In an analogical way we obtain for little permeable soil for the stress state in the first moment after loading with the equations (24) and (28) the expression for the critical load

$$q_m = \pi(k + \gamma g \, tg\varphi) \qquad (31)$$

(confer 5). Which decreasing hydrodynamical tensions of the interstitial water the values of q_m approach to the values q_r.

If the loading of the fundamental sole $q_o (q_o = q - \gamma g t)$ is smaller than q_r in permeable soil or smaller than q_m in little permeable soil, every test of the dangerousness of gliding is unnecessary. With the quotient $\frac{q_r}{q_o}$ or $\frac{q_m}{q_o}$ we determine also the safety factor therefore it is not necessary to investigate this factor with the gliding surfaces.

Not earlier than if the loading q_o becomes larger than q_r or q_m i.e. if there will develope a gliding field with regard to a supposed distribution of the tensions, a test of stability with the gliding surfaces resp. with the test surfaces, assumed by us as gliding surfaces and examined for the dangerousness of gliding, will be reasonable. The methods of such an investigation (e.g. after Fellenius or Krey, confer 6) are known. These methods suppose at any rate a fictitious state of tensions along the gliding surface. The soil does not glide in such a manner that the whole shear stress would act in the same measure along the gliding surface, it is shorn rather gradually - with respect to the different shear stresses and shear resistances - along the gliding surface - and the whole shear resistance also decreases gradually to the values of the gliding resistance.

With regard to the results of the investigations, collected in 2 and 3 we may propose a new criterion for the dangerousness of gliding. We construct a gliding field for the supposed distribution of tensions. The gliding surfaces crossing this field are divided in suitable elements and we determine for every element with the surface f the shear stresses τ and the shear resistance t. The larger the algebraic sum of the ficticious forces is $\Sigma[(t - \tau)f]$, the smaller is the danger of gliding. Gliding surfaces crossing the gliding field on a relative long way are to be examined.

The cases, when

$$\Sigma[(t - \tau)f] \leq 0. \qquad (32)$$

are doubtless taken as critical ones.

The construction of gliding fields and gliding surfaces as well as the determination of the values t and for different points of the gliding surfaces are in general taking up much time. For the first phase of loading in little permeable soil however the use of the criterion (32) for $n = 3$ and $\xi = 1$ is very simple and quick. With such suppositions we induce on the whole a security in the calculation, therefore the simplification for $n = 3$ and $\xi = 1$ and with the state at once after the loading in argillaceous soil is allowed.

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I e 15

PROTRACTED SLIDING SURFACE

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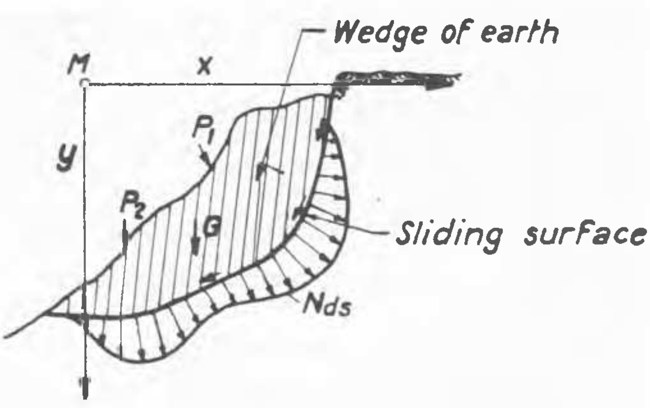
The protracted sliding surface occurs for instance with landslides. Bendel's process of calculation for determining the danger of landslides with extensive protracted sliding surfaces, is given hereafter.

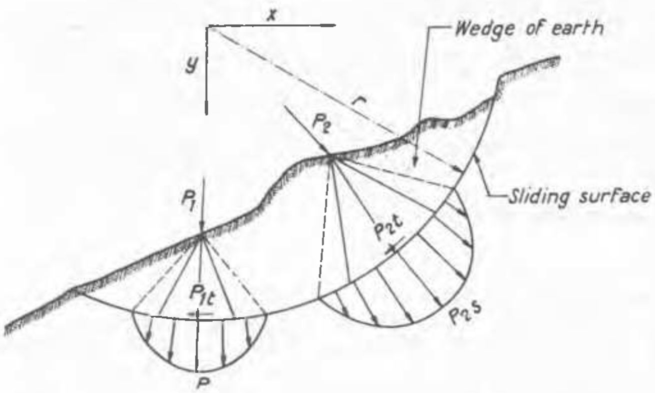
The action of the forces at the moment when, with a protracted sliding surface the sliding commence, is shown in Fig. 1.

