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principal settlement, the facts mentioned above under 4, make it possible essentially to reduce the laboratory equipment, since the number of oedometers required may be diminished proportionally to the reduction in the duration of the tests.

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THE FIRST LAW OF THERMODYNAMICS AND THE CONSOLIDATION PROCESS.

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INTRODUCTION

The first law of thermodynamics can be succesfully applied to the consolidation process, in order to evaluate the influence of loading speed on the amount of work to obtain a definite amount of consolidation. The influence of loading speed on the consolidation process has proved to be a practical problem of outstanding importance in soil mechanics.

One possibility of approaching this problem is the analytical method. Terzaghi's theory on consolidation 1) is an example of a mathematical method of analyzing the consolidation process. This theory was elaborated by Terzaghi and Fröhlich 2) to give the proper solution of the rate of settlement for a given relation between load and time. The findings of Buisman 3) a.o. in connection with secular time-effects (or secundary time-effects) complicated the consolidation properties of cohesive soils to such an extent, thet the application of the above-mentioned mathematical theories could not be maintained any longer. Buisman tried to give an approximative solution (I.c. par. 49, p. 119) based on the assumption of the soil constants, α_p resp. α_s expressing the direct and the secular effect of an instantaneous load on consolidation, and the retarding effect of hydrodynamic stresses, as expressed by a set of successive sinusoidal isochrones. One of the authors' principal objections to this point of view is the use of the secular effect α_s as a constant in the computation of settlement. So far no definite proof has been given of the existence of a secular effect of constant magnitude during the hydrodynamic phase of the consolidation process, though some effect must be supposed to be present. Investigations on the magnitude of α_s during the hydrodynamic phase are still in progress (see another report by the senior author for this Conference, entitled: "Consolidation characteristics of cohesive soils").

Meanwhile the authors tried to approach the subject by the application of a generally valid theory from an energetic point of view. The results of a series of consolidation tests with varying speeds of increasing loads gave the opportunity to study the effect of application of the first law of thermodynamics. Results of this study are given in the following report. Use has been made of this opportunity to prove the existence of the link between the above-mentioned process and the physico-chemical properties of soil. These latter are investigated by the authors simultaneously and will be the subject of later publications.

EXTERNAL WORK ON A SOIL SAMPLE.

The work done on a soil sample by a load P during a settlement dz is given by:

$$dW = Pdz$$

and in general, as found by integration:

$$W = \int_0^{z_t} P\,dz$$

Thus W can be easily obtained by graphical integration for any load-time relation. The work done on a sample, which is supposed to be saturated with water or a dilute solution is mainly used to overcome:

i. the side-friction between the sample and the ring in the consolidation apparatus.

ii. the internal frictional resistance during consolidation.

iii. a potential energy of the system because of physico-chemical properties of the sample.

In doing so we neglect in the first place the energy spent on compression of the solid particles in the soil and the water between them. Secondly kinetic effects because of consolidation speed will not be taken into account.

i. SIDEFRICTIONAL RESISTANCE.

Measurements of the pressures at the bottom of a consolidation sample gave an impression of the amount of work which is lost by sidefriction. An exact interpretation of the results of the consolidation test is possible only if the side-friction is decreased as much as possible by an efficient construction of the consolidation apparatus. If we call the heat evolved on overcoming sidefrictional resistance Q_f, the net work done on the sample will be given by $W_n = W - Q_f$.

ii. INTERNAL FRICTIONAL RESISTANCE.

Probably most of the net work done on the sample is used to overcome the internal frictional resistance. The internal frictional resistance is partly due to the displacement of the soil particles along each other, partly to the internal frictional resistance (viscosity) of the soil solution. If we accept the conception that the particles are completely surrounded by one or several layers of adsorbed molecules of water we may ascribe the internal friction practically to the viscosity of the soil solution only. The internal frictional energy is completely transformed into heat and transferred to the surroundings during an isotherm process. The experimental determination of the internal frictional energy seems to be a very laborious matter and would be possible only with a very sensitive isothermal calorimeter.

Influence of the velocity of settlement.

It is obvious that the velocity of settlement will have a great influence on the internal frictional resistance. In order to avoid a complicated, mathematical discussion which is of little use in explaining the results gained by our experiments, we will evaluate the relation between the velocity of settlement and the heat evolved by internal friction for a soil sample or a part of a soil sample of infinitesimal height.

