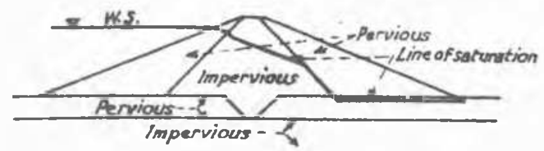
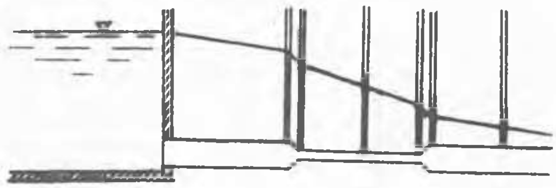


Percolation of water through earthdam



Flow of water through a pipe line of changing diameter.

FIG.35

putting $K = K_T K_S K_P K_V$ 9), and finally rewriting Darcy's formula in the following manner:

$V = KI = K_T\ K_S\ K_P\ K_V\ (J \pm J_{capillary})$.

SUMMARY

1) There are two main schools with regard to the classification of capillary water: 1) that of Jurin, relegating it to moleculary water; 2) that of Laplace, relegating it to gravitational water.
Which is preferable?

2) The static conditions are of considerable influence in any case of ground-water movement.

3) There are basic misconceptions introduced by Slichter into the theory of the movement of ground-waters.

4) Piezometric gradient, determined by observation on W.Ls of wells lowered to any arbitrary level does not represent the curve of depression.

5) Ground-water movement, even in case of a phreatic surface proceeds under conditions of compulsion, which are identical with the flow of water under pressure in pipes.

BIBLIOGRAPHY.

1) LEBEDEFF, A.,Soil & Ground-Waters, U.S.S.R. 1936 (Russian).
2) ISRAELSEN, O.,Irrigation Principles & Practices, New-York, 1945.
3) TERZAGHI, K.,Soil Moisture Hydrology, McGraw-Hill Book Co., Inc., 1942.
4) JUKOVSKY, N.,Filtration of water through dams, Moscow (Russian).
5) WYCKOFF, R.D; BOTSET, H.G; MUSKAT, M; Flow of liquids through porous media under the action of gravity.Physics, U.S.A., 1932.
6) SLICHTER, Ch.,Theoretical investigation on the motion of groundwaters, U.S. Geol. Survey, 1898.
7) HAZEN, A.,Some Physical Properties of sands and gravels, Mass. State, U.S.A., 1892/93.
8) CHERTOUSOFF, M.D.,Special course on Hydraulics, Leningrad-Moscow, 1937 (Russian).
9) IRMAY, S.,Sur le mouvement des eaux dans le sol.Revue Universelle des Mines (9e serie, tome III, N° 4), Liège, 1947.

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X a 2

ON THE PERMEABILITY OF HOMOGENOUS ANISOTROPIC SOILS

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SYNOPSIS

The permeability characteristics of homogenous anisotropic soils are discussed in this paper. It is shown that as a consequence of Darcy's law, the variation with respect to direction of the coefficients of permeability normal and tangential to an equipotential line can be represented by a circle similar to Mohr's circle of stress.

1. INTRODUCTION.

For two-dimensional problems of flow of water through homogenous anisotropic soils, suppose that the governing equations can be expressed by the following generalized form of Darcy's Law:

$$V_x = -K_1 \frac{\partial h}{\partial x} \qquad V_y = -K_2 \frac{\partial h}{\partial y} \qquad (1)$$

where v_x and v_y are components of the discharge velocity of water along two perpendicular directions x and y definitely oriented in the soil, h is the total head existing at a point, and k_1 and k_2 are the coefficients of permeability along the x and y directions respectively.

Equations (1) imply that the velocity of flow along each of the two directions x and y depends only on the hydraulic gradient in that direction. These two perpendicular directions will be called the principal directions of flow. k_1 and k_2 will be referred to as the principal coefficient of permeability.

