respect to over-consolidated natural clays which are sensitive to remolding. Induced expansion is liable to break down their brittle inner structure and as a result even increase their active lateral pressure in comparison to their "at rest" condition.

5) During the Princeton tests no generally valid relationships could be established between the results of lateral pressure measurements and the shearing strength of clays and of sand-clay mixtures as determined in the laboratory by various conventional kinds of strength tests carried to failure. No such generally valid relationships are to be expected in view of the fact that no rupture can be induced in a plastic backfill by normal bulkhead motions.

6) The conventional assumption that there is a zone of zero lateral pressure against the upper portion of retaining structures backfilled with cohesive soil does not hold for plastic clays.

7) "Arching" in submerged sand backfills does not decrease bending moments of flexible bulkheads to the extent sometimes assumed, since it cannot develop during normal backfilling operations. When induced by later bulkhead displacements, "arching" may even be accompanied by an increase of total pressure and a resulting increase of the maximum bending moments. Further, all "arching" has an unstable character and should not be relied upon in the design of flexible bulkheads.

8) Shearing stresses at relatively rigid boundaries of the backfill have an appreciable effect on the decrease of lateral pressures of the backfill when it undergoes deformations. To such boundary effects and not to "arching" should be attributed any cases of greater full-scale bulkhead stability in the field than would follow from conventional design procedures which neglect these boundary effects.

9) The suggested semi-empirical procedure for bulkhead design outlined in this paper, when fully developed, promises to provide a method for taking care of the many imponderables which affect bulkhead design. Final recommendations, must however, await completion of the present research program.

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EARTH PRESSURE ON FLEXIBLE WALLS

S. PACKSHAW, B.Sc.

A wall might be defined as a flexible wall if it deflects to the extent of ½% or more of its span when subjected to the loading for which it is designed. In general, this definition embraces sheet pile walls and to a lesser extent L-shaped reinforced concrete cantilever walls. Most of the retaining walls of any magnitude that are now designed fall within one or other of these categories and the importance of an accurate assessment of their behaviour therefore needs no further emphasis.

In evaluating the earth pressure on a flexible wall it is customary to assume that the magnitude of the total pressure is much the same as on a wall of any other type, provided the yield of the wall is sufficient to permit the soil to develop its full shear strength, so that the pressure drops from the "at rest" value to the active value. As this stipulation excludes only exceptionally rigid walls, tunnels and so on, it need be considered no further. For ordinary walls, including the flexible types, the active earth pressures can be calculated with reasonable accuracy from the classical earth pressure theories such as those of Coulomb, Rankine and so on, provided the limitations of these theories are recognised and the results modified accordingly. These theories are simple and convenient and the circumstances in which they can be used without introducing large errors - at any rate in comparison with the inaccuracies still inherent in all earth pressure calculations are fairly well defined. Where they are not applicable, as in the case of the passive resistance of the soil, methods developed from those of Krey and Terzaghi can be used. The equations and graphs in this paper are accordingly derived from whatever theory or method seems most appropriate to the conditions.

WALL FRICTION AND ADHESION.

One of the modifications that have to be applied to the classical theories, particularly to the very convenient algebraic methods of Rankine, to make them applicable to the design of flexible walls is in respect of wall friction and adhesion. The forces derived from friction and adhesion have relatively little effect on the active pressure and are therefore often neglected, though in fact adhesion may cause a substantial diminution of the pressure of cohesive soils. In calculating the passive resistance, however, wall friction and adhesion cause such large increases that they must not be left out of account 1).

Wall friction and adhesion cannot, however, always come into play. They cannot develop on the active pressure side of a timbered trench and they would not be effective in increasing the resistance developed in front of a cantilever sheet pile wall or of anchor walls securing the tie rods of sheet pile walls. This is because the passive wedge which forms in front of these walls would take the wall with it as it moved upwards, the weight of the wall being generally insufficient to hold it down. The full values of the forces developed by wall friction and adhesion can, however, be used for anchored sheet pile walls, though it would be prudent to neglect them if the wall is subjected to severe vibration and, in the case of cohesive soils, for the first 3 to 5 ft. of depth. Weathering may occur in this region and cause the soil to shrink away from the wall.

