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SOIL-GLIDING UNDER THE INFLUENCE OF AN INFINITE STRIP OF LOAD

LUJO SUKLJE

SUMMARY

The shape of the diagram between the gliding displacements and the shear stresses, determined with shear tests, show, that it is possible to fix the following analytic conditions for the two extreme field-limits, wherein the gliding surface with a determined starting point is formed

1) $\frac{\delta \tau}{\delta \alpha} = 0$ in

2) $\frac{\delta (t-\tau)}{\delta \alpha} = 0$;

τ is the shear stress acting in the direction α, t is the shear resistance in the same direction. The gliding surfaces are thereby defined as surfaces, connecting the directions of the gliding displacements. Using the law of Coulomb for t and analytic expressions for the shear and normal stresses deduced adter the principle of straight lines of the principal stresses under a point load on a semispace, we may for loading a semispace with a straight infinite load strip solve relations 1 and 2 and construct on their basis the extreme lines of the gliding surfaces.

At a uniform loading the construction of the gliding surfaces with the assumption of $n = 3$ (elastic isotropic soil) and $\xi = 1$ (the pressures of the unloaded soil are in all directions the same) becomes very simple. In a little permeable soil in the first moment after loading both the extreme lines fuse into the trajectory of the largest shear stresses. These trajectories accomodate well to the shape of the logaritmical spirals with the angle of inclination of $\pi/4$ especially in the rising part.

The soil gliding under the influence of the load becomes critical, as soon as the shear tensions τ in single places attain the values of the shear resistance t. The gliding fields, determined by the supposed distribution of stresses and defined with $\tau > t$, show, that the gliding becomes more intensive and that the stresses must change into a new state of equilibrium with $\tau < t$. Although the idea of gliding fields is only a ficticious one nevertheless it is very useful for the estimation of the dangerousness of the soil-gliding. The critical loading q_r follows on the condition, that the deepness of the gliding field be equal to zero. If q_o means the pressure on the fundamental sole in the deepness g minus the pressure of the digged soil ($\gamma . g$), if γ is the weight of unit volume of the soil and k and $\mu = tg\,\varphi$ are constants of the law of Coulomb, then

$$q_r = \frac{\pi}{A}(k + \gamma g \mu).$$

We get the criterion of Fröhlich for the end stage if $A = 1 - (\frac{\pi}{2} - \varphi)\mu$ permeable soil, and the criterion for the beginning state (unpermeable soil) if $A = 1$. The quotient $\frac{q_r}{q_o}$ is the safety factor. The investigation of gliding surfaces under the condition $q_o < q_r$ is not necessary. If $q_o < q_r$ we may estimate the degree of dangerousness of gliding by determining the algebraic sum of the ficticious forces $\Sigma[(t - \tau).f]$ (f ... is the area of an element of the gliding surface) along the gliding surfaces. To estimate the gliding of little permeable soil we may employ this criterion with security and in a simple manner with $n = 3$ and $\xi = 1$ and by taking into consideration only the shear resistance t of the unloaded soil. The stages of negative resultanta $\Sigma[(t-\tau).f]$ are especially critical ones.

1. GENERAL STRESS EQUATIONS.

Applying for the action of an infinite load strip with pressures, distributed equally, the principle that the trajectories of the principal stresses for a concentrated load are straight lines 1) we obtain for the coefficient of concentration 1) $n = 3$ in a given soil point, determined with the angles ε and ψ, for the direction α (figure 1) the following expressions for the normal stress ν and τ the shear stress

$$\nu = \frac{q_o}{\pi}\left\{2\varepsilon - \sin 2\varepsilon \cdot \cos 2(\alpha - \psi)\right\} + \frac{\gamma z}{2}\left\{(1+\xi) - (1-\xi)\cos 2\alpha\right\} \quad (1)$$

$$\tau = \frac{q_o}{\pi}\sin 2\varepsilon \sin 2(\alpha - \psi) + \frac{\gamma z}{2}(1-\xi)\sin 2\alpha \quad (2)$$