2. COEFFICIENTS OF PERMEABILITY NORMAL AND TANGENTIAL TO AN EQUIPOTENTIAL LINE.

In Figure 1, O represents a point in a mass of homogeneous anisotropic soil; x and y are axes along the principal directions of flow. The direction of the maximum hydraulic gradient is indicated by n making an angle α with the x-axis. The equipotential line e is perpendicular to the direction n. The resultant velocity v is directed along a direction s making an angle δ with n.

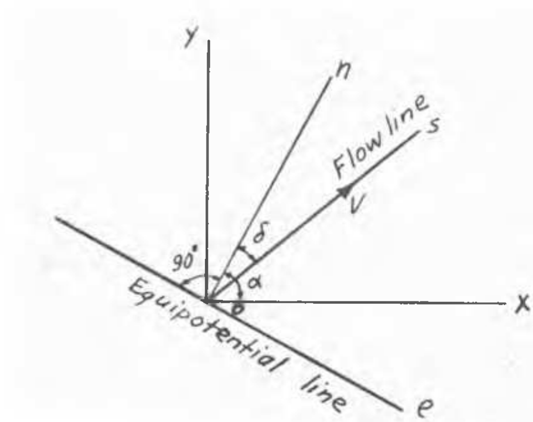

FIG.1

Along the direction n, the hydraulic gradient $\frac{dh}{dn}$ is a maximum and

$$\frac{\partial h}{\partial x} = \frac{dh}{dn}\cos\alpha \qquad \frac{\partial h}{\partial y} = \frac{dh}{dn}\sin\alpha \tag{2}$$

The component v_n of the velocity v along n is

$$V_n = v\cos\delta = v_x\cos\alpha + v_y\sin\alpha$$

Using Equations (1) and (2) one obtains

$$v_n = -K_1\frac{\partial h}{\partial x}\cos\alpha - K_2\frac{\partial h}{\partial y}\sin\alpha$$

$$= -\left(K_1\cos^2\alpha + K_2\sin^2\alpha\right)\frac{dh}{dn} \tag{3}$$

The coefficient of permeability k normal to the equipotential line e will be defined by the following equation

$$v_n = -K\frac{dh}{dn} \tag{4}$$

k thus represents the component of the discharge velocity normal to the equipotential line when the maximum hydraulic gradient is unity. With this definition, Equation (3) yields the following result:

$$K = K_1\cos^2\alpha + K_2\sin^2\alpha$$

$$= \frac{1}{2}\left(K_1+K_2\right) + \frac{1}{2}\left(K_1-K_2\right)\cos 2\alpha \tag{5}$$

where α is the angle the direction n makes with the x-axis along which $k = k_1$.

The component v_e of the velocity v along the equipotential line is

$$v_e = v\sin\delta = v_x\sin\alpha - v_y\cos\alpha$$

$$= -K_1\frac{\partial h}{\partial x}\sin\alpha + K_2\frac{\partial h}{\partial y}\cos\alpha$$

$$= -\left[\left(K_1-K_2\right)\sin\alpha\cos\alpha\right]\frac{dh}{dn} \tag{6}$$

The coefficient of permeability λ, along the equipotential line e may be defined by the following equation:

$$v_e = -\lambda\frac{dh}{dn} \tag{7}$$

λ thus represents the component of the discharge velocity along the equipotential line when the maximum hydraulic gradient is unity. With this definition, Equation (6) leads to:

$$\lambda = -\frac{1}{2}\left(K_1-K_2\right)\sin 2\alpha \tag{8}$$

Equations (5) and (8) show that the variation of k and λ with respect to α is the same as the variation of normal and shearing stresses at a point as given in mechanics of materials 1). Mohr's circle can therefore be drawn for k and λ to show their variation with respect to α.