Let Δh be the height of the sample, then the amount of energy dQ which is transformed into heat in the time dt, is equal to the product of the expelled volume of water dV and the difference of waterpressures at the boundaries Δp, so: $dQ = \Delta p . dV$ and as $\Delta p = \gamma . i . \Delta h$ where: γ = specific weight of the solution and i = hydraulic gradient, then

$$\frac{dQ}{dt} = \Delta p . \frac{dV}{dt} = i . \Delta h . \frac{dV}{dt} \qquad (1)$$

If we introduce the permeability k from the law of Darcy

$$k = \frac{1}{iF} \cdot \frac{dV}{dt} \qquad (2)$$

where F represents the cross-sectional area of the sample, then:

$$\frac{dQ}{dt} = \frac{\Delta h}{kF} \left(\frac{dV}{dt}\right)^2 = \frac{\Delta hF}{k} \left(\frac{dz}{dt}\right)^2 = \frac{\Delta V}{k} \left(\frac{dz}{dt}\right)^2 \quad (3)$$

Thus we find that the internal energy of friction is directly proportional to the square of the velocity of settlement. This highly important relation is analogous to the well known law of Poiseuille for the evolution of heat in a cylindrical capillary by laminary flow of a liquid. We get the law of Poiseuille, replacing the factor $\frac{\Delta V}{k}$ in formula (3) by

$\frac{8\eta l}{\pi r^4}$ η = viscosity of the liquid
l = length of the capillary
r = radius of the capillary.

Accordingly we have the law of Joule for the heat evolded in an electrical resistance R by an electrical current I:

$$\frac{dQ}{dt} = I^2R$$

The extension of the formula (1) for application to a sample of finitesimal height is complicated by the phenomenon of the hydrodynamic period and will not be discussed further.

iii. POTENTIAL ENERGY IN THE SAMPLE.

The fore-mentioned energy of internal friction Q is influenced to a great extent by the velocity of settlement and also because of the increase of the factor $\Delta V/k$ during the process, by the actual settlement z. It is important to know, whether there might also exist a form of energy, which is not influenced by the velocity of settlement, but a function of the settlement only. This energy would be a function of the density of the soil sample and be responsible for a swelling at a decrease of the load. It is a well known fact, that such an effect actually occurs in soil samples containing a large amount of very fine particles, such as colloidal clays and the fact can be explained in colloid chemistry by adsorption phenomena. The adsorption phenomena may be devided into two groups: those due to the physical adsorption of water and those due to the adsorption of ions. Each of these kinds of adsorption can be responsible for a potential-energy-effect which plays a role in the consolidation of soils. It is a well known fact that the great power of attraction of water for many clay minerals is responsible for the swelling power of these clays. Results of experimental investigations (not published) at the Delft Soil Mechanics Laboratory on sorption of water by clays, showed however that these attractive forces generally do not play an important role in offering a definite resistance against the outpressing of water. The amounts of physically adsorbed water, evaluated from sorption isotherms were found to be small as compared with the total amounts of pore water in the samples at the end of the consolidation tests. This is in accordance with the modern conception in physical chemistry of the dimensions of the layers of bound water surrounding clay particles which should be only one or a few layers of molecules thick. Besides the interaction between the solid particles and the watermolecules much attention has been paid to the effects resulting from the adsorption of ions.

By the adsorption of ions from the salt solution surrounding the clay particles, they will be charged electrically. This electrical charge is responsible for the formation of an electrical double layer around the particles, causing the electrokinetic - or ζ - potential which plays an important role in electrokinetical phenomena such as kataphoresis and electro-osmosis. Increasing density during consolidation will cause the double layers around each particle to penetrate into each other, leading to a repulsion between the particles which counteracts the Van der Waals-London attractive forces and prevents the flocculation of the clay. The evaluation of the double-layer-interaction in lyophobic colloids was done some time ago by Verwey and Overbeek 4). It was found that for a weak interaction between two flat plates of a substance of the silver-iodide type the repulsion energy could be written:

$$U_d = Ae^{-2xd}$$

The factors A and x are functions of the temperature, the concentrations and the electrical charges of the ions and d is the distance between the plates. The computation of this effect is not yet possible for a complicated system, like a poly-disperse clay suspension. It is however probably a factor of great importance, which could be investigated experimentally by consolidation tests on flocculated and peptizised samples of the same clay.

APPLICATION OF THE FIRST LAW OF THERMODYNAMICS.

