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ground water level or because they have been excavated and recompacted, exhibit an angle of shearing resistance always greater than zero.

3) Clay-shales and siltstones give a value of ϕ greater than zero. In some cases this may be due to partial saturation, but the evidence suggests that there may also be some other cause such as appreciable area of contact between the grains.

4) Some saturated silts give values of ϕ greater than zero even when tested under conditions of no water content change. These silts have liquid limits less than about 35%. The reason for this result is not known but it is suggested that it may be connected with the phenomenon of dilatancy which could cause an increase in effective pressure due to a decrease in the pore-water pressure.

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A STUDY OF THE IMMEDIATE TRIAXIAL TEST ON COHESIVE SOILS

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1. INTRODUCTION.

Two basic types of compression test are used for investigating the shear characteristics of cohesive soils. Firstly the immediate triaxial test, in which the specimen is stressed under conditions of no water content change. The unconfined compression test is a special case of the immediate triaxial test, where the applied lateral pressure is zero.

Secondly, the equilibrium triaxial test in which the specimens is allowed to attain equilibrium under an applied hydrostatic pressure before being tested in compression ($\bar{\sigma}$).

The immediate triaxial test is, from the practical point of view, the more useful and provides the basis for the $\phi = 0$ analysis of stability in saturated clays. The equilibrium test is less easily interpreted and may, indeed, be misleading in the evaluation of stability problems (Terzaghi 1947).

In the present paper an attempt is made to study the immediate triaxial test from three points of view:

i) to assess the significance of the inclination of the shear planes in compression specimens of cohesive soils

ii) to obtain a theoretical expression for the pore water pressure set up in a saturated clay when stressed under conditions of no water content change

iii) to obtain a theoretical expression for the compression strength of a saturated

clay, as measured in the immediate triaxial test, in terms of the true cohesion and true angle of internal friction as defined by Hvorslev (1937).

The treatment is approximate, but it is presented in the hope that it may prove useful in further research work.

2. THE IMMEDIATE TRIAXIAL TEST.

In the immediate triaxial test a cylindrical specimen initially in equilibrium at some particular water content under a capillary pressure x_a p, is placed between non-porous end pieces and covered with a thin rubber envelope. The specimen is subjected to a hydrostatic pressure σ_3 and the axial pressure is then progressively increased until failure occurs under a total applied axial pressure σ_1 . No water content change is allowed to take place during the test.

With saturated clays the compression strength ($\sigma_1 - \sigma_3$) is found to be a constant,

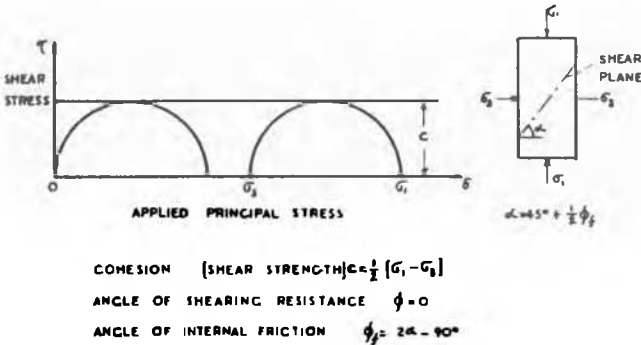
x) In the U.S.A. these two types are referred to as the "quick" and "consolidated" triaxial tests (v. Triaxial Shear Research 1947).

xa) In Laboratory work on remoulded clays p is the pressure under which the specimen has been consolidated prior to the test.

for any given water content, and the clay is therefore behaving with respect to the applied stresses at failure as a purely cohesive material, with an angle of shearing resistance ϕ equal to zero. The apparent cohesion or "shear strength" of the clay is thus (v. Fig. 1a).

$$c = \frac{1}{2} (\sigma_1 - \sigma_3) \quad (1)$$

and this criterion of failure is the basis of the $\phi = 0$ analysis of stability in saturated clays.