In these equations q_o is the pressure in the sole of the foundation minus the pressure of the digged soil ($q_o = q - \gamma . g$), ξ is the coefficient of the resting soil, expressing the relation between the horizontal and vertical pressures of the unloaded soil, γ is the weight of volume of the soil (taking into consideration the influence of the buoyancy. The meaning of z is given by equation

$$z = b\frac{\cos 2\varepsilon + \cos 2\psi}{\sin 2\varepsilon} + g \quad (3)$$

The largest shear stresses τ_m act in the directions α_m, determined after the condition, that

$$\frac{\partial \tau}{\partial \alpha} = 0 \quad (4)$$

$$tg\, 2\alpha_m = -\frac{\sin 2\varepsilon \cos 2\psi + \frac{\pi}{2 q_o}(1-\xi) z}{\sin 2\varepsilon \sin 2\psi} \quad (5)$$

$$\tau_m = \pm\sqrt{\left(\frac{q_o}{\pi}\sin 2\varepsilon \sin 2\psi\right)^2 + \left\{\frac{q_o}{\pi}\sin 2\varepsilon \cos 2\psi + \frac{\gamma z}{2}(1-\xi)\right\}^2} \quad (6)$$

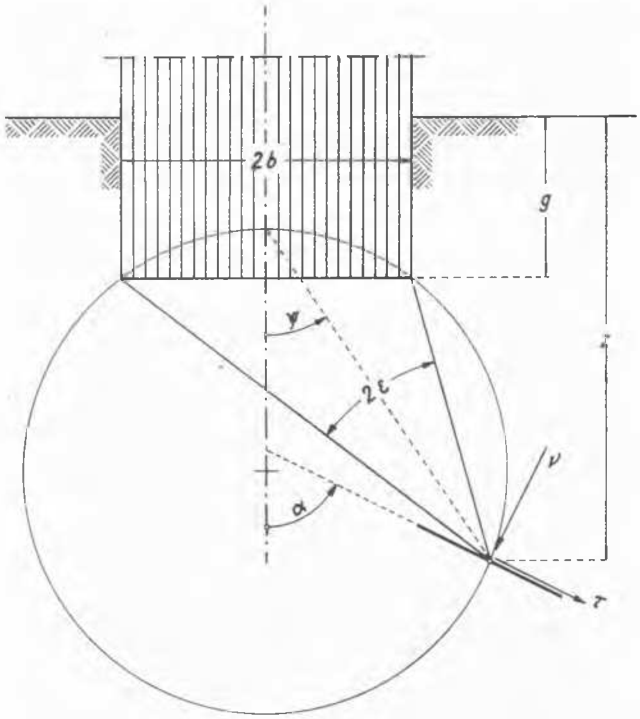

FIG.1

2. GLIDING-SURFACES.

Shear tests of soil materials show, that especially argillaceous soils are gliding at relatively low shear stresses (confer 2) and 3).) This gliding however is stopping asymptotically 3) at these stresses. As in the shear apparatus the soil is gliding also in the ground of the foundation at moderate loading of the soil. In a given point of the soil the grains will glide in the directions of the lowest resistance. In this paper we shall call as gliding-surfaces all surfaces, that connect these directions. The direction of the gliding will depend for a fixed point of the soil on the total state of stresses, represented by the ellipse of stress and the laws of gliding.

The diagram of the gliding displacements d and the shear stresses can't included in a uniform analytic relation (confer 2) and 3)). Only the first part of this diagram (one-two thirds) we are able to substitute approximately with a straight line

$$d = \lambda \tau \tag{7}$$

Supposing that the quotient does not depend on the normal pressure, the directions of the largest gliding-displacements, i.e. the directions of the gliding surfaces, would be identical with the trajectories of the largest shear stresses and the gliding surfaces given by the condition

$$\frac{\partial \tau}{\partial \alpha} = 0 \tag{4}$$

In fact however the gliding displacements are at equal shear stresses $\tau < t$ larger if the normal pressure ν' is low. Therefore it is to expect that the direction of the gliding displacements will decline from the direction of the largest shear stresses τ_m towards the large axis of the stress ellipse.