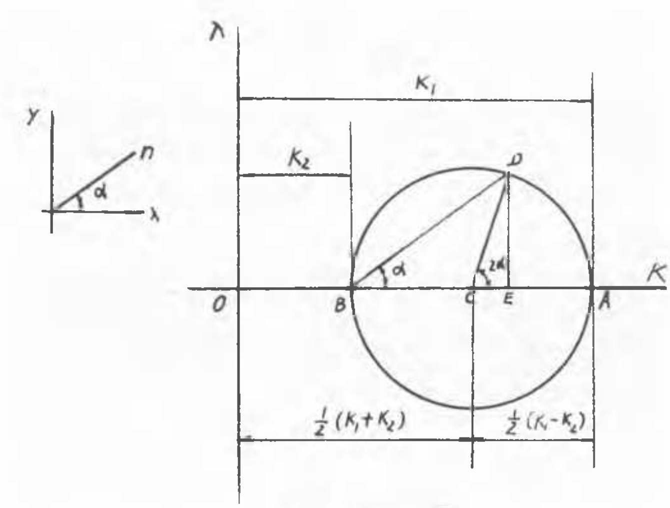

FIG.2

As shown in Figure 2, lay off $OA = k_1$, $OB = k_2$. With AB as a diameter construct a circle. For any direction n of the maximum hydraulic gradient making an angle α with the x-axis, draw the radius CD making an angle 2α with the abscissa. Then the coordinates of D represent k and λ. That is, OE = k, DE = λ. Similar to the circle for stress, for each circle there is a pole. 2). In Figure 2, B is the pole of the circle. For any direction N, one can simply draw a line through the pole B parallel to n to intersect the circle at D. The coordinates of D are the corresponding values of k and λ.

The quantity k as defined here agrees with the coefficient of permeability for isotropic soils defined conventionally as the factor by which the gradient in a given direction is to be multiplied in order to get the velocity in that direction. The quantity N, however, has a different meaning. It is the factor by which the maximum gradient is to be multiplied in order to give the velocity component in a direction perpendicular to the maximum gradient. The physical significance of k and N can be visualized in the following way. If a maximum gradient of unity occurs in an arbitrarity direction α, the corresponding values of k and N obtained from the circle of Fig. 2 represent respectively, the velocity component along the direction α and the velocity component perpendicular to that direction.

3. COEFFICIENTS OF PERMEABILITY TANGENTIAL AND NORMAL TO A FLOW LINE.

Denoting by ρ the reciprocal of the coefficient permeability, one can write Equations (1) as follows:

$$v_x = -\frac{1}{\rho_1}\frac{\partial h}{\partial x} \qquad v_y = -\frac{1}{\rho_2}\frac{\partial h}{\partial y} \tag{9}$$

where

$$\rho_1 = \frac{1}{K_1} \qquad \rho_2 = \frac{1}{K_2} \tag{10}$$

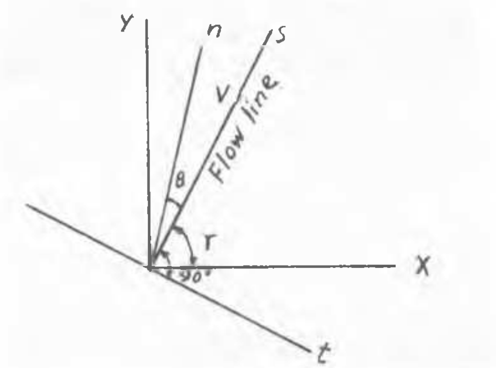

FIG.3

In Figure 3, the direction of the flow line s is assumed to be at an arbitrary angle γ with the x-direction. The direction of the maximum hydraulic gradient is represented by n, making an angle θ with the flow line. The hydraulic gradient along the flow line is

$$\frac{dh}{ds} = \frac{dh}{dn}\cos\theta = \frac{\partial h}{\partial x}\cos\gamma + \frac{\partial h}{\partial y}\sin\gamma$$

$$= -\left(\rho_1\cos^2\gamma + \rho_2\sin^2\gamma\right)v \tag{11}$$

The coefficient of permeability $1/\rho$ along the flow line will be defined by

$$v = -\frac{1}{\rho}\frac{dh}{ds} \tag{12}$$

Thus ρ represents the hydraulic gradient along the flow line when the resultant velocity is unity. Equation (11) then gives

$$\rho = \rho_1\cos^2\gamma + \rho_2\sin^2\gamma$$

$$= \frac{1}{2}(\rho_1+\rho_2) + \frac{1}{2}(\rho_1-\rho_2)\cos 2\gamma \tag{13}$$