Typical results of immedial triaxial testson saturated clay.

FIG.1a

It is clear from Fig. 1a that the unconfined compression strength (where $\sigma_3 = 0$) is identical with the strength found in the immediate triaxial test even when σ_3 is large. This result was proved experimentally by Terzaghi in 1932.

With saturated silts and partly saturated clays the angle of shearing resistance is greater than zero, in spite of the fact that no water content change takes place. The reasons for this behaviour are believed to be (i) that dilatancy occurs in silts and the pore water pressure can therefore decrease with a corresponding increase in effective pressure: and (ii) that the presence of air voids in a partially saturated clay results in an almost instantaneous increase in effective pressure, although the overall volume change may be small (Golder and Skempton 1948).

In all types of cohesive soils it is often observed that failure takes place by shearing along quite well defined diagonal planes (Luders' lines) and that, with very few exceptions, these planes are inclined at angles of more than 45° to the horizontal (σ_1 being taken as vertical).

3. INCLINATION OF THE SHEAR PLANES.

According to Mohr's theory of failure the shear planes in a compression specimen should be inclined at $(45^\circ + \frac{1}{2}\phi)$ where ϕ is the slope of the envelope curve, (see Fig. 1b). This is in reasonable agreement with observations on those materials which fail in shear, such as steel and (xb) marble and it might be expected that the shear planes in a saturated clay tested at constant water content would be inclined at 45° while the angle would be steeper than 45° for silts (and for saturated clays in the equilibrium triaxial test where ϕ is greater than zero). Terzaghi in 1936 demonstrated, however, that the inclination of the shear plane depends upon the true angle of friction of the soil and is entirely independent of the angle of shearing resistance.

This important fact is not widely appreciated and the following analysis has been presented to emphasise the point and also to provide a basis for the mathematical treatment

in sections 4 and 5. As the initial condition it will be assumed that the specimen is in equilibrium under the pressure p. During the applications of the pressures σ_1 and σ_3 pore water pressure will be established, as observed by Rendulic (1937) and Taylor (1947). If at the time of failure the pore water pressure is u then the effective principal stresses σ'_i will be

$$\left. \begin{aligned} \sigma'_1 &= p + \sigma_1 - u \\ \sigma'_3 &= p + \sigma_3 - u \end{aligned} \right\} \quad (2)$$

It will now be assumed that failure takes place on a particular plane when the shear stress on that plane is equal to the shear strength of

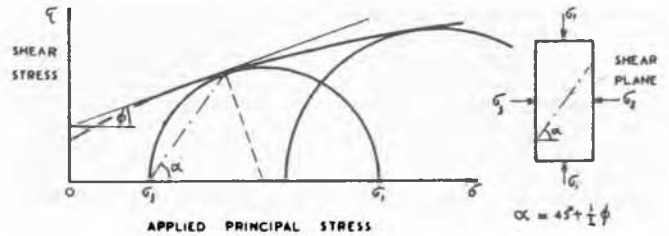


FIG.1b

the soil, defined by the equation

$$s = c_e + n' \tan \phi_f \quad (3)$$

This is the Coulomb criterion of failure in which, following Hvorslev (1937), c_e and ϕ_f are the true cohesion and true angle of internal friction of the soil at the water content, or porosity, of the test specimen, and n' is the effective pressure normal to the plane. Both c_e and ϕ_f will vary with water content but at any given water content they can be taken as constants.

