with the orientation of the principal directions are determined. The method of construction is illustrated below for the k-λ circle. The procedure was suggested by G. Murphy 4) in connection with the determination of principal strains from normal strains.

Suppose that k_a, k_b, and k_c along three directions as shown in Figure 4 are known. ϕ and ψ are known angles. On a line ed, lay off $\overline{ea} = k_a$, $\overline{eb} = k_b$, $\overline{ec} = k_c$. Draw perpendicular lines ee', aa', bb', cc'. From any point B on line bb' draw lines making ϕ and ψ respectively with bb' to intersect aa' at A and cc' at C. Determine the point of intersection D of the perpendicular bisectors of BC and BA. With D as center, draw a circle passing through A, B, and C. This is the k-λ circle desired. Through D draw a line parallel to ed to intersect ee' at O. Then O is the origin and OD is the k-axis. The principal coefficients of permeability are given by $OF = k_1$, $OE = k_2$. The principal direction along which $k = k_1$ makes an angle α with the direction of k_a. The angle α is given by angle AEF.

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X a 3

NOTES ON GROUND WATER LOWERING BY MEANS OF FILTER WELLS

R. GLOSSOP and V.H. COLLINGRIDGE

Ground water lowering, or the method of draining an excavation with bored wells, has long been known and used in the sandy coastal regions of north-western Europe, but it is only in the last 10 or 15 years that is has been adopted in the United States and in Britain. This is surprising for it is in many ways superior to direct pumping from the excavation itself, chiefly because the flow of water is away from the excavation and towards the wells, and thus piping cannot occur.

Two advances in technique have led to its increased use, they are the development of submersible electric pumps in Germany, these simplify the problem of pumping from deep wells; and the invention of the jetted "well point" in the United States, which by standardising plant gives an easy, but by no means fool proof method for the installation of a shallow well system.

Both these advances have provoked the study of the theory of groups of wells, for in estimating for deep well installations it is essential to know the number and depth of the wells that will be needed, and in planning excavation by well points it is at least necessary to know whether they will be successful or not. The subject has a voluminous literature to which the comparatively recent contributions of Sichardt and Weber are perhaps the most important. The present need is to check their theories against practice and thus to sort out non-essential factors and establish the formulae necessary to plan installations with sufficient accuracy to ensure their success. In Table I are listed details of twelve ground water lowering contracts carried out by the authors and their colleagues and it is hoped that the following notes on them may contribute to this end.

THE CAPACITY OF GROUPS OF WELLS

Given the following simplifying assumptions; that flow in the ground is laminar, that the water bearing stratum is of unlimited horizontal extent and is homogeneous, that there is uniform inflow from all directions and that the aquifer is underlain by an impermeable stratum to which the wells are sunk, then the following formula can be derived for the yield of a group of wells.

$$Q = \frac{\pi K(H^2 - h^2)}{\log_e R/\sqrt[n]{r_1 \cdot r_2 \cdots r_n}} \qquad (1)$$

for free water table conditions, or

for artesian conditions, where

$$Q = \frac{\pi K(H - h)2m}{\log_e R/\sqrt[n]{r_1 \cdot r_2 \cdots r_n}} \qquad (2)$$

Q is the yield of the well
K is the coefficient of permeability of the ground
H the depth from normal water level to the impermeable stratum
h the depth from the depressed water table to the impermeable layer
R the range, or distance from the centre of the area of depression to points at which no appreciable lowering of the water table has occurred
n is the number of wells
m the thickness of the aquifar for artesian conditions
r_1 etc., the distance from the centre of the area of depression to a well.
r_o is radius of a well
h_o is head of water outside the well.
k_m coefficient of permeability in cm per second
k_f coefficient of permeability in ft per second.

$\sqrt[n]{r_1 \cdot r_2 \cdots r_n}$ can be written as A and is the radius of the circle equivalent in area to the well layout.

The depth h_x at any point on the depressed water table can be obtained by substituting the expression $\sqrt[n]{x_1 \cdot x_2 \ldots . x_n}$

$$h_x = H^2 - \frac{Q \log_e R/\sqrt[n]{x_1 \cdot x_2 \ldots x_n}}{\pi K} \qquad (3)$$

This formula is obviously true for only a limited degree of lowering since in the formula Q varies with h, but in fact as h decreases a point is reached when the amount of water entering a well is limited by the depth of immersion of the well. This has led Sichardt to introduce the concept of "capacity". He finds from a study of actual water lowering installations that there is a limiting value to the hydraulic gradient in the immediate neighbourhood of the well and that this limiting value, i_{max}, is related to the permeability of the ground, so that in metric units:

$$i_{max} = \frac{1}{15\sqrt{K}} \qquad (4)$$

thus the capacity, or maximum possible yield of a single well is given by the formula

$$Q_{max} = 2\pi r_o h_o \frac{\sqrt{K}}{15} \qquad (5)$$

In Table I the slope of the water table at entry to the well has been computed from the measured yield, and from the immersed surface area of the wells.

