Here the equilibrium equations will take the following form:

$$\frac{pb}{2} = P, \qquad (24)$$

$$-\frac{p_h(1+\alpha)}{2}h(1-\alpha)+\frac{\alpha p_h}{2}\alpha h+\frac{\mu pb}{2}=R \qquad (25)$$

and

$$\frac{p_h(1+\alpha)}{2}h(1-\alpha)(k-z)-\frac{\alpha p_h}{2}\alpha h\left(k-h+\frac{2\alpha h}{3}\right)-\frac{\mu pb}{2}k+\frac{pb}{2}e=M \qquad (26)$$

where

$$P = S + G.$$

In the case under consideration it will be

$$e = \frac{a}{2} - \frac{b}{3},$$

where the distance -z-, as well as the α will be defined on the basis of the above mentioned formulae.

From the solution of the equations (24), (25) and (26) we will obtain

$$p=\frac{2P^2}{3\left\{\frac{p_h h}{2}\left[(1-\alpha^2)(k-z)-\alpha^2\left(k-h+\frac{2\alpha h}{3}\right)\right]-P\left(\mu k-\frac{a}{2}\right)-M\right\}} \qquad (27)$$

and

$$b=\frac{3}{P}\left\{\frac{p_h h}{2}\left[(1-\alpha^2)(k-z)-\alpha^2\left(k-h+\frac{2\alpha h}{3}\right)\right]-P\left(\mu k-\frac{a}{2}\right)-M\right\}$$

Thus when determining the necessary foundation depth exerting non uniform pressure on the soil, one may practically speaking, apply formula (17) or (18) (taking into account the safety factor 1,25), in which formulas the height of the column of earth H should be taken for the maximum pressure on the soil. This pressure is determined by the formula (23) or (27).

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I g 3

COMPUTATION OF BEAMS RESTING ON SOIL.

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INTRODUCTION

The computation of beams resting on soil is generally made by considering the soil as a perfectly elastic material, defined by a modulus of soil reaction K (kg/cm^3). Doing so, the soil is supposed to be a perfect liquid with a specific weight K, or an infinity of independent springs.

The method of the modulus of soil reaction has first been introduced in railway-construction, to define the settlement of the sleepers resting on the ballast.

The same method is currently used in the U.S.A. to determine the thickness required for concrete pavements of airfields. The modulus of soil reaction is defined by means of a loading test on a slab of 30" diameter which nearly corresponds to the contact area of the tires of large planes.

When it is necessary to compute beams with arbitrary contact areas and subjected to arbitrary loads, the problem is to find which value can yet be attributed to the figures obtained by the method of the modulus of soil reaction, this method being based on assumptions which differ largely from the real properties of the soil. Another problem is how, for arbitrary contact areas, arbitrary dimensions and arbitrary loads, the modulus of soil reaction can be deducted from real, directly measurable soil properties.

The question of the exactitude of the method of the modulus of soil reaction has already interested many technicians. For instance Wieghardt and Schiel believed to have found a more exact solution by assuming the soil to be a perfect liquid with non negligeable superficial tensions. On the other hand Borowicka gave the exact solution for a circular slab resting on a material with a constant modulus of elasticity.

GENERAL SOLUTION.

One can try to base the computation of beams resting on soil on the real properties of the soil itself. Consider a beam with an arbitrary stiffness, subjected to arbitrary forces, and resting on a soil with an arbitrary compressibility (fig. 1). Under the effect of the forces, the beam and the soil will deflect. These deflections are a priori unknown, but it is known that the deformations of the beam and the soil are to concord in each point of contact. Thus the reactions soil-beam must be so distributed that in each point this condition is fulfilled. Beside the usual equations of the equilibrium of forces, one disposes of an infinity of relations expressing for each point of contact the equality of the deformations of the soil and of the beam.

Theoretically the problem thus is solved. Practically one can proceed as follows: one adopts arbitrarely a distribution of the reactions soil-beam, but which satisfies the normal equation of equilibrium. For this sollicitation the deformations s_p of the beam and s_t of the soil are computed, on the base of the real properties of deformability of the latter. As the law of distribution has been arbitrarely adopted, the values of s_p will generally be different from those of s_t. Then a new law of distribution can be chosen, to obtain values s'_p and s'_t, which better agree, and by successive approximations each desired degree of exactitude can be obtained.