ACTIVE EARTH PRESSURE.

The pressure of cohesionless soils requires little comment. The theoretical investigations of Jaky 2), Terzaghi 3) and others have shown that no large error is made by basing the calculation on the classical Coulomb theory and taking the slip surface to be a plane. For vertical walls with a horizontal ground surface it is very convenient to use the Rankine theory, as this does not necessitate a graphical construction. If wall friction is to be taken into account the Rankine theory can still be used provided a correction factor is applied 1). Recent work has established an approximately linear relationship between the angle of internal friction and the dry density of the material. This holds good for a wide range of particle-size distributions for various sands and sand and gravel mixtures.

The pressure of cohesive soils can also be determined without considerable error by assuming that the slip surface is a plane. The total pressure can then be found from the equilibrium of the unstable wedge that forms behind the wall and for the particular case of a vertical wall with horizontal backfill the total thrust P_a can be expressed by the general equation:

$$P_a \cos\delta = \frac{1}{2}\gamma H^2 K_{a\gamma} - cHK_{ac} \quad (1)$$

Here $K_{a\gamma}$ and K_{ac} are coefficients which vary with ϕ, δ and c_w, $P_a \cos\delta$ being the normal component of the thrust on the vertical wall.

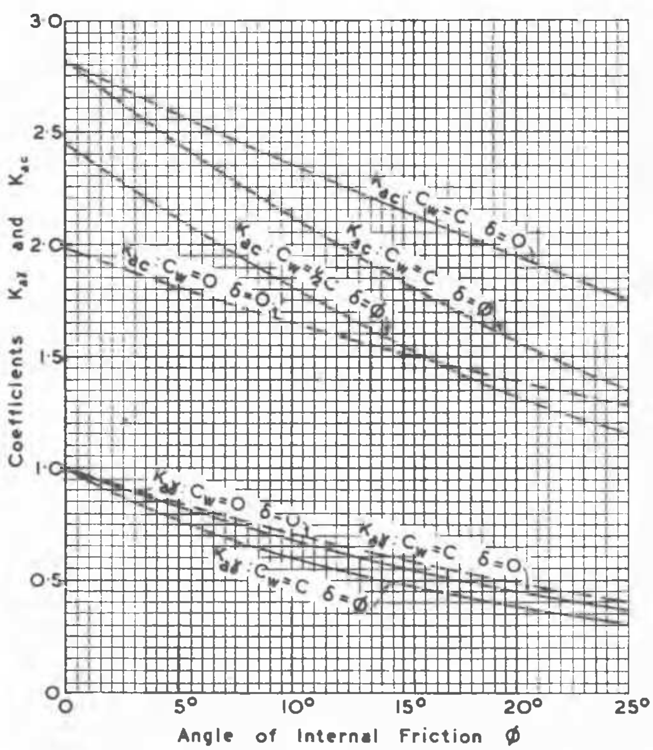
For the special case when ϕ and δ are both zero but c_w has some magnitude, this equation becomes

$$P_a = \frac{1}{2}\gamma H^2 - 2cH\sqrt{1+n} \quad (2)$$

where $n = c_w/c$. The second term in the equation may thus vary from 2cH when c_w and consequently n are zero, to 2.83 cH when c_w equals c and n is therefore equal to 1. On the other hand if δ and c_w are both taken to be zero, the general equation can also be written in the well known form first derived by Francais, (1820)

$$P_a = \frac{1}{2}\gamma H^2 tg^2\left(45° - \frac{\phi}{2}\right) - 2cH tg\left(45° - \frac{\phi}{2}\right)$$

It is possible to obtain a relation between ϕ and the coefficients $K_{a\gamma}$ and K_{ac}, taking various combinations of δ, ϕ and c_w into account 1). This involves a very slight approximation and is shown by the graphs of Fig. 1. Thus if ϕ is known and appropriate values have been selected for δ and the ratio c_w/c, the coefficients can be read off and introduced into equation (1).



Approximate values of Pressure Coefficients for Cohesive Soils.