Keeping up the simplification of the linear dependency between the gliding displacements d and the shear stresses τ, we accomodate better to the observations of laboratory using the relation

$$d = \frac{a}{t} \cdot \tau \tag{8}$$

where a is a constant. The directions of the largest gliding displacements would be then determined with the condition

$$\frac{\partial \frac{\tau}{t}}{\partial \alpha} = 0 \tag{9}$$

For the shear resistance we apply the law of Coulomb

$$t = k + tg\,\varphi \cdot \nu' \tag{10}$$

k is the cohesive resistance of the soil material on the surface of the soil $tg\,\varphi = \mu$ is the coefficient of the total shear resistance, considerating the augmentation of the cohesive and friction resistance at increasing acting pressure ν'.

Substituting the conditionel equation (9) with the simpler relation

$$\frac{\partial (t - \tau)}{\partial \alpha} = 0 \tag{11}$$

we may see comparing the sizes of the diagram $d - \tau$ for different ν' that the directions with larger ν' are less estimated than the directions with lower ν' (for the same difference $e = t - \tau$ the gliding displacements d are larger when ν' is larger). The direction of the gliding surfaces declines therefore against the large axis of the stress -ellipse more than in the condition (9) and more than it is in agreement with the magnitude of the gliding displacements. Therefore we may use the condition (11) to determine the lower limit, below which the gliding surfaces cannot be more inclined against the trajectories ν_{max} whilst the upper limit of the gliding surfaces is given by the trajectories τ_m after the equation (4).

The trajectories τ_m are as the first extreme lines of the gliding surfaces for n = 3, determined with the equation (5). They are constructed in the known manner determining the isoclines α_m = constant. The construction is the more exact, the denser the set of the isoclines is. We construct the curves α_m=const. by determining for different points of the soil (given with the angles ε and ψ) the values α_m, by finding with interpolation the sets of points α_m = const. and by connecting them with continuous curves.

If we take $\xi = 1$ the equations of the trajectories α_m become simpler. The directions α_m are no more in dependence of the constants γ, q_o, b and g, but depend only of the concentration coefficient n. For n = 3 is

$$\alpha_m = \psi + (1 + 2u)\frac{\pi}{4} \quad , (u = 0\ ,\ \pm 1\ ,\ \pm 2\ , \cdots) \tag{12}$$

The curves α_m = const. are therefore identical with the curves ψ = const., when n = 3.

The shear resistance t, occuring in the equation (11) of other extreme lines of the gliding surfaces is after the relation (10) in a linear dependency from the acting pressure ν' In permeable soil we may substitute ν' by the whole normal stress ν after the equation (1). In little permeable soil however the additional pressures are taken by the interstitial water in the first moment after the loading. It is only by and by that these pressures are given to the solid parts of the soil after the outflow of the interstitial water, causing the increase of the acting pressures ν' and with them the value of the shear resistance t. In the first moment there may against the shear stresses τ, determined with the equation (2) the initial shear resistance of the unloaded soil (q_o = 0) only.

$$t = k + \frac{\gamma z}{2}\left\{(1+\xi) - (1-\xi)\cos 2\alpha\right\} \qquad (13)$$

So we deduce for the other extreme forms of gliding surfaces with the condition (11) and with n = 3 the following analytic expressions.
Permeable soil:

$$tg\,2\alpha = \frac{\sin^2 2\varepsilon(\cos 2\psi + \mu \sin 2\psi) + A(\cos 2\varepsilon + \cos 2\psi) + B\sin 2\varepsilon}{\sin^2 2\varepsilon(\mu\cos 2\psi - \sin 2\psi) + \mu\{A(\cos 2\varepsilon + \cos 2\psi) + B\sin 2\varepsilon\}} \qquad (14)$$

$$A = \frac{\pi\gamma}{2q_o}(1-\xi)\,b, \qquad B = \frac{\pi\gamma}{2q_o}(1-\xi)\,g \qquad (15)$$

$$\xi = 1 \qquad \alpha = \psi + (1+2u)\frac{\pi}{4} - \frac{\varphi}{2} \qquad (16)$$

Little permeable soil at the first moment after loading:

$$tg\,2\alpha = \frac{\sin^2 2\varepsilon\cos 2\psi + A(\cos 2\varepsilon + \cos 2\psi) + B\sin 2\varepsilon}{-\sin^2 2\varepsilon\sin 2\psi + \mu\{A(\cos 2\varepsilon + \cos 2\psi) + B\sin 2\varepsilon\}} \qquad (17)$$

When $\xi = 1$ the expression (17) turns into the equation of the trajectories τ_m (12).