Now the effective normal pressure and the shear stress on any plane inclined at α to the horizontal are:

$$\left. \begin{aligned} n' &= \sigma'_1 \cos^2 \alpha + \sigma'_3 \sin^2 \alpha \\ \tau &= \frac{1}{2} (\sigma'_1 - \sigma'_3) \sin 2\alpha \end{aligned} \right\} \quad (4)$$

Thus from equation (2)

$$\left. \begin{aligned} n' &= p + \sigma_1 \cos^2 \alpha + \sigma_3 \sin^2 \alpha - u \\ \tau &= \frac{1}{2} (\sigma_1 - \sigma_3) \sin 2\alpha \end{aligned} \right\} \quad (5)$$

For limiting equilibrium along this plane

$$\frac{1}{2} (\sigma_1 - \sigma_3) \sin 2\alpha = c_e + (p + \sigma_1 \cos^2 \alpha + \sigma_3 \sin^2 \alpha - u) \tan \phi_f \quad (6)$$

and failure will take place along that plane where σ_1 in equation (6) is a minimum. The value of α satisfying this condition can be

found by equating $\frac{d\sigma_1}{d\alpha}$ to zero, and is

$$\alpha = 45^\circ + \frac{\phi_f}{2}$$

xb) v. Nadai (1931), Mohr's theory does not strictly apply to materials such as concrete which fail partly by shear and partly by splitting. (Terzaghi 1945).

xc) As defined by Terzaghi (1936).

Hence the shear plane is inclined at $(45^\circ + \frac{1}{2} \phi_p)$ to the horizontal; and this result is independent of the pore water pressure u and the angle of shearing resistance ϕ . Equation (4) therefore applies, theoretically, to any cohesive soil.

In tests on individual undisturbed specimens it is often difficult to obtain consistent values of α owing to lack of homogeneity αd). The author has, however, plotted in Fig. 2 the results of tests known to him where this angle was reasonably consistent in a number of specimens of the particular clay. It will be seen that there is a marked tendency for ϕ_p to decrease with increasing porosity. The relationship is particularly clear with remoulded soils where anisotropy and structural effects are reduced to a minimum.

Four points may particularly be noted:

1) Bentonite consists almost exclusively of very small particles of the clay mineral montmorillonite which are probably surrounded by layers of adsorbed water. It would be difficult to see how any appreciable frictional forces could develop in such a material and, as will be seen, the shear planes in the specimens of Bentonite are, alone of all the cases recorded in Fig. 2, inclined at very nearly 45° . It is possible, however, that any clay at water contents approaching the liquid limit will show angles close to 45° . This requires further investigation.

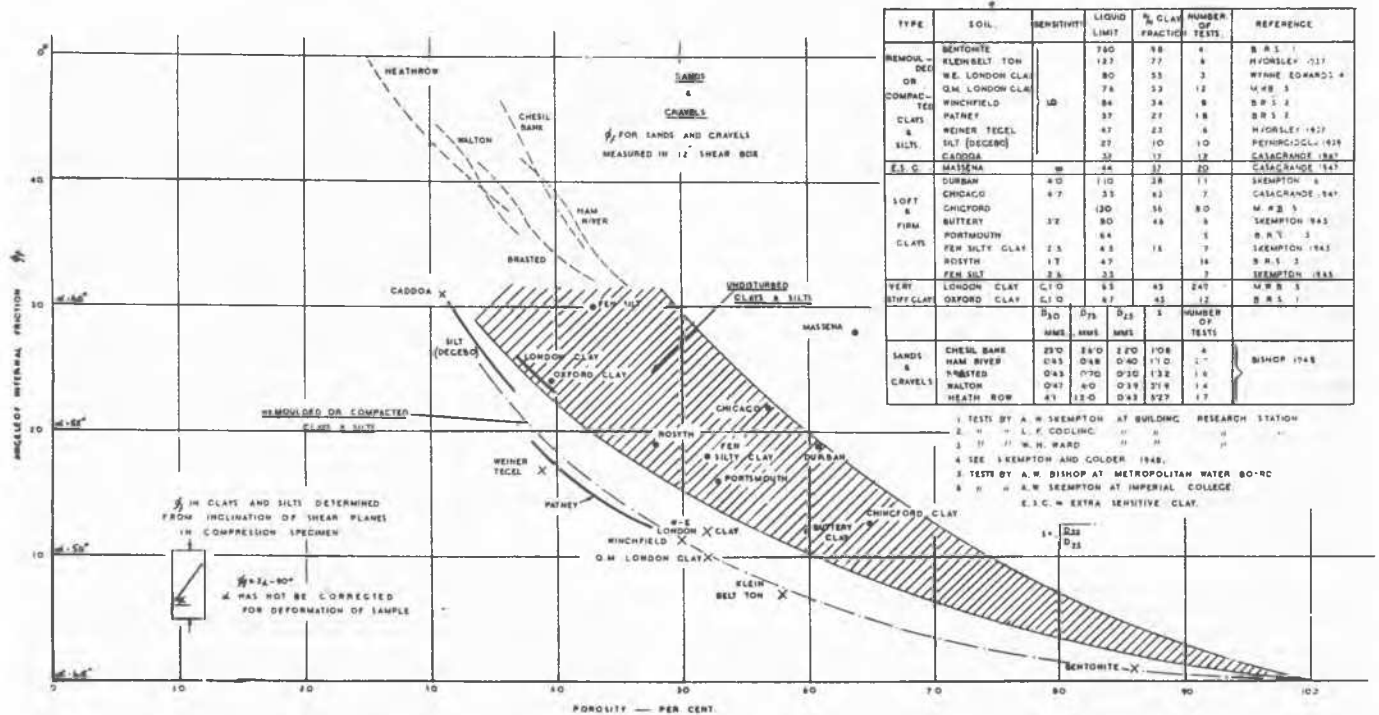
ii) Other factors than porosity are obviously involved. There is for example a definite tendency for clays with a greater sensitivity to αe) remoulding to exhibit a higher ϕ_p , at any given porosity, than those with a less sensitive structure. The Massena clay is an extreme example, showing a complete loss in strength on remoulding, and there may be another zone in Fig. 2 characteristic of these extra sensitive clays.