Along the direction t normal to the flow line, the hydraulic gradient is

$$\frac{dh}{dt} = -\frac{dh}{dn}\sin\theta = \frac{\partial h}{\partial x}\sin\gamma - \frac{\partial h}{\partial y}\cos\gamma$$

$$= -\left[(\rho_1-\rho_2)\sin\gamma\cos\gamma\right]v \tag{14}$$

The coefficient of permeability $1/\omega$ along the direction t will be defined by

$$v = -\frac{1}{\omega}\frac{dh}{dt} \tag{15}$$

Thus ω represents the hydraulic gradient normal to the flow line when the resultant velocity is unity. Equation (14) gives therefore

$$\omega = (\rho_1-\rho_2)\sin\gamma\cos\gamma$$

$$= \frac{1}{2}(\rho_1-\rho_2)\sin 2\gamma \tag{16}$$

As shown by Equations (13) and (16), the variation of ρ and ω with respect to γ can also be represented by a circle, similar to the circle for k and λ. It is interesting to note that Equation (13) has been demonstrated experimentally by R. Dachler 3) as to be valid for stratified soils.

The quantities $1/\rho$ and $1/\omega$ introduced in this section are factors relating to a flow line. They are different from the quantities k and N which are related to an equipotential line. It is true only for the two principal directions that $1/\rho$ becomes equal to **k** while ω and N are equal to zero. If a resultant velocity of unity occurs along an arbitrary direction γ, the corresponding values of ρ and ω determined from a circle represent respectively, the gradient along the direction γ and the gradient perpendicular to that direction.

4. DETERMINATION OF PRINCIPAL DIRECTIONS AND PRINCIPAL COEFFICIENTS.

Many statements regarding stresses or strains and obtainable from Mohr's circle apply equally well in the present cases. For example, along any two perpendicular directions the sum of the coefficients of permeability k is a constant,

$$K_\alpha + K_{\alpha+\frac{\pi}{2}} = K_1 + K_2 \tag{17}$$

Similarly, along any two perpendicular directions γ and $\gamma+\frac{\pi}{2}$, one has

$$\rho_\gamma + \rho_{\gamma+\frac{\pi}{2}} = \rho_1 + \rho_2 = \frac{1}{K_1} + \frac{1}{K_2} \tag{18}$$

Whenever the values of k (or of ρ) along three arbitrary directions are known, it is immediately possible to construct the k-λ circle (or the ρ-ω circle) and the principal coefficients of permeability k_1 and k_2 together

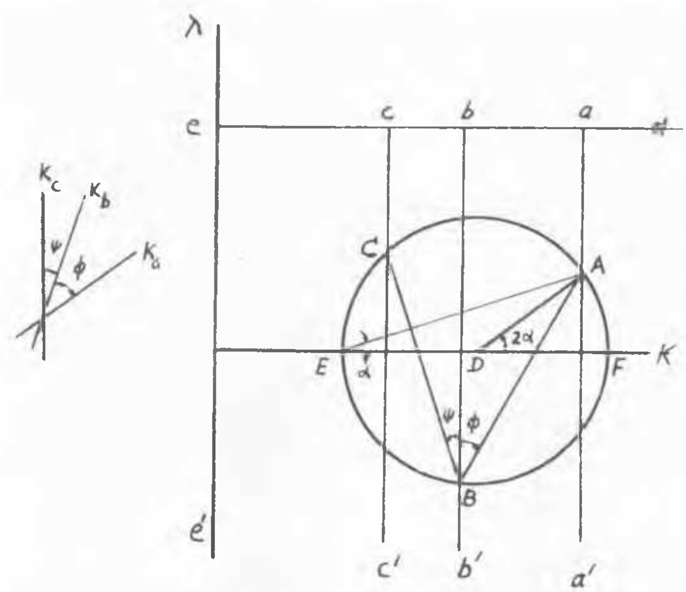

FIG.4

with the orientation of the principal directions are determined. The method of construction is illustrated below for the k-λ circle. The procedure was suggested by G. Murphy 4) in connection with the determination of principal strains from normal strains.

