}-\varphi \qquad (14)$$

solves, as it can be seen, the above equation. The geometrical place of points where the maximum shear stresses occur is the circumference of a circle to which the straight line AB inclined at an angle φ is tangent.

The specific weight of earth in its turn should be taken into account.

In connection with the above it must be stated that for a certain point the corresponding stresses will have the following values:

$$N_1=N_2=\gamma y$$

and

$$T=0$$

Thus Rankine's and Coulomb's conditions because of

$$N_1-N_2=T=0,$$

and

$$N_1+N_2=2\gamma y$$

will have one symbol more $-2\gamma y\sin\varphi$

Finally should the position of the foundation footing be at a given depth -h-, reason-

ing analogical to the above should be made.
Thus considering the above results in Rankine's and Coulomb's conditions, we will finally obtain these equations in the following form:

$$\frac{2(p-\gamma h)}{\pi}\left[\sin\left(\frac{\pi}{2}-\varphi\right)-\left(\frac{\pi}{2}-\varphi\right)\sin\varphi\right]-2\gamma(h+y)\sin\varphi=0 \qquad (15)$$

$$\frac{1}{\cos\varphi}\left\{\frac{(p-\gamma h)}{2}\left[\sin\left(\frac{\pi}{2}-\varphi\right)-\left(\frac{\pi}{2}-\varphi\right)\sin\varphi\right]-\gamma(h+y)\sin\varphi\right\}=C \qquad (16)$$

It will be noticed that a maximum -h- will be obtained when $y = 0$ in the expressions (15) and (16).
If we introduce here the notation $p = \gamma H$, then after suitable transformation the necessary depth of a foundation can finally be expressed by the following formula:

$$h=H\frac{1-(\pi/2-\varphi)\operatorname{tg}\varphi}{1+(\pi/2+\varphi)\operatorname{tg}\varphi} \qquad (17)$$

in the case of loose earths, and

$$h=\frac{H[1-(\pi/2-\varphi)\operatorname{tg}\varphi]-\pi C}{1+(\pi/2+\varphi)\operatorname{tg}\varphi} \qquad (18)$$

in the case of firm, cohesive soil. Moreover on the basis of the equations (15) and (16) pressures at the -h- depth (at the moment of the extreme equilibrium of the earth) could be defined:

$$p=\gamma h\frac{1+(\pi/2+\varphi)\operatorname{tg}\varphi}{1-(\pi/2-\varphi)\operatorname{tg}\varphi}$$

in the case of loose earths,

$$p=\frac{\gamma h[1+(\pi/2+\varphi)\operatorname{tg}\varphi]+\pi C}{1-(\pi/2-\varphi)\operatorname{tg}\varphi}$$

in the case of cohesive soil.
For practical purposes the formulae (17) and (18) defining the necessary depth of a foundation should be applied with the 1,25 factor of safety against a possible existence of the extreme equilibrium of the soil.
The question of the necessary depth of a foundation with a not uniformly spread load will be treated somewhat further (fig. 7).
In the case under discussion only active earth pressure is taken into account as exerting influence on those side parts of the foundation block which due to a certain votation exert the corresponding pressure on the ground. The passive earth pressure is better not to be taken into account at all, as it appears only when caused by a previous condensation of soil under the influence of relatively considerable movement of the foundation block, which is altogether inadmissible in buildings.

That's why friction on the surfaces of the foundation block which in certain kinds of soil takes place only during the movement of the foundation is not taken into consideration. Moreover this friction is often diminished by the application of slightly inclined side surfaces of the foundation block and by loosening the adhering parts of soil.
To define the value of pressure on the foundation base, the equilibrium of the foundation block as a solid body must be born in mind, when writing three principal equations of equilibrium, i.e.:

$$\frac{p_1+p_2}{2}a=P \qquad (19)$$

$$-\frac{p_h(1+\alpha)}{2}h(1-\alpha)+\frac{\alpha p_h}{2}\alpha h+\frac{\mu(p_1+p_2)}{2}a=R \qquad (20)$$

$$\frac{p_h(1+\alpha)}{2}h(1-\alpha)(k-z)-\frac{\alpha p_h}{2}\alpha h\left(k-h+\frac{2\alpha h}{3}\right)-\frac{\mu(p_1+p_2)}{2}ak+\frac{p_1+p_2}{2}ae=M \qquad (21)$$