Values of i_o are plotted on Figures 1 against the corresponding values of K as logarithms. Examples Nos. 2, 3, 4, 6, 7, 9, 10 and 11 all lie on, or close to a line representing the equation

$$\log i_o = -\tfrac{1}{2} \log K - 0.92$$

or

$$i_o = \frac{1}{8.3\sqrt{K}} \quad \text{where K is in feet/sec.}$$

This is in very close agreement with Sichardt's conclusion that

$$i_o = \frac{1}{15\sqrt{K}} \quad \text{where K is in meter/sec.}$$

The two exceptions, cases Nos. 1 and 8 can be explained thus. Although in both instances the water table was depressed sufficiently for the purpose in hand, in the case of No. 8 the length of the header pipe was such that the leakage of air through the many pipe joints proved greater than the capacity of the vacuum pump running in conjunction with the pumping system. In case No. 1 the delivery of the pump was at a maximum, so that in both installations the suction limit of the pumping equipment was reached before the wells were drawn down to their ultimate capacity.

PERMEABILITY.

The first step in planning an installation of wells is to arrive at a figure for K, the average permeability of the soil. The best method is to carry out a pumping test on a well, but time is rarely allowed for this and the Authors' usual practice is to make mechanical analyses of samples from borings and use the formula suggested by Professor Skempton, that

$$K = d_{10}{}^2$$

where K = permeability in cm/sec.
d_{10} = effective (10%) size in mm.

An examination of Table I shows that within a fairly wide range of circumstances the arrangements made for dewatering were successful, and exception to this was in case No. 4 where the required lowering was not obtained due to an unexpectedly low K value, more wells were then added (case 5). In this instance, which may serve as a warning, preliminary investigations had not been made.

Six out of the twelve cases are of work carried out in the Thames Valley gravels, (Nos. 1, 2, 3, 8, 10, 12) these are well graded sandy gravels, and the 10% size is remarkably constant at about 0.25 mm throughout any stratum, except at the base where a coarse layer is usually found.

Nos. 1, 2, 8, 11 and 12 are in a comparatively thick gravel deposit, whereas Nos. 3 and