Comparing the shape of the gliding surfaces, constructed after the mentioned equations for n = 3, and n = 2 and n = 4 (which are not mentioned here but are given together with other detailed analysis in 4) we obtain the following conclusions. (In the brackets there are given specific conditions under which the conclusions are made).

1) The cohesive resistance (k) has no influence on the shope of the gliding surfaces.
2) The concentration coefficient n in the limits between 2 and 4 has only little influence on the shape of the gliding surfaces. If n are smaller the gliding surfaces originating from the same points of the fundamental sole are flatter and shorter (Little permeable soil with $\xi = 1$, resp. the trajectories τ_m).
3) The smaller the number of the resting soil pressure ξ is, the deeper and longer are the gliding surfaces. (Little permeable soil, n = 3, $\frac{\gamma}{q_o} = 0{,}05\ m^{-1}, \mu = 0{,}50$).
4) The gliding surfaces are the deeper and the longer, the larger the breadth of the fundament 2 b, the deepness g, the weight of unit volume of the soil γ and the smaller the specific load q_o is. (Special conditions as sub 3). For $\xi = 1$ this deduction is of no value.)
5) Taking for $\xi = 1$ the size of the initial gliding surfaces in little permeable soil does not depend on the characteristic of the soil and the fundament. Both kinds of extreme lines of the gliding surfaces unite in the trajectory τ_m.
6) Taking for $\xi = 1$ the size of other extreme lines of the gliding surfaces in permeable soil depends (except from n) only on the quotient of the shear resistance μ (n = 3). The gliding surfaces are the deeper and longer the larger μ is. Nor does the size of the first extreme lines of the gliding surfaces depend from μ.
7) The difference between both the extreme lines of the gliding surfaces in permeable soil is much larger than in unpermeable soil. (Specific conditions as sub 3 for unpermeable soil and n = 3, $\xi = 1$ for permeable soil).
8) At the same n and the same ξ, the gliding surfaces in permeable soil are deeper and longer than the initial gliding surfaces in little permeable soil (n = 3, $\xi = 1$).
9) In little permeable soil the difference between both the extreme lines of the gliding surfaces is not large in the first moment of loading. The trajectories τ_m are somewhat flatter but nearly as long as the other extreme lines of gliding surfaces (Specific conditions as sub 3).
10) In the fundamental sole the trajectories τ_m have the slope $\alpha_m = \frac{\pi}{4}$, other extreme lines of gliding surfaces however the slope $\alpha = \frac{\pi}{4} - \frac{\varphi}{2}$. In the level of the fundamental sole outside of the fundament the trajectories τ_m have the slope $\alpha_m = \frac{3\pi}{4}$, other extreme lines of gliding surfaces however the slope $\alpha = \frac{3\pi}{4} - \frac{\varphi}{2}$. Thereby is to take into consideration that in little permeable soil the coefficient $\xi = 1$ is in agreement with the angle $\varphi = 0$.
11) The trajectories τ_m cross the axis of the load-strip under the angle $\alpha_m = \frac{\pi}{4}$, the other extreme lines of gliding surfaces in permeable soil under $\alpha = \frac{\pi}{4} - \frac{\varphi}{2}$. So the first extreme lines of gliding surfaces form under the fundamental soil simmetric convexical shaped wedges with the angle $\frac{\pi}{2}$ in the axis of the load, the other extreme lines form similar but sharper wedges. In permeable soil the sharpness of these wedges is $\frac{\pi}{2} - \varphi$.
12) Among all discussed shapes of gliding surfaces the trajectories τ_m for n = 3 and $\xi = 1$ are the fattest and shortest, if we exclude similar lines for n = 2; however these are only a little shallower and shorter. The construction of these trajectories, which are at the same time the glidingsurfaces in little permeable soil for n = 3 and $\xi = 1$, is a simple one.
13) The trajectories τ_m for n = 3 and $\xi = 1$ in the rising part are well accomodating to the shape of logarithmical spirals

$$r = a\,e^{m\varphi} \qquad (18)$$

with m = 1 and the pole in the line of the fundamental soil; in the decreasing part they are shallower. The trajectories τ_m for n = 4 or $\xi < 1$ and the other extreme lines of gliding surfaces correspond better to logarithmical spirals with m > 1.