where

$$P = S + G,$$

and μ coefficient of friction between the foundation block and the soil.
Taking into consideration in equations (19), (20) and (21) relative distances of centres of gravity

$$e=\frac{a}{6}\cdot\frac{p_2-p_1}{p_2+p_1},$$

$$z=\frac{h}{3}\cdot\frac{(1-\alpha)(1+2\alpha)}{1+\alpha}$$

we obtain from the solution of these equations the pressure values p_1, p_2 in the form of the following formulae:

$$p_1=\frac{P}{a}-\frac{6}{a^2}\left\{M+\mu Pk+\frac{p_h h}{2}\left[\alpha^2\left(k-h+\frac{2\alpha h}{3}\right)-(1-\alpha^2)(k-z)\right]\right\} \qquad (22)$$

$$p_2=\frac{P}{a}+\frac{6}{a^2}\left\{M+\mu Pk+\frac{p_h h}{2}\left[\alpha^2\left(k-h+\frac{2\alpha h}{3}\right)-(1-\alpha^2)(k-z)\right]\right\} \qquad (23)$$

and besides

$$\alpha=\sqrt{\frac{1}{2}+\frac{R-\mu P}{p_h h}}$$

Finally the value of pressure on the foudation base should be defined in the case shown on fig. 8.

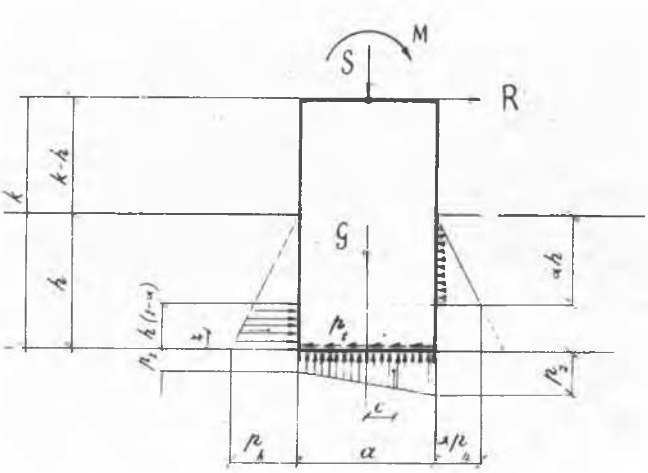

FIG.7

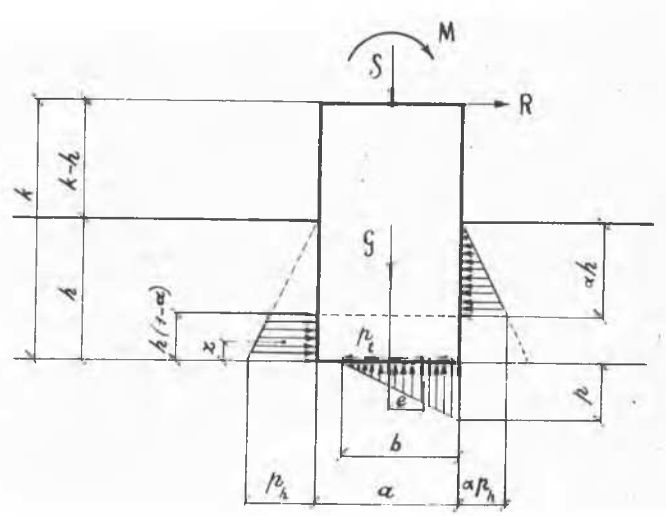

FIG.8

Here the equilibrium equations will take the following form:

$$\frac{pb}{2} = P, \qquad (24)$$

$$-\frac{p_h(1+\alpha)}{2}h(1-\alpha)+\frac{\alpha p_h}{2}\alpha h+\frac{\mu pb}{2}=R \qquad (25)$$

and

$$\frac{p_h(1+\alpha)}{2}h(1-\alpha)(k-z)-\frac{\alpha p_h}{2}\alpha h\left(k-h+\frac{2\alpha h}{3}\right)-\frac{\mu pb}{2}k+\frac{pb}{2}e=M$$

where (26)

$$P = S + G.$$

In the case under consideration it will be

$$e = \frac{a}{2} - \frac{b}{3},$$

where the distance -z-, as well as the α will be defined on the basis of the above mentioned formulae.