FIG.1

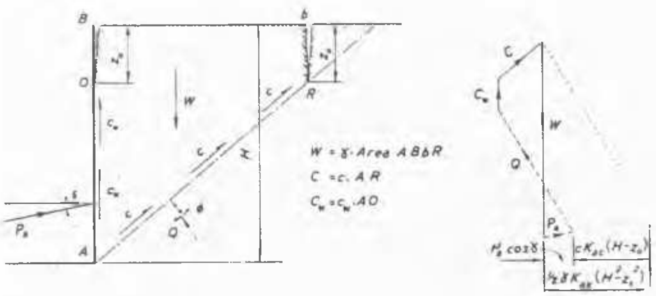
All these equations neglect the effect of the tension cracks which may form in the ground behind the wall. If these are taken into account the forces on the unstable wedge are as shown in Fig. 2 and equation (1) becomes

$$P_a \cos\delta = \frac{1}{2}\gamma K_{a\gamma}(H^2 - z_o^2) - cK_{ac}(H - z_o) \quad (3)$$

Here z_o is the depth of the tension cracks, determined by observation or from the depth $2c/\gamma$ down to which the intensity of pressure (not the total pressure) is zero.

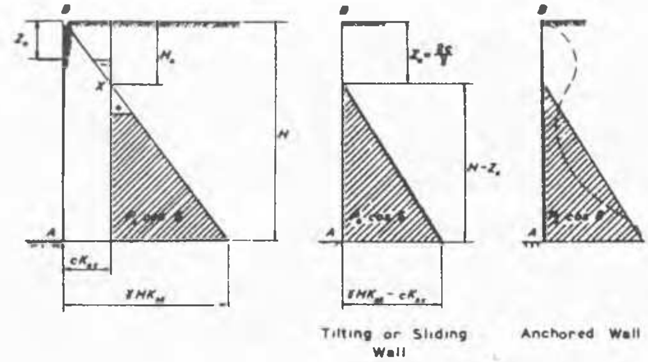
For all cases likely to occur in practice equation (3) will give greater pressures than equation (1) and should therefore be used in preference to the latter.

In the majority of cases it is advisable to calculate the pressure of cohesive soils for two sets of conditions. As a rule the



Forces on Wedge with Tension Cracks.

FIG.2



Probable Pressure Diagram for Tilting or Sliding Wall and for Flexible Wall.

FIG.3

soil properties will not adjust themselves as rapidly as the wall is built and dredging or excavation carried out. The first investigation should therefore be based on the assumption that the soil is a purely cohesive and frictionless material with $\phi = 0$, the value of c being obtained from unconfined compression or immediate shear tests which do not allow time for the soil to consolidate. The calculation should then be repeated with values of ϕ and c determined from tests which give time for the water content to adjust itself to the final conditions which may not be realized in the actual retaining wall until a period of several years has elapsed.

The effect of a uniformly distributed superload can be taken into account on the same principles.

All the foregoing is applicable only to vertical walls with horizontal backfill. For other cases it is necessary to carry out the construction of Fig. 2 with different inclinations of the slip plane to determine the maximum pressure. The effect of different strata can also be dealt with in the same manner 1).

DISTRIBUTION OF ACTIVE PRESSURE.

It is generally agreed 3)4), that although the active pressure on a flexible cantilever wall, that is a wall without anchorages, increases uniformly with the depth, some redistribution occurs when the wall is secured near the top by one or more sets of tie rods. This is due to the arching of the soil between the tie rods at the top and the support provided by the ground at the bottom of the wall. The shape of the resulting active pressure diagram has never been determined from experimental observations. It is assumed, however, that the magnitude of the total pressure remains the same as for other types of walls but that the redistribution causes a substantial reduction of the bending moment in the wall and some increase in the pressure at the upper and lower supports of the wall. As arching is apparently dependent only on the internal friction of the soil, it is unlikely to be very effective in cohesive soils that have only a small value of ϕ. Little numerical data is available on the subject but on the basis of Stroyer's tests and from theoretical considerations the following figures may be suggested to assist the designer until further evidence is obtained:

$\phi = 30°$: reduction in B.M. 20%
$\phi = 40°$: reduction in B.M. 30%

In view of the probable concentration of pressure at the upper support it is advisable to allow for an increase of 10% to 15% or even more in the tie rod loads. This allowance should be made even if the calculated deflection is less than $\frac{1}{2}$% of the span, in case the increase also occurs with smaller deflections. An equally conservative view might be taken in the case of tie rods for retaining walls in cohesive soils.