The application of the first law of thermodynamics on a consolidation process would be exact only if the final state of the system was a state of equilibrium. This is essentially never the case, particularly not before the end of the hydrodynamic period. However, after the hydrodynamic period has ended, when the velocity of settlement has decreased to a low value, we will assume, that the system is passing a series of states of equilibrium. Hence, application of the first law gives the equation

$$W_n = Q + \Delta U \qquad (4)$$

where W_n, Q and ΔU refer to total quantities since the beginning of the process. The increase of potential energy ΔU of the sample is a function of its final state only, defined by the settlement z i.e. by the final density of the sample in this process. The total heat, evolved by internal friction Q is both a function of the settlement z and the velocity of settlement during the process. As we found in equation (3) the heat evolved by internal friction is at each moment proportional to the square of the velocity of settlement. Thus dQ/dt should approach to zero, if the velocity of settlement at each moment should approach to zero. We may approximate this case by increasing the load on the sample from zero by slight quantities so realising an extremely slow velocity of settlement. We are dealing with a reversible or quasi-static process as it is called in thermodynamics. In a quasi-static process of settlement the heat evolved by internal friction may be neglected with respect to the increase of potential energy of the sample.

If we call $\varphi = \frac{\Delta U}{W_n}$ the "effect" of the consolidation process: $\varphi = \frac{W_n - Q}{W_n} = 1 - \frac{Q}{W_n}$ then the effect will increase with a decreasing value of Q. In this case also the settlement obtained with a definite quantity of net work W_n is greater. Thus in an almost quasi-static consolidation process with very slow velocity of settlement (governed by a very slow increase of loading) the settlement gained by a definite quantity of work is greater as in a not quasi-static process with high velocity of settlement.

EXPERIMENTAL RESULTS.

The results of a series of consolidation tests with different loading speeds on samples of a pottery clay offered the opportunity to verify this statement. The tests were done in the normal consolidation apparatus, the dimensions of the samples being height: 2,0 cm diameter: 6,5 cm. The initial watercontents of the samples amounted to 40-42% of the dry weight x)

The time-loading and time-settlement curves are given in fig. 1. In each test the amount of work W_i was determined for a number of settlements z_i by graphical integration of the loading-settlements curves (fig. 2). The settlements z_i were plotted vs. W_i in fig. 3. The results are in full accordance with our statement: The slowest loading speed yields the largest settlement for a definite amount of work. Moreover the phenomenon of retardation by hydrodynamic effects is without a marked influence on the results.

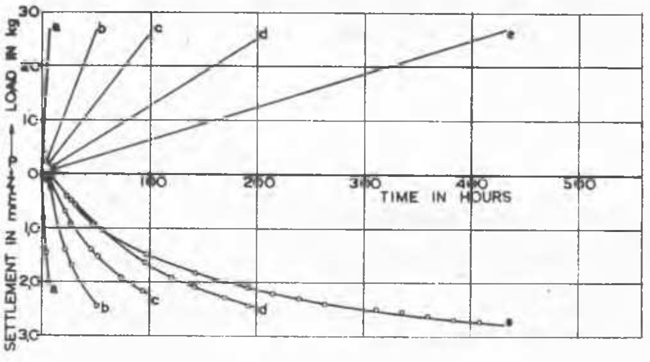

FIG.1

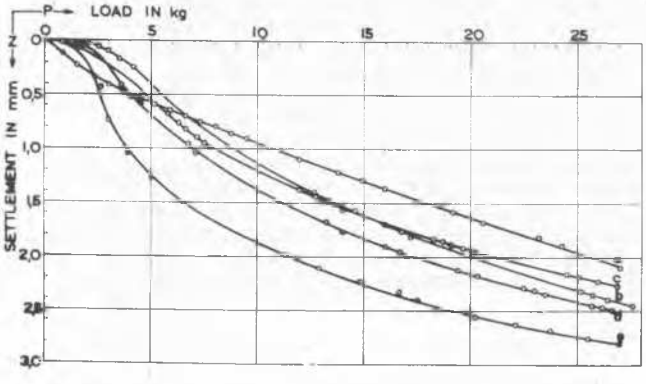

FIG.2

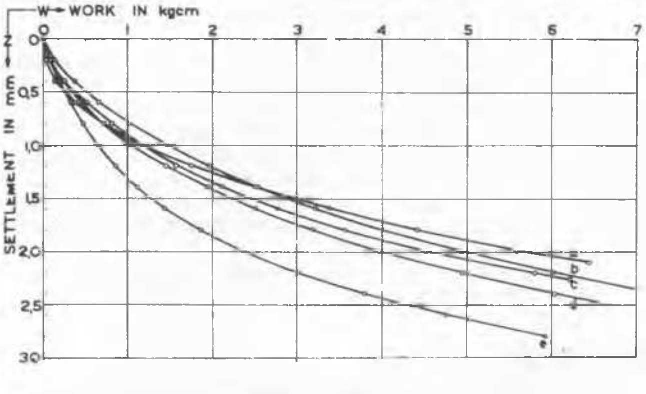

FIG.3

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x) The device for continuous increasing loading was designed by Mr. R. van de Beld and will be described in a subsequent paper.

