V b 4

COMPUTATION OF A QUAY WALL.

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INTRODUCTION.

Buisman considered this computation scheme for these and similar problems to be a solution, which, although it may not be mathematically perfect, might give an insight into the various forces with their resulting stresses and strains and which provides a result, in which at least the essential properties of the foundation soil are expressed.

I feel that, although the computation dates from 1931, the contents are as yet insufficiently known. Therefore the basic thought is here given for publication without paying undue attention to the calculation of stresses in the various parts of the construction.

T.K. Huizinga.

DESIGN OF THE QUAY WALL.

The design of the quay wall is given in fig. 1. The slab, carried by timber piles, was laid on the original ground surface, which was afterwards raised to 1.50 m + NAP. An overburden of 4 ton/m2 is allowed for.

DATA NEEDED FOR THE COMPUTATION.

The information necessary for a complete computation on stability and strength consists of: specific gravities of the foundation soil layers and of the future sand fill, friction properties of these soils, and, in view of the combined action of soil and pile groups, the elastic properties of the piles and of the soil after the driving of the piles. Another factor which has considerable influence on the result of the computation is the horizontal pressure in the soil prior to the dredging in front of the quay wall. One can only indicate, on the basis of theoretical considerations, the higher and lower limiting values of this pressure, and only investigations in the field can extend our knowledge in this respect.

HORIZONTAL PRESSURE.

Owing to the 5 m fill on the area behind the quay wall and also to the expected overburden, the soil mass behind the wall is loaded with about 10 ton/m2. The active earth pressure in the sand fill, occurring with a slight yielding of the superstructure can be fairly accurately determined (cohaesion = 0, $\phi = 30^o$). The active earth pressure in the soil below 3.50 m - NAP will be more difficult to calculate. Probably one is on the safe side if one neglects cohaesion and keeps to the measured angle of internal friction of 27°. In the usual way a diagram can now be drawn of the active earth pressure, assumed to act horizontally on a vertical plane through the back of the slab (fig. 1) (Also in this plane an appreciable vertical friction will be acting. This will influence the horizontal forces only to an extent of e.g. 10-20 %. This favourable effect is here only mentioned pro memoria). Any other diagram showing the same or greater surface area above any point of the vertical will be equally consistent with the maintenance of equilibrium. The stress distribution is in fact statically undetermined and depends on the possibilities of deformation. Where the pile groups show elastic bending under the rigid superstructure the stress distribution will tend to concentrate at the rigid superstructure, and along the piles it will be as small as is consistent with the local conditions of equilibrium. These conditions require a relationship between the principal stresses of $tg^2(45^o \frac{\phi}{2})$, but it is proved that this ratio, which gives the limiting condition for a slip ϕ_1, is different from the ratio limiting a remolding ϕ_2, which is the only deformation locally possible. Terzaghi pointed to this difference and for sand he gave ratios between the principal stresses of 0.288 to 0.106 for slip, and ratios of o.15 to 0.16 for remolding. For colloidal clay he finds practically equal values for both cases. For sand, direct measurements for the determination of the ϕ_2 for remolding is not feasible, and the only way out is an experimental test for the case of a flexible wall with lateral earth pressure. Even with very slight deflections it appears that the pressure in the middle of the wall is very small so that one is inclined to substitute the usual straight-line earth pressure diagram by a curved one. In this case the values of the bending moments drop to half the values obtained by the straight-line method. For clay direct measurements may be feasible. Here the difference is small.

Therefore, with regard to the lateral loads of piles, it is of great importance to find out if there exists a great difference between ϕ_1 and ϕ_2.

COMPUTATION OF THE AXIAL PILE LOADS.

Computations of the axial loads on the piles have been carried out by means of two methods:
firstly by the approximate method, dividing the piles in 3 groups and assuming equal forces in the piles of each group, secondly by the elastic method, assuming a rigid superstructure on piles of equal elastic properties (method Nökkentved). It appears that both methods give almost the same result (20 tons compression, 10 tons tension in the most unfavourable cases). Only for the purpose of checking the dimensions of the superstructure the elastic method is preferable.

Next, the bending stresses in the piles and in the sheet piling have to be investigated, and new and important questions arise in this respect. To simplify, a rigid layer (sand) is assumed below 12 m - NAP and the soil above this sand is assumed to be homogeneous.