The effect of tension cracks in cohesive soils, is another source of uncertainty in the distribution of pressure. The total thrust corresponding to equation (3) is shown diagrammatically by the shaded area in the left hand illustration of Fig. 3. The shape of this area does not necessarily represent the distribution of pressure, but for the particular case when z_o is taken to be $2c/\gamma$ it equals the depth H_o and the small positive and negative areas disappear. The diagram of total pressure then becomes a triangle with its apex at X which probably does correspond to the actual pressure distribution on a tilting wall. With anchored sheet pile walls, particularly if the tie rods are tightened before the full load comes on them, there is some doubt whether it is possible for tension cracks to form. Theoretically, equation (1) should then be used and in that case the whole triangular area above X becomes negative and can be deducted from the total pressure. Since, however, very little is known on this subject at present it seems best to neglect the negative area and to use equation (3) and the same diagram for both anchored walls and for cantilever walls.

EARTH RESISTANCE - COHESIONLESS SOILS.

In calculating the passive pressure or resistance it is not permissible to neglect the effects of wall friction and adhesion; nor can the slip surface be taken to be a plane. It should be assumed that the slip surface is curved at the bottom and plane at the top of the wedge. Wall friction is particularly important in the case of cohesionless soils and can increase the resistance to several times the Rankine value, which does not take wall friction into account. x) It makes little difference whether the curved portion is tak-

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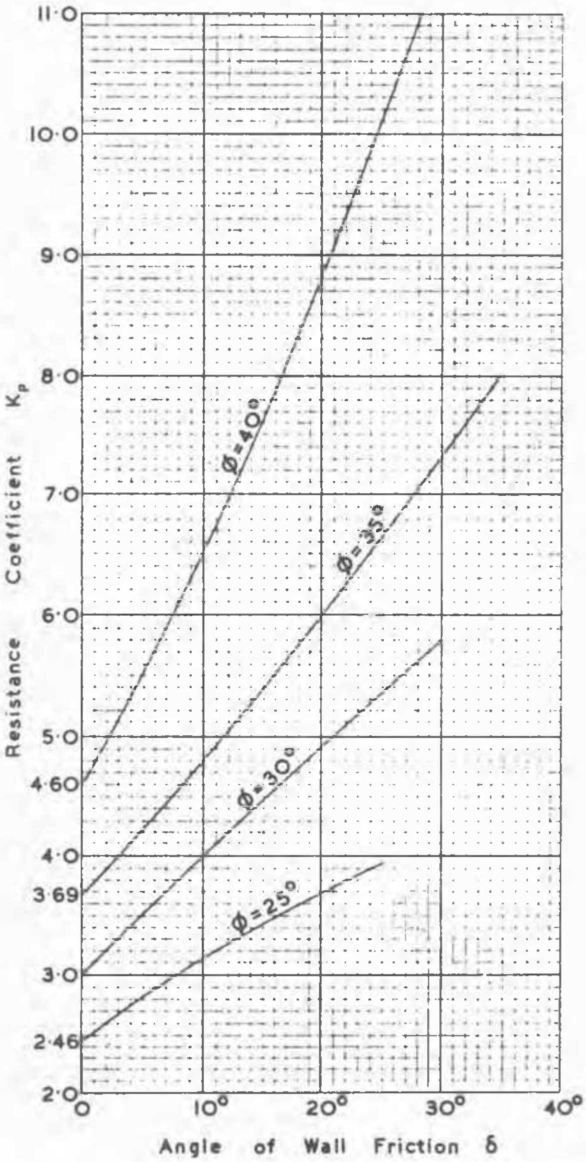
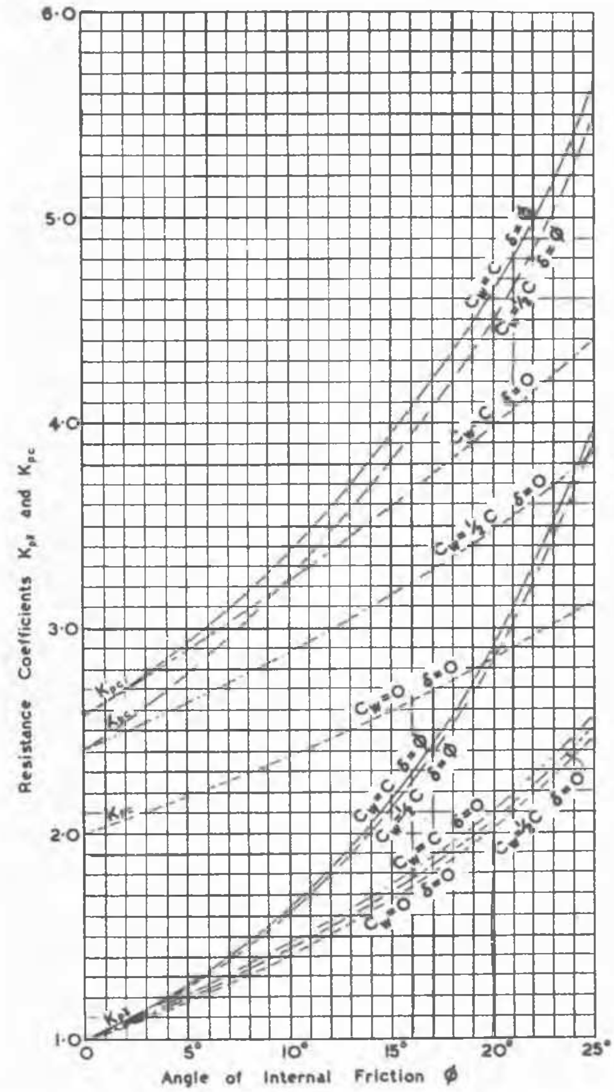
x) It should be noted, however, that some movement - the magnitude of which is dependent on the state of compaction and the characteristics of the soil - is necessary before the full passive pressure is developed. For design it may, therefore, be necessary to use a reduction factor on the full calculated passive pressure if appreciable movement is to be avoided.