iii) It is perhaps significant that the sands and gravels appear to link in with and form an extension of the relationship shown by the undisturbed silts and clays. This suggests that in cohesive soils the frictional properties are of a similar character to those in sands, although less pronounced owing to the greater porosities. Terzaghi's views on clay structure (Terzaghi 1947) seem to be in accord with this suggestion.

iv) Most of the clays (LL greater than 40) have been proved to show a zero angle of shearing resistance in the immediate triaxial test.

xd) The inclination of the shear planes is also influenced by anisotropic strength properties. (Casagrande and Carrillo, 1944).

xe) Sensitivity = $\frac{\text{undisturbed strength}}{\text{remoulded strength}}$



2 Relation between angle of internal friction and porosity.

FIG. 2

The total evidence presented in Fig. 2 indicates that, in spite of the inevitable difficulties of determination and interpretation, the inclination of the shear planes in cohesive soils has a real physical significance and is a measure of the angle of internal friction. Internal friction is a directional property. The angle of shearing resistance is not. It is simply the rate of increase in shear strength with pressure.

4. PORE WATER PRESSURES.

In the foregoing analysis, which applied to all cohesive soils, it was sufficient to assume the presence of a pore water pressure u . In the case of non-dilating saturated clays, however, it is possible to derive an expression for this pressure from the condition that no volume change takes place when the clay is stressed under conditions of no water content change. In the following treatment it is necessary to realize that a saturated clay is a 2-phase system consisting of a compressible structure of mineral particles (the "clay structure") and water, filling the voids of the clay structure. The water can be taken as incompressible in comparison with the clay structure.

It will be seen from equation (2) that the changes in effective pressure during the test are

$$\begin{aligned}\Delta\sigma_1' &= (\sigma_1 - u) \\ \Delta\sigma_3' &= -(u - \sigma_3)\end{aligned}\quad (8)$$

The change $\Delta\sigma_3'$ is actually a decrease in pressure since, as will be shown, $\sigma_3 < u < \sigma_1$

Now a change in effective pressure is accompanied by deformations in the clay structure, and the pore water pressure u must consequently be such that the specimen, although subject to deformations, remains at constant volume. With respect to increasing effective pressure (causing consolidation) the modulus of compression and Poisson's Ratio, of the clay structure, will be taken as E_c and μ_c while with respect to decreasing effective pressure (causing swelling) the modulus and Poisson's Ratio will be E_s and μ_s .