COMPUTATION OF THE SHEET PILING.

Be the pressure in vertical planes directly behind the sheet piling p; then the

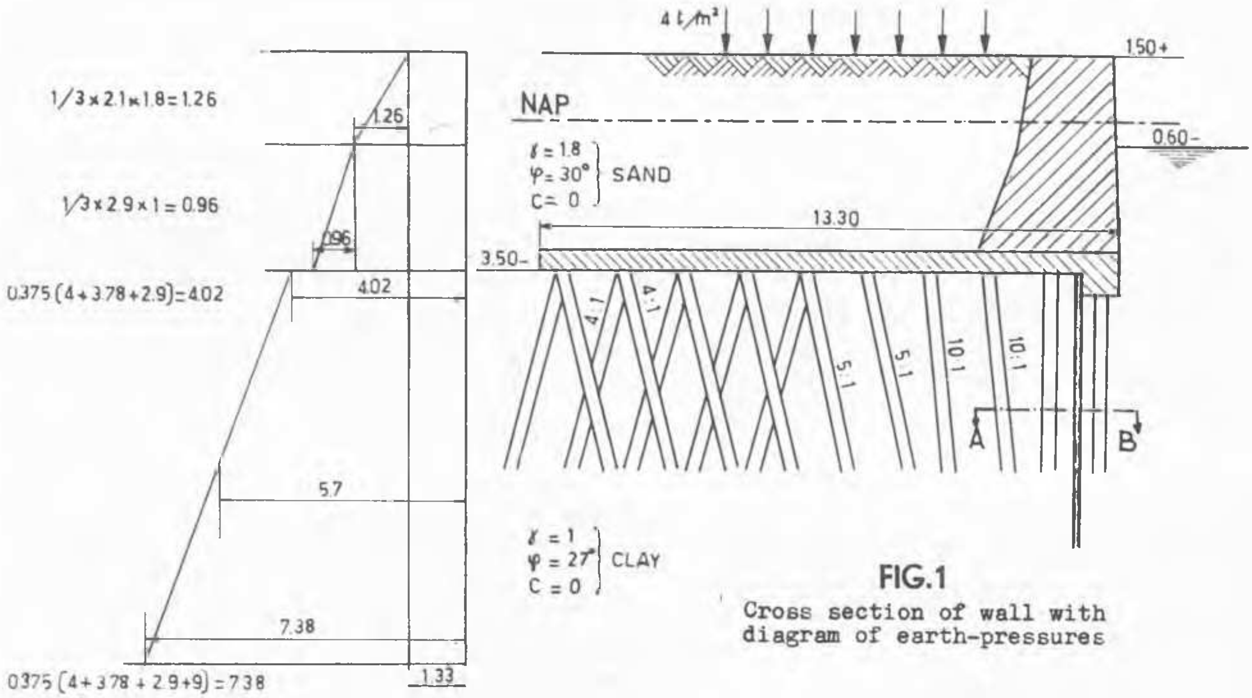

FIG.1
Cross section of wall with diagram of earth-pressures

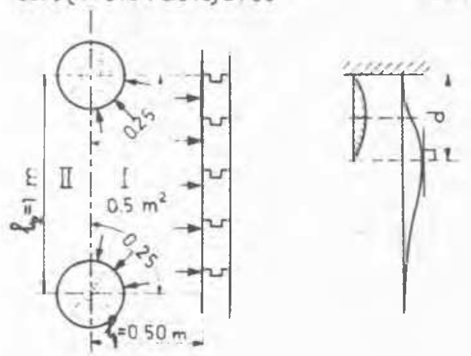

FIG.2
Section AB.