Suppose that k_a, k_b, and k_c along three directions as shown in Figure 4 are known. φ and ψ are known angles. On a line ed, lay off $\overline{ea} = k_a$, $\overline{eb} = k_b$, $\overline{ec} = k_c$. Draw perpendicular lines ee', aa', bb', cc'. From any point B on line bb' draw lines making φ and ψ respectively with bb' to intersect aa' at A and cc' at C. Determine the point of intersection D of the perpendicular bisectors of BC and BA. With D as center, draw a circle passing through A, B, and C. This is the k-λ circle desired. Through D draw a line parallel to ed to intersect ee' at O. Then O is the origin and OD is the k-axis. The principal coefficients of permeability are given by $OF = k_1$, $OE = k_2$. The principal direction along which $k = k_1$ makes an angle α with the direction of k_a. The angle α is given by angle AEF.

REFERENCES.

1) Timoshenko, S. "Theory of Elasticity", McGraw-Hill Book Co., Inc., 1934, P.16.
2) Terzaghi, K., "Theoretical Soil Mechanics", John Wiley and Sons, Inc., 1943, p. 18.
3) Dachler, R. "Über Sickerwasserstromungen in Geschichtetem Material", Die Wasserwirtschaft, No. 2, 1933, pp. 13-16.
4) Murphy, G., "A Graphical Method for the Evaluation of Principal Strains from Normal Strains", Journal of Applied Mechanics, December 1945.

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NOTES ON GROUND WATER LOWERING BY MEANS OF FILTER WELLS

R. GLOSSOP and V.H. COLLINGRIDGE

Ground water lowering, or the method of draining an excavation with bored wells, has long been known and used in the sandy coastal regions of north-western Europe, but it is only in the last 10 or 15 years that is has been adopted in the United States and in Britain. This is surprising for it is in many ways superior to direct pumping from the excavation itself, chiefly because the flow of water is away from the excavation and towards the wells, and thus piping cannot occur.

Two advances in technique have led to its increased use, they are the development of submersible electric pumps in Germany, these simplify the problem of pumping from deep wells; and the invention of the jetted "well point" in the United States, which by standardising plant gives an easy, but by no means fool proof method for the installation of a shallow well system.

Both these advances have provoked the study of the theory of groups of wells, for in estimating for deep well installations it is essential to know the number and depth of the wells that will be needed, and in planning excavation by well points it is at least necessary to know whether they will be successful or not. The subject has a voluminous literature to which the comparatively recent contributions of Sichardt and Weber are perhaps the most important. The present need is to check their theories against practice and thus to sort out non-essential factors and establish the formulae necessary to plan installations with sufficient accuracy to ensure their success. In Table I are listed details of twelve ground water lowering contracts carried out by the authors and their colleagues and it is hoped that the following notes on them may contribute to this end.

THE CAPACITY OF GROUPS OF WELLS

Given the following simplifying assumptions; that flow in the ground is laminar, that the water bearing stratum is of unlimited horizontal extent and is homogeneous, that there is uniform inflow from all directions and that the aquifer is underlain by an impermeable stratum to which the wells are sunk, then the following formula can be derived for the yield of a group of wells.

$$Q = \frac{\pi K(H^2 - h^2)}{\log_e R/\sqrt[n]{r_1 \cdot r_2 \ldots r_n}} \qquad (1)$$

for free water table conditions, or

for artesian conditions, where

$$Q = \frac{\pi K(H - h)2m}{\log_e R/\sqrt[n]{r_1 \cdot r_2 \ldots r_n}} \qquad (2)$$

Q is the yield of the well
K is the coefficient of permeability of the ground
H the depth from normal water level to the impermeable stratum
h the depth from the depressed water table to the impermeable layer
R the range, or distance from the centre of the area of depression to points at which no appreciable lowering of the water table has occurred
n is the number of wells
m the thickness of the aquifar for artesian conditions
r_1 etc., the distance from the centre of the area of depression to a well.
r_o is radius of a well
h_o is head of water outside the well.
k_m coefficient of permeability in cm per second
k_f coefficient of permeability in ft per second.

$\sqrt[n]{r_1 \cdot r_2 \ldots r_n}$ can be written as A and is the radius of the circle equivalent in area to the well layout.

The depth h_x at any point on the depressed water table can be obtained by substituting