Resistance Coefficients for Non-cohesive Soils.

FIG. 4



Approximate Values of Resistance Coefficients for Cohesive Soils.

FIG. 5

en as an arc of a circle or part of a logarithmic spiral. The method is already well known 3) but for convenience the earth resistance coefficient K_p in the basic equation

$$P_p \cos\delta = \frac{1}{2}\gamma H^2 K_p \qquad (4)$$

has been worked out for a vertical wall with horizontal backfill for various combinations of ϕ and δ 1). The results are shown in Fig. 4. The coefficients are smaller than those derived from the Coulomb theory, which assumes that the slip surface is a plane, and agree well with such practical tests of earth resistance as have been carried out.

EARTH RESISTANCE - COHESIVE SOILS.

The passive resistance developed by a cohesive soil with a horizontal ground surface against the thrust of a vertical wall can be expressed by the general equation

$$P_p \cos\delta = \frac{1}{2}\gamma H^2 K_{p\gamma} + cHK_{pc} \qquad (5)$$

For the particular case when wall friction and adhesion are neglected, the slip surface becomes a plane and the equation can then also be written in the following well-known form:

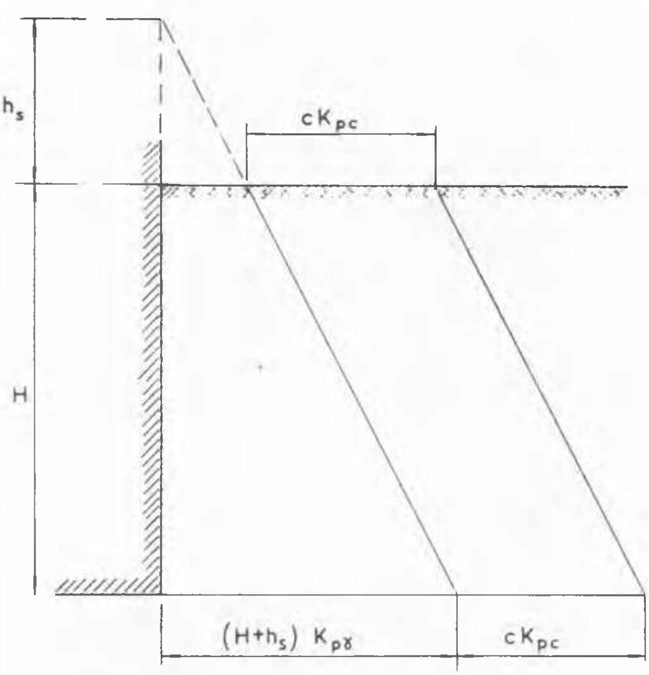
$$P_p = \frac{1}{2}\gamma H^2 tg^2\left(45° + \frac{\phi}{2}\right) + 2cH\,tg\left(45° + \frac{\phi}{2}\right) \qquad (6)$$