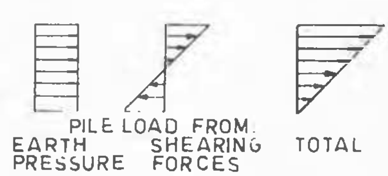

FIG.3

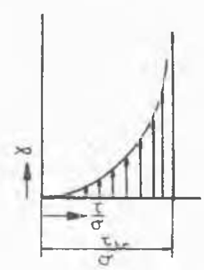

FIG.4

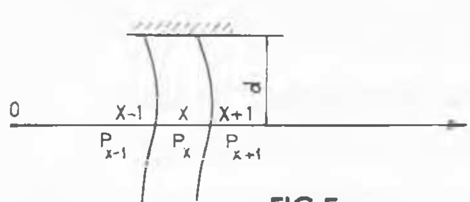

FIG.5

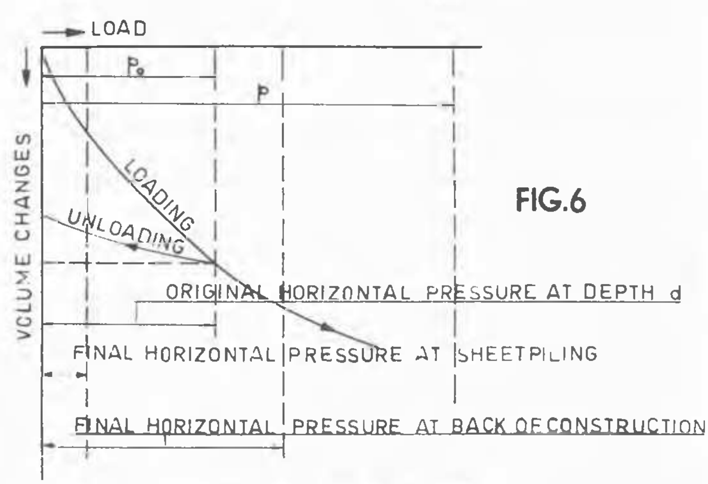

FIG.6

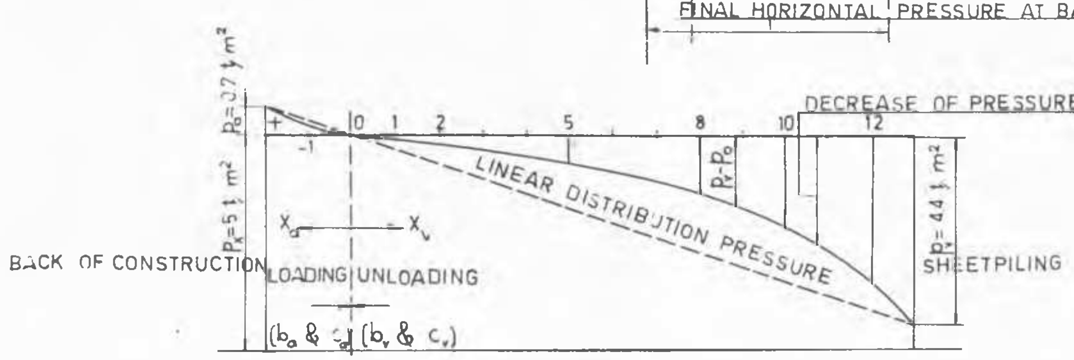

FIG.7

value attainable by $\underline{p}$ at a certain depth will be limited, because $\overline{p}$ cannot exceed a certain value. For it is clear that with increasing depth, the increment of the weight of the earth mass I (fig. 2) is partly carried by friction, so that the horizontal pressure from a certain point downwards does not increase any more. This is the case as soon as

$$p \cdot \left(\ell_1 + 2\frac{\pi}{4} \cdot d\right) \cdot f = \ell_1 \cdot \ell_2 \cdot \gamma$$

from which it follows that

$$p = \frac{\ell_1 \cdot \ell_2 \cdot \gamma}{(\ell_1 + 2\frac{\pi}{4}d)f} = \frac{0{,}5 \times 1 \times 1}{(1 + 0{,}5) \times 0{,}6} = 0{,}6 \text{ ton/m}^2$$

In bay II, between the first and the second row of piles, the horizontal pressure will be greater, because the first row is deflected by the excess pressure of II over I, and also because the ratio between area and perimeter is less favourable for II than for I. If the pressure in the bays between the successive rows of piles would decrease linearly from E = 5,7 ton/m^2 at the back to p = 0,6 ton/m^2 in front and if this pressure was evenly distributed over the 14 pile groups, then there would be a pressure in bay II of

$$p + \frac{E}{14} = 1 \text{ ton/m}^2$$

which pressure per m' depth does not provoke sufficient friction along the piles to carry the weight of a 1 m thick soil layer.