TABLE I

	SITE	GROUND WATER CONDITIONS	DESCRIPTION OF SITE AND LOCAL CONDITIONS	GROUND	WELL TYPE	H (feet)	h (feet)	m (feet)	h_o (feet)	R (feet)	A (feet)	r_o (feet)	n	Q ft³/sec	k_m cm/sec	k_f ft/sec	i_o	log k_f	log i_o	REMARKS
1.	BATTERSEA	WATER TABLE	160'×42' RECTANGULAR PLAN, ADJACENT AND BELOW RIVER LEVEL.	THAMES GRAVEL	SBW	23	12	-	11.7 G	736 C	42	0.54	12	0.305	0.068	0.002	0.86	-2.6576	-0.0635	i_{max}: NOT ATTAINED DRAW-DOWN IN WELL LIMITED BY PUMP SUCTION.
2.	BECKTON	ARTESIAN	ROUGHLY CIRCULAR, WELLS EQUALLY SPACED. THICKNESS OF AQUIFER VARIES 7' TO 25'. 80% PENETRATION.	THAMES GRAVEL	DBW	30	20	17.5 AVERAGE	17.5 AVERAGE	1700 C	181	0.75	5	1.305	0.060	0.002	2.21	-2.6990	0.3454	YIELD OF ONE WELL VERY SMALL, CHOKED BY FERRIC HYDROXIDE.
3.	BENTALLS, KINGSTON	WATER TABLE	110'×80' RECTANGULAR PLAN, 400' AWAY FROM RIVER.	THAMES GRAVEL	SBW	12.5	1.0	-	2.0 E	332 C	66	0.54	15	0.447	0.118	0.004	[illegible]	-2.3979	0.2768	WELLS PUMPING TO FULL CAPACITY.
4.	CAMBERLEY I	WATER TABLE	40'×30' L-SHAPED, WELLS IRREGULARLY SPACED. SMALL RIVER 300' DISTANT.	RIVER GRAVEL OVER BAGSHOT SAND.	SBW	14	4	-	1.1 G	287 C	20	0.67	5	0.080	0.020	0.0007	4.82	-3.1549	[illegible]	WELLS PUMPING TO FULL CAPACITY.
5.	CAMBERLEY II	WATER TABLE	SAME SITE AS ABOVE WITH ADDITION OF SMALL DIAMETER WELLS.	AS ABOVE	SBW	11	1.5	-	0.6 E	444 C	24	0.67 0.20	3 12	0.122	0.028	–	–	–	–	YIELD INCLUDED WATER FROM A TILE-DRAIN LAID WITHIN THE EXCAVATION.
6.	GRIMSBY	ARTESIAN	200'×140' CRESCENT SHAPED ARTESIAN STRATUM THINNING TO ZERO ON ONE SIDE.	SAND AND GRAVEL	DBW	74	58	10 AVERAGE	7.5 AVERAGE	1080 C	36	0.87	6	0.433	0.036	0.0012	3.04	-2.9208	0.4833	CONCLUSIONS NOT VERY RELIABLE DUE TO EXTREME VARIATIONS IN AQUIFER.
7.	OLD KENT ROAD	WATER TABLE	164' DIAMETER CIRCLE, WELLS SPACED UNIFORMLY.	THANET SAND	SBW	26	13	-	3.8 G	220 C	82	0.54	8	0.067	0.0012	0.00004	16.26	-4.3979	1.2108	SOUNDINGS TAKEN IN WELLS GIVE h_o = 2' TO 3'. COMPARE WITH h_o = 3.8'.
8.	RUSSIA YARD, SURREY DOCKS	WATER TABLE	290'×270' RECTANGULAR SITE BOUNDED ON OPPOSITE SIDES BY DOCK WALLS. WATER LEVEL IN DOCK 7.5' ABOVE GROUND-WATER LEVEL.	THAMES GRAVEL	SBW	27	12	-	8.1 G	1066 C	156	0.67	20	1.126	0.078	0.0026	1.28	-2.5850	[illegible]	i_{max}: NOT ATTAINED DRAW-DOWN IN WELLS LIMITED BY SUCTION LIFT.
9.	SOUTHAMPTON	ARTESIAN	1000'×360' RECTANGULAR SITE. AQUIFER IN THREE HORIZONS TOTALLING 28' OUT OF 48', REMAINDER CLAY.	BRACKLESHAM SANDS	DBW	180	36	28	28 G	1065 C	350	1.0	8	1.868	0.006	0.0002	6.63	-3.6990	0.8215	h_o IS IMMERSED DEPTH, NOT HEAD OF WATER AT WELL.
10.	UXBRIDGE	WATER TABLE	180'×80' RECTANGULAR SITE, GRAVEL VERY COARSE AT BOTTOM, CANAL 100' DISTANT.	RIVER GRAVEL	SBW	14	3	-	5.0 E	1610 C	68	0.56	13	1.830	0.218	0.0073	1.36	-2.1367	0.1335	WELLS AT FULL CAPACITY.
11.	PLAISTOW WHARF I	ARTESIAN	60'×40' RECTANGULAR PLAN 37% PENETRATION OF AQUIFER.	THAMES GRAVEL UNDER SOFT CLAY	JWP	37	30	23	8.5 I	C 600 O 500	28	0.10	28	0.67	0.056	0.0017	2.59	-2.7696	0.4133	8.7 GALLONS/MINUTE PER WELL POINT.
12.	PLAISTOW WHARF II	ARTESIAN	SAME SITE EXTENDED TO 60'×120' IN PLAN.	AS ABOVE	JWP	37	28	23	8.8 I	C 600 O 800	48	0.10	48	1.33	0.067	0.0022	2.36	-2.6576	0.3729	10.4 GALLONS/MINUTE PER WELL POINT.