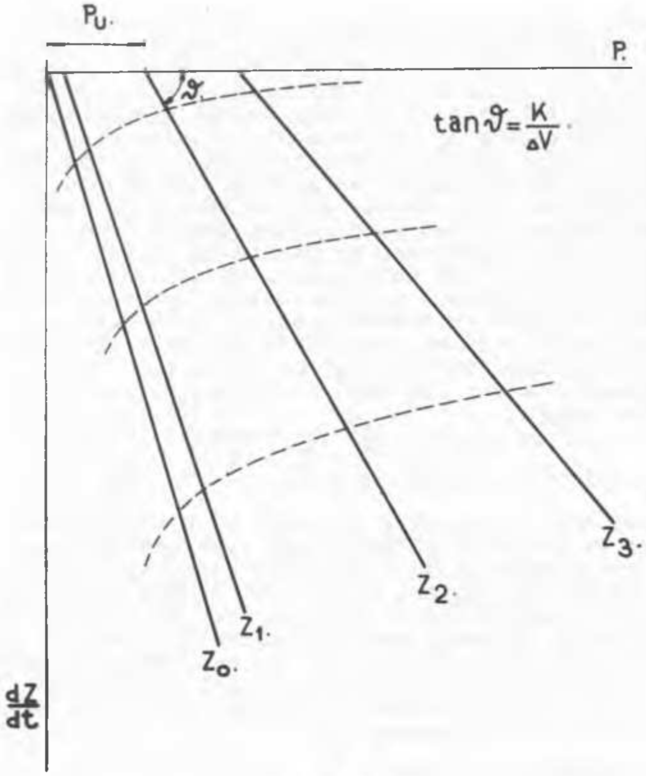

FIG.4

PROSPECTS FOR FURTHER INVESTIGATION.

Theoretically it should be possible to evalute from the results of a series of consolidation tests with a number of identical samples, both the permeability coëfficient k or its substitute for a sample of definite height and the quantity U as a function of the settlement.
At each moment the formula holds:

$$\frac{dW_n}{dt} = \frac{\Delta V}{k}\left(\frac{dz}{dt}\right)^2 + \frac{dU}{dt} \qquad (5)$$

Neglecting the side frictional resistance Q_s ($Q_s << W; W_n \approx W$) we may write:

$$\frac{dW}{dz}\cdot\frac{dz}{dt} = \frac{\Delta V}{k}\left(\frac{dz}{dt}\right)^2 + \frac{dU}{dz}\cdot\frac{dz}{dt}$$

or ($dW = Pdz$):

$$\frac{dz}{dt} = \frac{k}{\Delta V}(P - P_u) \qquad (6)$$

if $P_u = \frac{dU}{dz}$.

Applicated to a series of identical samples at the same value of the settlement z, formula (6) presents a straight line while $\frac{k}{\Delta V}$ is given by the slope and P_u by the intercept of the P-axis.

For a number of settlements z_i, formula (6) would present a number of straight lines so giving a set of values of $\frac{k}{\Delta V}$ and P_u(fig.4).

Though it was not yet possible because of lack of experimental data to check this method of verification it should be worth further attention.

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A FORMULA COMBINING THE TERZAGHI LOAD-COMPRESSION RELATIONSHIP AND THE BUISMAN SECULAR TIME EFFECT.

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SYNOPSIS.

The empirical law given by Prof. Dr. K. TERZAGHI in 1925, indicating the relation between vertical pressure and void ratio of laterally confined artificial samples of some clays and sands, implies that a final compression (complete consolidation) is reached for any finite load. However many types of undisturbed soils turn out to show a secular ("secondary") compression, which proceeds in a way that cannot be explained by the existing theory of consolidation and which does not end within times of observation. The late Prof. Ir. A.S. KEVERLING BUISMAN gave in 1936 a formula for this secular time effect, based on observed settlements of embankments and structures and on test results of undisturbed soil samples. In 1942 the author showed that the test results could be expressed by a settlement formula, containing the secular time effect as well as the load-settlement relation. This paper gives a brief exposition of this development, an illustration on the basis of laboratory tests and an example of a settlement computation. In addition some comment and conclusions are given.

INTRODUCTION.

The author prefers to use the term "consolidation" for the whole process of settlement due to the application of any finite load. A hydrodynamic and a secular phase of consolid-