A favourable circumstance is that the second row of piles is battered and therefore with increasing depth approaches the first row. If, starting at the top, one tries to find out whether at a certain depth the vertical pressure will exceed 2.66 ton/m^2 and therefore the horizontal pressure will exceed 2.66(tg^2 45 - 27/2) = 1 ton/m^2, it appears that at a pressure of slightly more than 1 ton/m^2 a state of equilibrium will exist. Also from the computation of the pile forces (as shown below) it follows by elastic considerations, that in bay II a pressure slightly higher than 1 ton/m^2 will occur, so that in this bay and even more so in the bays further shoreward the soil will be carried by pile friction to such extent that it seems sufficient to assume a lateral pressure on the sheet piling of only 0.6 ton/m^2 as a result of the "bin effect".

If, therefore, the horizontal earth pressure was evenly distributed over all the piles, whereby each pile would carry 400 kg/m', then the sheet piling would, under the influence of the bin effect, deflect more than the adjoining row of piles, so that the sheet piling has to resist ultimately only the "bin pressure". With uneven distribution of the earth pressure over the piles and low initial horizontal stresses in the soil mass, the piles at the back would have to resist more than the piles in front and the latter would show less deflection. In that case the first reasoning with regard to the load on the sheet piling is even more applicable.

While computing the strength of the sheet piling the question arises to what extent it may be considered to have a fixed end in the deep sand. From a calculation (theory of elastically supported beams) it appears that the bending stress in the sheet piling amounts to about 150 kg/cm^2 under the most unfavourable conditions.

HORIZONTAL PILE FORCES.

In order to solve the problem of the pile deflections, we know the values of the horizontal pressures in front and behind, but it remains to find a sensible approximation of the distribution of the pressures over the piles. This depends on the flexibility of the piles, the shearing modulus of the soil and the horizontal compressibility of the soil after the driving of the piles. To this end an investigation into the elastic properties of the soil in the undisturbed state (E and G) is required, and also an insight into the horizontal stresses of the soil after the driving of the piles. The higher the horizontal stresses were, prior to dredging in front and filling up of the area behind, the better it is: the future horizontal compression of the soil will be smaller, but not the horizontal relaxation in front. This relaxation however is not so essential, so that generally speaking a greater prestress will result in smaller deflection differences of the piles and to an equalization of the individual pile reactions against the total pressure difference. It is obvious that low compressibility of the layers in horizontal sense in itself already (e.g. sand) would promote this equalization and also that a high shearing modulus will lead to a shifting of the stresses to the soil and away from the piles. Here also, as was the case at the sheet piling, there is the uncertainty about the position of the point of contraflexure of the pile. Without field observations one can only go by one's personal technical feeling.

The soil mass under the foundation slab will be pushed forward if the resistances of the soil and of the piles do not counteract this. At the top the only resistance comes from the piles; at the bottom there is the resistance of the soil in front of the wall, but mainly the shearing resistance of the heavily loaded soil mass. The piles will bend while trying to keep the soil mass in place and every pile has a point where the deflection is greatest and the slope of the pile axis the same as before. In this particular layer, run through by piles at an unchanged angle, square angles have suffered no change and shearing stresses do not exist!

It is of prime importance to know at what depth $\underline{d}$ under the foundation slab this layer is situated because - apart from eventual small shearing forces in the piles which may be acting at the points of the greatest deflection - the total earth pressure of the layers above must be resisted entirely by the superstructure.

Between this plane at depth $\underline{d}$ and the bottom of the foundation slab, the soil layers will be able to transmit shearing forces. At the bottom of the foundation slab these will be very small, either because the soil may be free from the foundation slab, or because of the low vertical effective stress in the soil with the resulting very small shearing strength.

The diagram of the shearing forces with the depth can be approximated by a parabolic curve, the ends of which, above and below, are determined by the prevailing conditions viz. below by the gradually inclining pile (for calculation purposes all piles are assumed vertical), above by the gradually increasing power to transmit shearing forces. The diagram of the pile loads is then triangular (fig. 3) and it diminishes from the value 2T/d above, to zero at depth $\underline{d}$, where T represents the total horizontal reaction of all the piles at the top, in as much as it is needed for the equilibrium of the soil mass. The bending moment M_{min} of the piles at the fixed end at the top, is then 1/4.Td and at depth d (M_{max}) 1/12.Td and the corresponding deflection is $\frac{1}{40}\frac{T\,d^3}{EI_{piles}}$. (T and d are as yet unknown in this expression). In order to determine them, it is proposed to neglect at first the lateral compressibility or expansion of the soil and to consider exclusively deformations caused by shear. In that case all the piles will deflect an equal amount and consequently offer the same resistance, so that the reaction T at the top of the piles will be distributed evenly over all the piles.