For laterally confined soils the deformability is expressed by the law of compress-

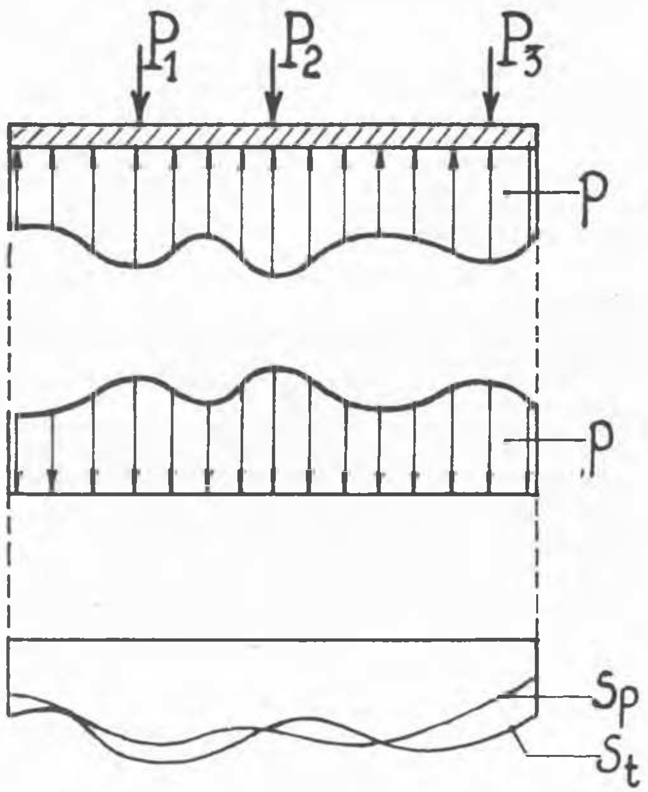


FIG. 1

ibility of Terzaghi. The foundation beams are generally located at a sufficient depth underneath the soil surface; on the other hand, as long as the effective loads are but a fraction of the ultimate bearing capacity of the soil underneath the beam, the settlements of the latter produced by lateral displacements of the soil-particles are small in comparison with those produced by the compression without lateral displacement.

A simple derivation of the formula of Terzaghi indicates that, if the lateral displacements are prevented, the modulus of elasticity E_s of the soil can be expressed by :

$$E_s = C\sigma_k \qquad (1)$$

where E_s = the modulus of elasticity (kg/cm^2)
C = the constant of compressibility, which can be determined by consolidation tests
σ_k = the effective stress at the considered point and on the considered plane (kg/cm^2).

The law (1) indicates that, according to the nature and the history of the soil, it is possible to have a material with a constant modulus of elasticity, or a material that is heteregeneous and/or anisotropic in relation with its compressibility.

If, in view of the simplicity, the tangential reactions soil-beam are neglected, the increase of the stresses in the soil produced by the arbitrarely chosen law of sollocitation can be easily computed on the base of the law of Boussinesq or any other similar law. For this purpose the diagrams of Newmark are very useful.

APPLICATION TO RECTANGULAR BEAMS SUBJECTED TO CENTRAL LOADS.

The described general solution has been systematically used for the computation of rectangular beams with an arbitrary length l and a width b, subjected to a central load P and resting on a soil with a constant modulus of elasticity E_s. Assuming that the unknown distribution of the reactions soil-beam is a parabola of the 2nd degree, one relation of deformability is sufficient. It is obtained by expressing that the maximum deflection of the beam has to be equal to the difference in settlement of the centre and the borders. An example of calculation is given on fig. 2. It appears that the deformed surfaces of the soil So and of the beam Po only concord at the centre and at the borders; thus the parabolic distribution of the 2nd degree is only a first approach to the problem.

By defining the law of distribution by the formula

$$p = A\left(\frac{x}{\ell}\right)^4 + B\left(\frac{x}{\ell}\right)^2 + C - D\ln\left(\frac{p_c}{p_{c,\infty}} - \frac{2x}{\ell}\right) \qquad (2)$$

it is possible, by a judicious choice of 3 parameters, to obtain practically the exact distribution.

In the formula (2) are:

p = the reaction soil-beam in the point with abscissa x
x = the abscissa of the considered point, the origin being at the centre C of the beam.
ℓ = the length of the beam.
p_C = the reaction at the rentre.
$p_{C,\infty}$ = the reaction at the centre under the same beam and the same load, the stiffness of the beam being supposed infinite.
A,B,C,D = coefficients having the dimension of a stress.

As it is necessary to know $p_{c,\infty}$, the formula (2) must first be applied to the case of the beam, supposed to have an infinite stiffness.

The fig. 3 represents an application of the formula (2).

Although the parabolic distribution of the second degree is not an exact one, the values of the moment at the centre given by this distribution differ only very little from the moments corresponding to the distribution of the formula (2).