3. GLIDING-FIELDS.

With an increasing pressure q_o the equations of stress (f.i. 1 and 2) in connection with the law of shear resistance (10) dictate fields, where the shear stresses are larger than the shear resistance. The boundaries of these fields, called after Fröhlich (1) plastic fields, are determined with the equation

$$t = \tau \qquad (19)$$

Such a field, dictated by the supposed equations of stresses of course cannot exist. With rising stresses the gliding displacements increase also. These gliding displacements, accompanied in permeable soil by the consolidation of soil by reason of the added pressures may cause such a change of the stress distribution on one side and such an augmentation of the shear resistance under certain conditions on the other side, that with $t > \tau$ another state of equilibrium is established. The gliding may however increase in an accelerated way so that the soil together with the building glide along the gliding surfaces in a oathastrophical manner and there is not earlier than after the cathastrophe a new state of equilibrium possible. Nor do exist in this case "plastic fields", which we shall prefer ably call here gliding fields. Nevertheless the idea of these fields - as shown by Fröhlich - is very useful for the estimation of

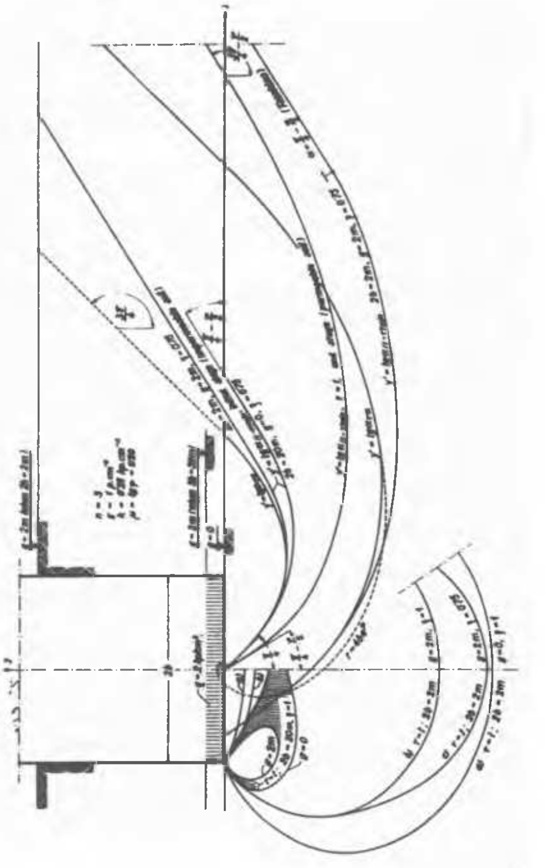

FIG. 2

the soil stability.

In the discussed two-dimensional problem we obtain for the boundary lines of the gliding fields with $n = 3$ the following analytical expressions.

Permeable soil:

$$z = \frac{2q_o\left[-\mu 2\varepsilon + \sin 2\varepsilon\left\{\sin 2(\alpha-\psi) + \mu\cos 2(\alpha-\psi)\right\}\right] - 2\pi k}{\pi\gamma\left\{\mu(1+\xi)-(1-\xi)(\mu\cos 2\alpha + \sin 2\alpha)\right\}} \qquad (20)$$

Little permeable soil (the first moment after loading);

$$z = \frac{2q_o \sin 2\varepsilon \sin 2(\alpha-\psi) - 2\pi k}{\pi\gamma\left[\mu\left\{(1+\xi)-(1-\xi)\cos 2\alpha\right\}-(1-\xi)\sin 2\alpha\right]} \qquad (21)$$

are the directions of the gliding surfaces. We give them the value of their extreme lines after the equations sub 2. The points of the boundery line of the gliding field we may find as follows : Along the curves $\alpha =$ const., necessary for the construction of the gliding surfaces we try to find for different ε and ψ the values "z" on the righty side of the equations (20) resp. (21). The intersection of the curve for the calculated "z" with the curve $\alpha =$ const. determines the point of the boundary line.