One might proceed conveniently by assuming first that the soil mass containing the piles is free to move forward, as if all the piles were cut, and then to determine for this case the deformation caused by the pressure of the soil mass below 3.50 m - NAP at the back. To this end the various shearing forces have to be determined first. A difficulty here is the choice of the value for the passive earth pressure. However, it appears that this has not much effect on the result as the influence of the shearing forces diminishes sharply with the depth (greater shearing modulus). In this way a certain displacement of the layer at depth d is found. Now we introduce an imaginary counter force T with a corresponding couple, which, applied at the head of the piles, bend these back to their original position, and during this action not only the deflection of the piles above the layer concerned comes in, but also the pushing-backward-again of the soil layers. Thus a certain value of T may be found. However, while determining the displacements by shear, one should have taken T into account from the very beginning, because shear strains are not directly proportional to the forces but follow more complicated laws, and this precludes the application of the principle of superposition. From tests on undisturbed samples it is found that the shear can be approximately expressed by

$$\gamma = \alpha\left(\frac{\frac{\tau}{\sigma}}{\frac{\tau_{kr}}{\sigma}-\frac{\tau}{\sigma}}\right) \qquad \text{(fig 4)}$$

The results of this investigation may best be embodied in a table and afterwards illustrated by plotting the successive deformations of the layers at an estimated value of T on an exaggerated scale, in order to determine the deformation of the soil mass. The deformation must check with the deflection of the pile at this estimated value of T. If not, the same process must be repeated for another value of T.

DISTRIBUTION OF T OVER THE PILES.

Up till now we assumed the soil mass to be undeformable in horizontal sense and we have only dealt with the elastic relative deformations of the layers caused by shear. Under these conditions, and also when local rigid sandlayers act as a bracing between the piles, resulting in equal deflections, all piles would carry an equal share of the horizontal earth pressure.

This is not the case, if we take account of the fact that the soil, before the dredging in front of the sheet piling and before the filling up of the area at the back, was in a state of horizontal stress, which stress might be determined by field tests and which later on will decrease at the front and possibly increase at the back, resulting in definite expansions and contractions in a horizontal sense. Consequently the deflections of the piles will be unequal and therefore more dangerous for the piles that deflect most. This points to the necessity to try to evaluate the unevenness of the distribution.

If one now assumes that the dredging in front and the filling behind is done, but that the soil is still undeformable i.e. the coefficients A and B in the formules for the elasticity moduli of the soil for compression and expansion ($E_c = A.\sigma$ and $E_e = B\sigma$) are infinitely large, then the diagram of the horizontal earth pressures between the successive rows of piles (which here again are assumed vertical) from back to front will be a straight line and all the piles will show at depth d an equal deflection forward with respect to pile head and pile point.

If one assumes next that the soil suddenly acquires the deformations resulting from the changed conditions of stress, then the distances between the pile heads will remain constant owing to the foundation slab, and also between the pile points, because of their location in sandlayers with very large A. Any change in the distances of the pile points would for that matter cause only smaller stresses in the piles so that the fixing of heads and points is the most unfavourable assumption. The elastic lines of the various piles will now not be identical any more.

Next we shall try to determine which ratio will obtain for any pile between the deflections at the depth d for the case of equal deflections of all piles (W) and for the case of the real deflections ($W' = W + \Delta W$), or, between the original linearly assumed diagram of horizontal pressures and the real one, where deformation is taken into account. The difference of the deflections of two contiguous piles (dW') must check with the horizontal strain of the soil in between. Again, each deflection in particular depends on the pressure difference at both sides of the pile.