When wall friction or adhesion are taken into account, however, the slip surface is not a plane. Values of $K_{p\gamma}$ and K_{pc} have been worked out for the ϕ - circle construction, as in the case of cohesionless soils, and the results for various combinations of ϕ, δ and c_w are shown in Fig. 5, 1). Some approximations had to be made to present the results in the form of a simple series of graphs, but the errors are well within the range of the uncertainties in assessing the properties of the ground.

Some restrictions may have to be made on the calculated resistance of soft or medium clays if the permissible yield of the wall is limited. On the other hand, in contrast to active pressures, the full shear strength of the soil must be mobilized and become effective before failure occurs. The theoretical resistance due to each of several different soil strata can be estimated on the same principles. The only source of doubt is the extent to which materials with different stress-strain diagrams can develop their maximum resistances simultaneously and not one after another, when some of the layers have passed into a state of plastic equilibrium and thus lost some of their shear strength.

DISTRIBUTION OF PASSIVE RESISTANCE.

The distribution of the passive pressure or resistance in front of a wall is not subject to the same uncertainties as the distribution of the active pressure, and there are no complications due to the possibility of tension cracks. For cohesionless soils a hydrostatic distribution, giving a uniformly increasing resistance, can be relied upon provided the movement of the wall is sufficient. In the case of cohesive soils the cohesion adds a constant amount cK_{pc} to the diagram (Fig. 6). In the design of sheet pile retaining walls it is general to base the calculations on the assumption that the actual resistance diagram will be in accordance with the theoretical. Since, however, the deformation of the ground in front of a sheet pile wall is greatest at the ground surface, reducing to zero at some lower point, it is probable that the lower part of the diagram is rounded off and that the peak is not realised in practice. The error caused by including the whole diagram for calculation purposes is not large, though of course it diminishes the factor of safety.



Resistance Diagram for Cohesive Soils.

FIG.6

The difference between the active pressure on the back of a wall and the passive resistance of the ground in front of it has a very great influence on the design of the wall. For example, it can be shown that for the simplest case of a frictionless clay and no adhesion against the wall, it is impossible to develop any net resistance if the height of the wall is greater than $4c/\gamma$ 5). If this critical height is exceeded the pressure on the back of the wall will always be greater than the resistance in front of it, however great the penetration. If adhesion is taken into account the critical height is increased and if the ground possesses some internal friction a net passive resistance will be developed if the wall is deep enough. Clearly the relative shapes of the pressure and resistance diagrams is very important and it is regrettable that this is still largely a matter of conjecture.

FUTURE RESEARCH.

There is much scope for research into the behaviour of flexible walls, especially those built of sheet piling. Several lines of approach can be suggested. The loads in the tie rods should be measured by means of strain gauges or extensometers; the pressure and resistance at various depths should be determined by pressure cells and the results then compared with the theoretical pressures and the shapes of the pressure diagrams. The provision of strain gauges on the sheeting would provide a check on the calculated bending moment and show the extent to which it is affected by the flexibility of the sheeting in cohesive and non-cohesive soils. The effectiveness of driving the sheeting deep enough to ensure that it acts as a beam "fixed" in the ground by the earth resistance in front and behind it should be ascertained, also the yield of the ground necessary to develop the calculated resistance.

Observations should be continued for a considerable period until the soil has completely adjusted itself to the new conditions.

Very little has been done on these lines up to now and as the annual cost of flexible walls in various parts of the world runs into millions of pounds, there is clearly plenty of opportunity for worth-while research.

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