CODE FOR WELL TYPE:

SBW = SHALLOW BORED WELL WITH SUCTION LIFT
DBW = DEEP BORED WELL WITH SUBMERSIBLE PUMP
JWP = JETTED WELL-POINTS

G. CALCULATED FROM GEOMETRY OF LAY-OUT, $h_o = \sqrt{H^2 - \frac{Q \log_e R/A}{\pi k}}$
E. ESTIMATED BY OBSERVATION
I ACTUAL IMMERSED DEPTH
C CALCULATED FROM $R = 300(H-h)\sqrt{k_{cm/sec}}$, k BEING 1st APPROXIMATION
O. OBSERVED FROM CONTROL WELLS

$$k = \frac{Q \log_e R/A}{\pi (H^2 - h^2)} \quad \text{OR} \quad = \frac{Q \log_e R/A}{2\pi m (H-h)}$$

$$i_o = \frac{Q}{2\pi r_o h_o n k} \qquad A = \sqrt[n]{r_1 r_2 r_3 \ldots r_n}$$

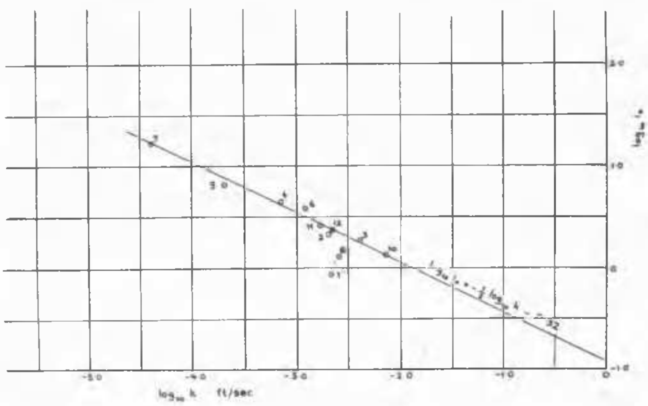

FIG.1

10 are shallow, and in these two cases the water level was drawn down into the lower beds. In the former group the values of K as calculated from the output of the wells was found to be:-

1)	Battersea	K=0.065 cm/sec.
2)	Beckton	K=0.060 cm/sec.
8)	Russia Yard	K=0.079 cm/sec.
11)	Plaistow Wharf (1)	K=0.055 cm/sec.
12)	Plaistow Wharf (2)	K=0.067 cm/sec.

Mean Value for Thames Gravel = 0.065 cm/sec.

in the latter group where the water was drawn down into the coarse bed the values of K were:

3)	Kingston	K = 0.119 cm/sec.
10)	Uxbridge	K = 0,218 cm/sec.

RANGE OF INFLUENCE.

The value of R, the range of influence of a group of wells is the most difficult quantity to assess in water lowering calculations. In cases 11 and 12 the limits of appreciable lowering were at 500 feet and 800 feet respectively. In case 10 where pumping was carried on for many months the level of water in a market gardener's well 1,000 feet from the site and on the opposite side of a canal was lowered.

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SUB-SECTION Xb

SEEPAGE PROBLEMS OF DAMS AND LEVEES.

Xb1 CRITICAL HEAD FOR THE EXPANSION OF SAND ON THE DOWNSTREAM SIDE OF WEIRS

ZDENĚK BAŽANT JR.
Assoc. Professor of Foundations, University of Technical Sciences, Prague, Czechoslovakia

1. INTRODUCTION.

The purpose of the following investigations is to obtain reliable data from which we can derive rules for designing weir structures. These structures are in danger to be destroyed by water percolating through the sand beneath them, which leads to the formation of piping. The mechanics of piping were tested many times. The review of this problem can be found in Terzaghi, Theoretical soil mechanics (1943, p.257).

The author has studied experimentally and mathematically piping in his article, Grundbruch unter der Spundwand (Die Bautechnik 1940), which deals with piping under an impermeable diaphragm. Further tests concerning piping beneath weirs, were studied in his article Critical head for piping beneath weirs in the Proceedings of Congress of Large Dams, in Stockholm 1948. Now he presents the result of tests concerning the determination of critical head with respect to expansion of sand on the downstream side of weir which preceeds the piping. Tests were made in the foundation laboratory of the University of Technical Sciences of Prague.

2. NOTATION. (see fig. 1a)

h_p = critical head with respect to piping
h = head, expressing the difference of the levels upstream and downstream
h_e = head for the beginning of expansion of sand
D_1 = depth of foundation below upstream surface
D_2 = depth of foundation below downstream surface
$2B$ = width of foundation
e = void ratio
$N_p = \frac{h_e}{h_p}$

3. DESCRIPTION OF SOIL AND TESTING APPARATUS.

The sand used for tests had grain sizes from 0,2 to 0,5 mm. Sand originated from diluvial deposits of the river Vitava near