If we now consider the bays numbered x-1, x and x + 1 (fig. 5), then the pile x-1, x will show a deflection of

$$\frac{P_{x-1} - P_x}{C} . W \text{ , and pile x, x+1 : } \frac{P_x - P_{x+1}}{C} . W,$$

so that the difference of deflection will be

$$\frac{P_{x-1} - 2P_x + P_{x+1}}{C} \; W,$$

where C represents the pressure increment in the soil from pile to pile, so that a deflection W' will correspond with a pressure increment of $\frac{W'}{W}$. C and with a reaction of the pile head of $\frac{W'}{W}\,\frac{T}{\text{number of piles}}$

P_x is a function of x, the horizontal distance to a fixed origin in the plane at depth d. By means of the Mac Laurin series for P_{x+1} and P_{x-1}, a deflection difference of $d\,W' = \frac{f''(x)}{C} \cdot W$ is found. The differential equation of the force distribution, expressing that the compression is equal to the deflection difference, follows:

$$\frac{a}{A} \lg \frac{P_x}{P_o} = p_x''\,\frac{W}{C}$$

(compression = deflection difference) where a is the distance between the piles.

It will be sufficiently accurate, if the range of p_x is not too great, to assume a constant E, which however must have 2 different values viz. for the area where pressure increase is expected (at the back) and for the area where pressure decrease takes place. The typical stress-strain diagram of a soil sample (fig. 6) may illustrate this.

The differential equation for the area of pressure increase is then:

$$a \frac{p_x - p_o}{E_c} = p''_x \frac{W}{C}$$

and for the area of pressure decrease

$$a \frac{p_x - p_o}{E_e} = p''_x \frac{W}{C}$$

Solved:

$$p_x - p_o = C_1 e^{bx} + C_2 e^{-bx}$$

which gives after differentiating twice for x

$$(p_x - p_o)'' = b^2 (C_1 e^{bx} - C_2 e^{-bx}) = b^2(p_x - p_o)$$

where b^2 represents $\frac{C.a}{W.E}$

If we choose the zero point for x at a point of constant pressure, then $0 = C_1 + C_2$ or $C_1 = - C_2$.

The pressure in front must be 0,6 ton/m^2 and at the back 5,7 ton/m^2. Therefore (fig. 7)

$$-0.44 = c.e^{b_v x_v} - c.e^{-b_v x_v}$$

$$+0.07 = c.e^{b_a x_a} - c.e^{-b_a x_a}$$

Dividing the former by the latter:

$$-6.3 = \frac{e^{b_v x_v} - e^{-b_v x_v}}{e^{b_a x_a} - e^{-b_a x_a}}$$

which condition is satisfied by choosing the origin in the second bay from the back:

$$x_a = -2 \qquad x_v = 13$$

Since the result does not check exactly, we must use the values c_a and c_v, which will not differ much.

c_v follows from s

$$-0.44 = c_v\left(e^{13\sqrt{\frac{Ca}{WE_e}}} - e^{-13\sqrt{\frac{Ca}{WE_e}}}\right)$$

c_a follows from

$$0.07 = c_a\left(e^{-2\sqrt{\frac{Ca}{WE_e}}} - e^{+2\sqrt{\frac{Ca}{WE_e}}}\right)$$

Now the state of stress at any point in the soil can be computed and plotted on a diagram. It is obvious that the computation is most essential in the outside bays. It appears from the calculation that the stresses in the bay behind the sheet piling, rise so high that the stresses are not determined by equilibrium conditions, because they exceed 1 ton/m^2.

This same computation can be carried through also in the case of piles of which the heads are not wholly fixed in the foundation slab.

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SUB-SECTION Vc

EARTH PRESSURE AGAINST UNDERGROUND CONSTRUCTIONS

Vc 1 EXPERIENCE WITH FLEXIBLE CULVERTS THROUGH RAILROAD EMBANKMENTS

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INTRODUCTION.

In about 1926, installation of large-diameter flexible steel culverts was initiated on the Denver & Rio Grande Western Railroad in Colorado, Utah and New Mexico. About thirty such culverts ranging in diameter from 7.5 to 15 feet were placed beneath fills varying in depth from 2 to 50 feet. The behavior of these culverts has been closely observed since that time, deflection measurements have been made on a number of the structures, and measurements made in detail on two that were subjected to extreme conditions of backfilling. This paper describes the results of the observations.

GENERAL DISCUSSION.

The culverts consist of corrugated steel or iron plates bent to a circular shape before delivery, and assembled into a cylindrical unit in the field. The thickness of the steel varies from .1719 inches to .2812 inches. The