The strains are therefore

$$\left. \begin{aligned}\delta_1 &= \left[\left(\frac{\sigma_1 - u}{E_c} \right) + 2\mu_s \left(\frac{u - \sigma_3}{E_s} \right) \right] \\ \delta_3 &= - \left[\mu_c \left(\frac{\sigma_1 - u}{E_c} \right) + (1 - \mu_s) \left(\frac{u - \sigma_3}{E_s} \right) \right]\end{aligned} \right\} \quad (9)$$

Consequently the volume change is

$$\delta_1 + 2\delta_3 = \frac{C_c}{3} \left[(\sigma_1 - u) - 2(u - \sigma_3) \frac{C_s}{C_c} \right] \quad (10)$$

where $C_c = \frac{3(1 - 2\mu_c)}{E_c}$ is the compressibility (volume decrease per unit all-sided effective pressure increase) and $C_s = \frac{3(1 - 2\mu_s)}{E_s}$

is the expansibility (volume increase per unit all-sided effective pressure decrease) of the clay structure. But the volume change is approximately zero, and, since the compressibility of the clay structure with respect to an increase in effective pressure is not zero (if) it is seen that the following relationship must exist:

$$(\sigma_1 - u) - 2(u - \sigma_3) \frac{C_s}{C_c} = 0 \quad (11)$$

or if $\frac{C_s}{C_c} = \lambda$

then

$$u = \frac{\sigma_1 + 2\lambda\sigma_3}{1 + 2\lambda} \quad (13)$$

The pore water pressure can therefore be evaluated if λ is known. For soft clays λ approaches zero, since the compressibility is far greater than the expansibility, while the upper limit of λ is unity. The condition $\sigma_3 < u < \sigma_1$ is therefore satisfied.

The effective principal stresses in a saturated clay, stressed under conditions of no water content change by applied principal stress σ_1 and σ_3 , are therefore

$$\left. \begin{aligned}\sigma_1' &= p + (\sigma_1 - \sigma_3) \frac{2\lambda}{1 + 2\lambda} \\ \sigma_3' &= p - (\sigma_1 - \sigma_3) \frac{1}{1 + 2\lambda}\end{aligned} \right\} \quad (14)$$

For most clays λ probably does not exceed about 0.5 and the practical limits of the effective principal stresses are thus

$$\begin{aligned}\lambda = 0: & \quad \sigma_1' = p \\ & \quad \sigma_3' = p - (\sigma_1 - \sigma_3) \\ \lambda = \frac{1}{2}: & \quad \sigma_1' = p + \frac{1}{2}(\sigma_1 - \sigma_3) \\ & \quad \sigma_3' = p - \frac{1}{2}(\sigma_1 - \sigma_3)\end{aligned}$$

These conclusions are in agreement with experimental observations by Rendulic (1937) and Taylor (1947) and are, moreover, in general accord with the "working hypothesis" put forward by Casagrande (1947).

Equations (14) are approximate in so far as linear stress-volume relationship have been assumed, see equation (10). Where $(\sigma_1 - \sigma_3)$ is small compared with p , as in a clay beneath a foundation, the approximation is probably reasonable and equation (14) provides the basis for calculating immediate settlements. In the triaxial test, however, a more exact solution would have to be based on finite stresses and non-linear stress-volume relationships.

It should be noted that when the hydrostatic pressure σ_3 is applied to the test specimen the principal effective pressures remain unaltered at the initial value p . This provides an explanation of the experimental result that $\phi = 0$ for saturated clays in the immediate triaxial test.