The case of rectangular beams resting on soils with a modulus of elasticity linearly increasing with depth has also been considered. One gets practically the same conclusions as for a soil with a constant modulus of elasticity.

COMPARISON OF THE RESULTS OBTAINED BY THE METHOD OF SUCCESSIVE APPROXIMATIONS WITH THOSE OBTAINED BY THE METHOD OF THE MODULUS OF SOIL REACTION.

The modulus of soil reaction K being no physical constant for a given soil, it is finally necessary to define this quantity.

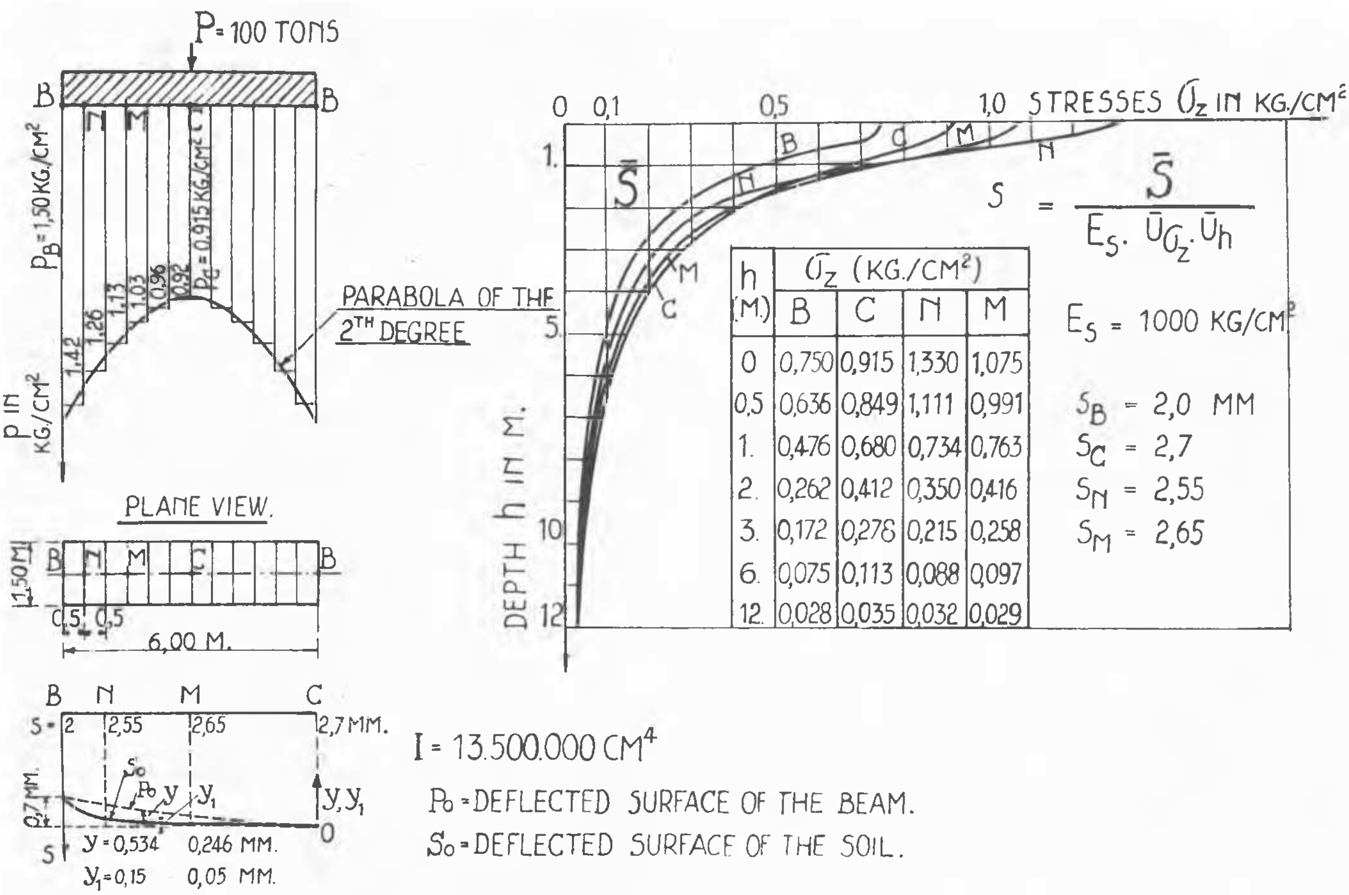
In cases of a soil with a constant modulus of elasticity K is defined by:

$$K = \frac{4}{3}\frac{E_s}{\sqrt[3]{b^2\ell}} \qquad (3)$$

In case of a soil with a variable modulus of elasticity, K is defined by

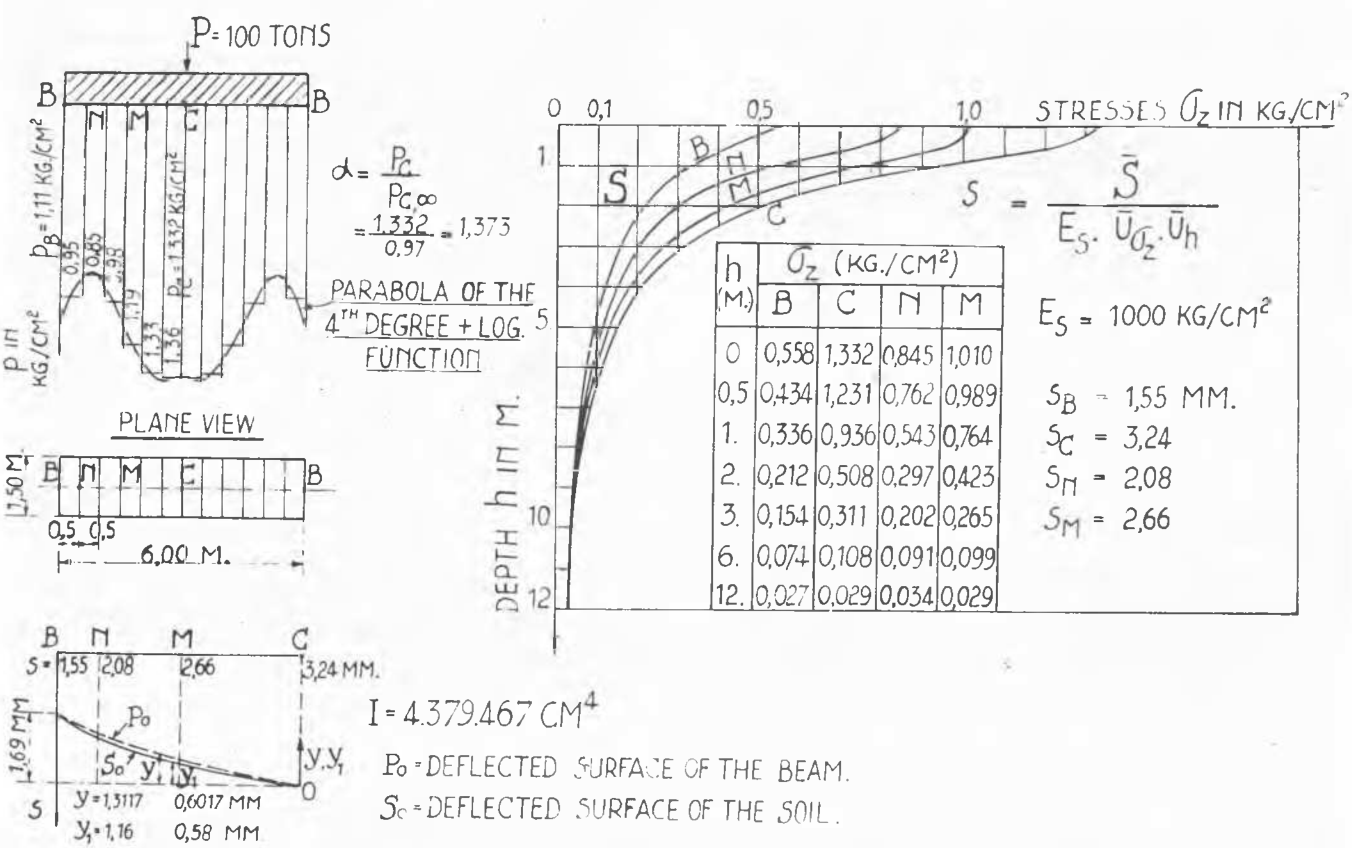
$$K = \frac{p_m}{s_\infty} \qquad (4)$$

where p_m = mean value of the reactions soil-beam.
s_∞ = the settlement of the beam, supposed to be of infinite stiffness. This settlement is computed on the



h (M.)	σ_z (KG./CM²) B	C	Л	M
0	0,750	0,915	1,330	1,075
0,5	0,636	0,849	1,111	0,991
1.	0,476	0,680	0,734	0,763
2.	0,262	0,412	0,350	0,416
3.	0,172	0,278	0,215	0,258
6.	0,075	0,113	0,088	0,097
12.	0,028	0,035	0,032	0,029

FIG. 2



h (M.)	σ_z (KG./CM²) B	C	Л	M
0	0,558	1,332	0,845	1,010
0,5	0,434	1,231	0,762	0,989
1.	0,336	0,936	0,543	0,764
2.	0,212	0,508	0,297	0,423
3.	0,154	0,311	0,202	0,265
6.	0,074	0,108	0,091	0,099
12.	0,027	0,029	0,034	0,029

FIG. 3

base of the real properties of compressibility of the soil.