Fröhlich has already given some examples of gliding fields in permeable soil for $\xi = 1$ 1). Here we mention only the reproduction of analytical expressions of the boundary lines in little permeable soil (for the first moment of loading) for $n = 3$ and $\xi = 1$. In this case the equation (21) becomes very simple. (For α we substitute the expression 12 with $u = 0$).

$$z = b\frac{\cos 2\psi + \cos 2\varepsilon}{\sin 2\varepsilon} + g = \frac{q_o \sin 2\varepsilon - \pi k}{\pi\gamma\mu} \qquad (22)$$

The deepest points of the boundary-lines of the gliding fields are arranged in a semicircle

$$2\varepsilon_m = \frac{\pi}{2} \qquad (23)$$

So we receive for the deepness of the gliding field the expression

$$z_m = \frac{q_o - \pi k}{\pi\gamma\mu} \qquad (24)$$

If the gliding field extends beyond the circle $2\varepsilon_m = \frac{\pi}{2}$ its deepness in the symmetral of the loaded strip is given with the condition

$$z_m = b\cot\varsigma\,\varepsilon_m + g = \frac{q_o}{\pi\gamma\mu}\sin 2\varepsilon_m - \frac{k}{\gamma\mu} \qquad (25)$$

If $n = 3$ and $\xi = 1$ also the indirect graphical construction of the boundary line of the gliding field in little permeable soil after condition (19) is very simple. For ε_m follows from the equations (2) the simple expression

$$\tau_m = \frac{q_o}{\pi}\sin 2\varepsilon \qquad (26)$$

The curves $\tau_m =$ const. are identical with the circles $\varepsilon =$ const. (figure 1), meanwhile the expression (13) for the shear resistance t becomes simpler

$$t = k + \mu\gamma z \qquad (27)$$

so that t is a linear function of f (z). We find therefore the points of the boundary-line of the gliding field as intersections of the horizontal lines t = const. with the circles τ_m = const.

In the figure 2 there are constructed the gliding fields for the first moment of loading in little permeable soil with constant values for n, γ, μ, k and q_o and for different values of b, g and ξ.

We may establish:

1) Broader fundaments have absolutely deeper and broader gliding fields. The relation between the dimensions of these fields and the breadth of the fundament 2 b however is more advantageous with larger breadth 2 b. The gliding surfaces cross the gliding fields of broader fundaments on a relatively shorter path.
2) An absolutely equal deepening of the fundamental soil has not only a larger influence on the absolute, but also on the relative size of the gliding fields at broader fundaments. Nevertheless the influence of such a deepening on the length of the gliding fields is a better one at narrower fundaments.
3) At a decreasing quotient ξ the gliding field deepens. In the same manner the gliding field becomes narrower and deeper if the quotient n increases (1).

4. ESTIMATION OF THE DANGEROUSNESS OF GLIDING.

The smaller the pressure of the fundamental sole q_o is, the smaller is the field of gliding. At a determined pressure the gliding field disappears. The analytic expression for this critical pressure q_r is

$$z_m = g. \qquad (28)$$

For permeable soil it follows from the equation (23) for the deepness of the gliding field after the condition $\partial z/\partial\varepsilon = 0$ and with the directions of the second extreme lines of gliding surfaces in consequence of the equation (16)

$$z_m = \frac{q_o}{\pi\gamma}\left[\cot g\,\varphi - \left(\frac{\pi}{2} - \varphi\right)\right] - \frac{k}{\gamma\, tg\varphi} \qquad (29)$$

and with the condition (28) results the equation for the "critical load in the edge" of Fröhlich

$$q_r = \frac{\pi(k + \gamma g\, tg\varphi)}{1 - \left(\frac{\pi}{2} - \varphi\right) tg\varphi} \qquad (30)$$

In an analogical way we obtain for little permeable soil for the stress state in the first moment after loading with the equations (24) and (28) the expression for the critical load

$$q_m = \pi(k + \gamma g \operatorname{tg}\varphi) \qquad (31)$$

(confer 5). Which decreasing hydrodynamical tensions of the interstitial water the values of q_m approach to the values q_r.