5. COMPRESSION STRENGTH IN TERMS OF c_e , ϕ_f AND λ

It will now be seen from Equation (5) and (14) that

$$\begin{aligned}n' &= p + (\sigma_1 - \sigma_3) \frac{2\lambda \cos^2 \alpha - \sin^2 \alpha}{1 + 2\lambda} \\ \tau &= \frac{1}{2}(\sigma_1 - \sigma_3) \sin 2\alpha\end{aligned}\quad (15)$$

Assuming, as before, the Coulomb-Hvorslev equation, failure will take place on any plane when

$$\frac{1}{2}(\sigma_1 - \sigma_3) \sin 2\alpha = c_e + \left[p + (\sigma_1 - \sigma_3) \frac{2\lambda \cos^2 \alpha - \sin^2 \alpha}{1 + 2\lambda} \right] \tan \phi_f$$

xf) This must not be confused with the compressibility of the whole specimen with respect to applied pressure. This is of course equal to zero.

The condition $\frac{d\sigma_1}{d\alpha} = 0$ again leads to the solution $\alpha = 45^\circ + \frac{\phi_f}{2}$

and putting this value of α in Equation (16) it is found that

$$\frac{1}{2}(\sigma_1 - \sigma_3) = \frac{c_e \cos \phi_f + p \sin \phi_f}{1 + \sin \phi_f \left(\frac{1-2\lambda}{1+2\lambda} \right)} \quad (17)$$

This equation is the criterion of failure, based on the assumption of linear stress-volume relationships for the clay structure, for saturated clays in the immediate triaxial test.

Since c_e , ϕ_f and λ are constants for the clay at any given water content, corresponding to a definite value of p , it follows that

$$\frac{1}{2}(\sigma_1 - \sigma_3)$$

is a constant at any given water content: which agrees with test results.

It is interesting to note that the effective normal pressure on the shear plane is

$$n' = p - \frac{1}{2}(\sigma_1 - \sigma_3) \left[\sin \phi_f + \frac{1-2\lambda}{1+2\lambda} \right] \quad (18)$$

Thus this pressure does not remain unaltered during the test; but the change is directly proportional to $(\sigma_1 - \sigma_3)$ which is itself constant, and the change in effective normal pressure is therefore independent of σ_3 .

7. A METHOD OF DETERMINING c_e AND ϕ_f .

The author would suggest the following method of determining c_e and ϕ_f for saturated clays. A sample is consolidated under a pressure p and then subjected, at constant water content, to an increasing axial pressure until failure occurs.

Thus $\sigma_3 = 0$ and

$$\frac{1}{2} \sigma_1 = \frac{c_e \cos \phi_f + p \sin \phi_f}{1 + \sin \phi_f \left(\frac{1-2\lambda}{1+2\lambda} \right)} \quad (19)$$

But if $\sigma_3 = 0$ then

$$u = \frac{\sigma_1}{1+2\lambda} = j \sigma_1 \quad (20)$$

and

$$\frac{1}{2} \sigma_1 = \frac{c_e \cos \phi_f + p \sin \phi_f}{1 + \sin \phi_f (2j-1)} \quad (21)$$

Both λ and j can be measured experimentally, thus providing a check on equation (20) and providing two methods of evaluating the compression strength in terms of c_e and ϕ_f . In addition ϕ_f can be found, at least approximately, from the inclination of the shear planes, and thus c_e can be determined.

The above procedure can be compared with results obtained by the method of Hvorslev (1937). It can not be used for dilating silts or partially saturated clays.

8. CONCLUSIONS.

1) The inclination of the shear planes in a compression specimen of cohesive soil is a function of the angle of internal friction and

is independent of the angle of shearing resistance, which is a non-directional property.

2) For saturated clays, stressed under conditions of no water content change, the pore water pressure can be evaluated in terms of the applied stresses and the ratio of expansibility and compressibility of the clay structure.

3) Knowing the pore water pressure, either from the theoretical expression or by direct measurement, the compression strength of a saturated clay in the triaxial test can be expressed in terms of the true cohesion and internal friction of the clay at the water content of the test specimen.

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