The value of K having been defined, it is now possible to utilise the method of the modulus of soil reaction to computé the moment at the centre.

Comparing the values obtained for the moment at the centre by the method K with those obtained with the formula (2) or with the parabolic distribution of the 2nd degree, it is found that for a high stiffness I the method K gives for the moment M_C values smaller than the exact ones, while for very low stiffnesses I it is the contrary. Thus for high stiffnesses the application of the method K can furnish values which are to low and thus could become dangerous.

The negative divergence being maximum for I $=\infty$, fig. 4 gives in case $E_s = c^t$, the values

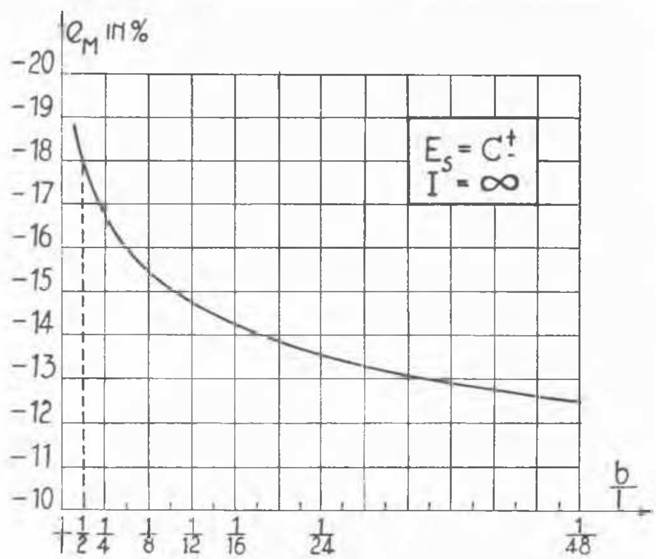


FIG.4

of the divergence e_M in function of the ratio $\frac{l}{b}$ for I $=\infty$. The divergence becomes larger, when the ratio l/b decreases, thus when the shape of the beam approaches that of a square slab. Since only "beams" where considered, for which the length is much larger than the width, the curve on fig. 4 is limited to values $l/b \geqslant 2$.

For these values, the maximum negative divergence between the moment at the centre given by the method K and the one obtained by the parabolic method is smaller than 18 %.

In case of a beam subjected to an axial single load and resting on a soil with a constant E_s, a safe value of the moment at the centre can easily be found. Indeed, it is sufficient to compute the mean value of the modulus of soil reaction by means of the formula (3). One applies the method K to obtain M'_C. For the ratio l/b of the given beam, the fig. 4 gives the value of e_M. Finally M_C is computed by the formula

$$M_C = M'_C \frac{100}{100 + e'_M} \qquad (5)$$

The value M_C so computed will be larger than the real moment, whatever the value of the moment of inertia I and of the ratio l/b will be.

Another way, which is a little less rapid, but gives a more exact value, is to assume that the reactions soil-beam are distributed according to a parabola of the 2nd degree, and to apply the general method.

In case of a soil with a variable modulus of elasticity, the computations show that the maximum value of the negative divergence between the moment at the centre given by the method K and the exact moment is smaller than 30 %; the negative divergence attains its maximum value, when the stiffness of the beam is infinite. For every small stiffnesses the method K gives safe values. A more exact value for the moment at the centre can again be obtained by assuming the law of distribution of reactions soil-beam to be a parabola of the second degree.

Finally for rectangular beams, subjected to a single central load, the moment at the centre obtained by applying the method of the modulus of soil reaction - this latter being defined by the formulas (3) or (4) - is never more than 30 % smaller than the exact value. Since the safety factors adopted for the computation of the dimensions of a beam are at least 2, it follows that, by using the method of the modulus of soil reaction for the considered case, dangerous errors in the choice of the dimensions of the beam are excluded.

For the rest it is worthwhile to note that this conclusion is not longer valid, when it is necessary to compute the moments and the shear forces in an arbitrary section.

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