If the loading of the fundamental sole q_o ($q_o = q - \gamma g\,!$) is smaller than q_r in permeable soil or smaller than q_m in little permeable soil, every test of the dangerousness of gliding is unnecessary. With the quotient $\frac{q_r}{q_o}$ or $\frac{q_m}{q_o}$ we determine also the safety factor therefore it is not necessary to investigate this factor with the gliding surfaces.

Not earlier than if the loading q_o becomes larger than q_r or q_m i.e. if there will develope a gliding field with regard to a supposed distribution of the tensions, a test of stäbility with the gliding surfaces resp. with the test surfaces, assumed by us as gliding surfaces and examined for the dangerousness of gliding, will be reasonable. The methods of such an investigation (e.g. after Fellenius or Krey, confer 6) are known. These methods suppose at any rate a fictitious state of tensions along the gliding surface. The soil does not glide in such a manner that the whole shear stress would act in the same measure along the gliding surface, it is shorn rather gradually - with respect to the different shear stresses and shear resistances - along the gliding surface - and the whole shear resistance also decreases gradually to the values of the gliding resistance.

With regard to the results of the investigations, collected in 2 and 3 we may propose a new criterion for the dangerousness of gliding. We construct a gliding field for the supposed distribution of tensions. The gliding surfaces crossing this field are divided in suitable elements and we determine for every element with the surface f the shear stresses τ and the shear resistance t. The larger the algebraic sum of the ficticious forces is $\Sigma[(t - \tau)f]$, the smaller is the danger of gliding. Gliding surfaces crossing the gliding field on a relative long way are to be examined.

The cases, when

$$\Sigma[(t - \tau)f] \leqq 0. \qquad (32)$$

are doubtless taken as critical ones.

The construction of gliding fields and gliding surfaces as well as the determination of the values t and for different points of the gliding surfaces are in general taking up much time. For the first phase of loading in little permeable soil however the use of the criterion (32) for n = 3 and ξ = 1 is very simple and quick. With such suppositions we induce on the whole a security in the calculation, therefore the simplification for n = 3 and ξ = 1 and with the state at once after the loading in argillaceous soil is allowed.

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PROTRACTED SLIDING SURFACE

Dr.L. BENDEL

Ing, Privatdozent

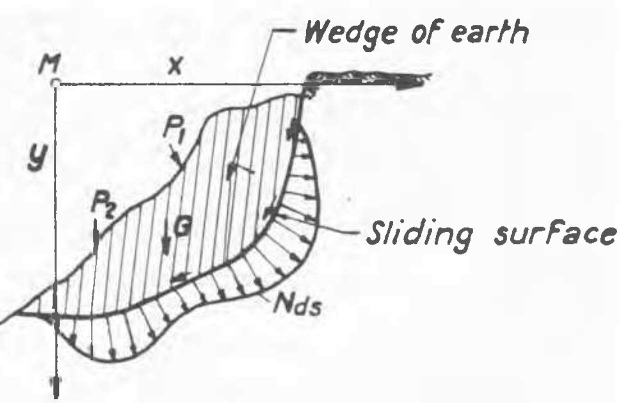
an der Ecole Polytechnique de l'Université de Lausanne

Luzern.

The protracted sliding surface occurs for instance with landslides. Bendel's process of calculation for determining the danger of landslides with extensive protracted sliding surfaces, is given hereafter.

The action of the forces at the moment when, with a protracted sliding surface the sliding commence, is shown in Fig. 1.

The pressure working on the sliding surface through the outside forces P_1 and P_2 on the one hand and through its own weight G on the other hand, is shown in Figs 2 and 3. In Fig. 4 the action of the forces on a wedge of earth is more closely looked into. It is assumed that the force W of the part of earth is in the same direction as the force E of the ground B. When equilibrium exists, the force W must be equally great or greater than E (W = E). For further mathematical considerations it is assumed that W = E, i.e. the for-



(P_1 ; P_2 = Forces on the surface)
Forces operating on sliding surface

FIG. 1