From the solution of the equations (24), (25) and (26) we will obtain

$$p=\frac{2P^2}{3\left\{\frac{p_h h}{2}\left[(1-\alpha^2)(k-z)-\alpha^2\left(k-h+\frac{2\alpha h}{3}\right)\right]-P\left(\mu k-\frac{a}{2}\right)-M\right\}} \qquad (27)$$

and

$$b=\frac{3}{P}\left\{\frac{p_h h}{2}\left[(1-\alpha^2)(k-z)-\alpha^2\left(k-h+\frac{2\alpha h}{3}\right)\right]-P\left(\mu k-\frac{a}{2}\right)-M\right\}$$

Thus when determining the necessary foundation depth exerting non uniform pressure on the soil, one may practically speaking, apply formula (17) or (18) (taking into account the safety factor 1,25), in which formulas the height of the column of earth H should be taken for the maximum pressure on the soil. This pressure is determined by the formula (23) or (27).

REFERENCES.

1) S. Betzecki. "Statyka ciat sypkich i ścian podporowych." 1914.
2) A. Caquot. "Equilibre des massifs à frottement interne." Paris 1934.
3) M. Ritter. "Grenzzustände des Gleichgewichtes in Erd- und Schüttmassen." Intern. Ver. für Brück. und Hoch. Zweiter Kongress. Berlin 1936.
4) G. Pigeaud. "Résistance des matériaux et élasticité." Paris 1934.

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I g 3

COMPUTATION OF BEAMS RESTING ON SOIL.

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INTRODUCTION

The computation of beams resting on soil is generally made by considering the soil as a perfectly elastic material, defined by a modulus of soil reaction K (kg/cm^3). Doing so, the soil is supposed to be a perfect liquid with a specific weight K, or an infinity of independent springs.

The method of the modulus of soil reaction has first been introduced in railway-construction, to define the settlement of the sleepers resting on the ballast.

The same method is currently used in the U.S.A. to determine the thickness required for concrete pavements of airfields. The modulus of soil reaction is defined by means of a loading test on a slab of 30" diameter which nearly corresponds to the contact area of the tires of large planes.

When it is necessary to compute beams with arbitrary contact areas and subjected to arbitrary loads, the problem is to find which value can yet be attributed to the figures obtained by the method of the modulus of soil reaction, this method being based on assumptions which differ largely from the real properties of the soil. Another problem is how, for arbitrary contact areas, arbitrary dimensions and arbitrary loads, the modulus of soil reaction can be deducted from real, directly measurable soil properties.

The question of the exactitude of the method of the modulus of soil reaction has already interested many technicians. For instance Wieghardt and Schiel believed to have found a more exact solution by assuming the soil to be a perfect liquid with non negligeable superficial tensions. On the other hand Borowicka gave the exact solution for a circular slab resting on a material with a constant modulus of elasticity.

GENERAL SOLUTION.

One can try to base the computation of beams resting on soil on the real properties of the soil itself. Consider a beam with an arbitrary stiffness, subjected to arbitrary forces, and resting on a soil with an arbitrary compressibility (fig. 1). Under the effect of the forces, the beam and the soil will deflect. These deflections are a priori unknown, but it is known that the deformations of the beam and the soil are to concord in each point of contact. Thus the reactions soil-beam must be so distributed that in each point this condition is fulfilled. Beside the usual equations of the equilibrium of forces, one disposes of an infinity of relations expressing for each point of contact the equality of the deformations of the soil and of the beam.

Theoretically the problem thus is solved. Practically one can proceed as follows: one adopts arbitrarely a distribution of the reactions soil-beam, but which satisfies the normal equation of equilibrium. For this sollicitation the deformations s_p of the beam and s_t of the soil are computed, on the base of the real properties of deformability of the latter. As the law of distribution has been arbitrarely adopted, the values of s_p will generally be different from those of s_t. Then a new law of distribution can be chosen, to obtain values s'_p and s'_t, which better agree, and by successive approximations each desired degree of exactitude can be obtained.

For laterally confined soils the deformability is expressed by the law of compress-