The pressure working on the sliding surface through the outside forces P_1 and P_2 on the one hand and through its own weight G on the other hand, is shown in Figs 2 and 3. In Fig. 4 the action of the forces on a wedge of earth is more closely looked into. It is assumed that the force W of the part of earth is in the same direction as the force E of the ground B. When equilibrium exists, the force W must be equally great or greater than E ($W = E$). For further mathematical considerations it is assumed that $W = E$, i.e. the for-



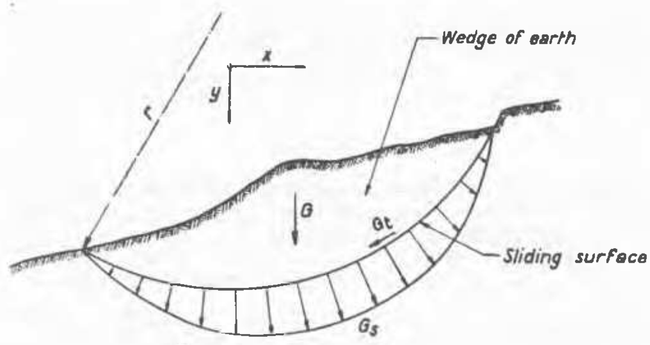
(P_1; P_2 = Forces on the surface)
Forces operating on sliding surface

FIG. 1



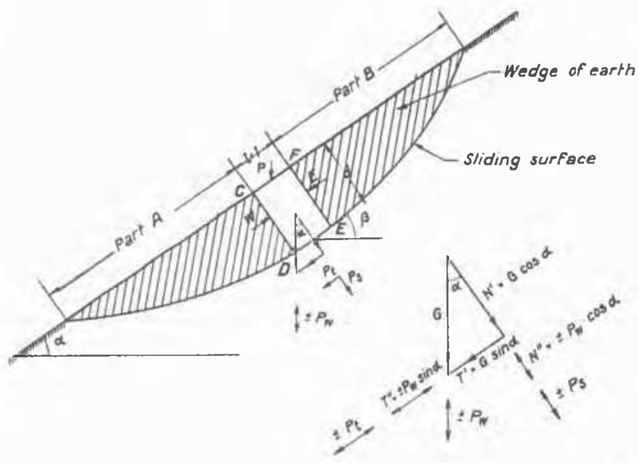
Forces operating on sliding surface

FIG. 2



Forces operating on sliding surface.

FIG. 3



Action of forces on a wedge of earth CDEF.

FIG. 4

ces neutralize each other mutually.

The forces which operate in the sliding surface and on the sliding surface are shown on Fig. 4. Herewith it is assumed that the sliding surface in the wedge of ground under consideration runs parallel with the surface of the ground i.e. it is $\sphericalangle\beta = \sphericalangle\alpha$

On the basis of this assumption for $\ell = 1$ and $\gamma = \gamma_s(1-n)$ for dry material G becomes $= \gamma_s(1-n)$ further is $N' = G \cos\alpha = d\,\gamma \cos\alpha$

$N'' = (\pm p_w)\cos\alpha$ x)

The forces operating on the sliding surface are vertically to the sliding surface.

Equilibrium exists when the driving force T in the sliding surface can be matched by the force S, developed by the mass of earth, i.e. for equilibrium the following condition must be fulfilled

When $T \leqq S$

The following holds good:

$$S = k + n\,tg\rho = k + \left[(d\gamma \pm p_w)\cos\alpha \pm P_s\right] tg\rho \qquad (5)$$

ρ = angle of inner friction

After Equations (3), (4) and (5) becomes, as $T = S$ (6)

$$(d\gamma \pm p_w)\sin\alpha \pm P_t = k + \left[(d\gamma \pm p_w)\cos\alpha \pm P_s\right] tg\rho$$

xa)

$$s = \frac{K + N\,tg\rho}{T} \qquad (6a)$$

referring to an element is introduced, the Author derives the Equation (1) as also the special cases for the Equations (8) to (20)

It is

$$s\left[(d\gamma \pm p_w)\sin\alpha\right] \pm sP_t = k + \left[(d\gamma \pm p_w)\cos\alpha \pm P_s\right] tg\rho \qquad (7)$$

The Equation (7) is the generally valid Slide-equation for an element with protracted sliding surfaces.

Consideration of an Element.

If it is assumed that the force $\pm p \sin\alpha$ in the direction of the current parallel to the line of slope can be neglected then the Equation (7) may be written

$$s\,d\gamma\sin\alpha \pm sP_t = k + \left[(d\gamma \pm p_w)\cos\alpha \pm P_s\right] tg\rho$$

From this can be calculated the admissible angle of slope until a landslide occurs. It is

$$\cos\alpha = \frac{-ab\,tg\rho \pm \sqrt{1 + b^2 tg^2\rho - a^2}}{1 + b^2 tg^2\rho} \qquad (8)$$

In Equation (8) signify:

$$a = \frac{k \pm P_s tg\rho \pm sP_t}{s\gamma d} \quad ; \quad b = \frac{d\gamma \pm p_w}{s\gamma d}$$

Equation (8) shows the relation between the angle α on the one hand and the existing qualities of the soil (as : binding force, angle ρ of the inner friction) the pressure of pore-water p_w and the existing pressure on the other hand. When in Equation (8) the safety number $s = 1$ we obtain the equation of the critical angle of the slope against sliding.

(Special cases of Equation (8).

1) When no outward force P operates then $P = 0$ and $a = \frac{k}{s\gamma d}$

and Equation (8) becomes

$$\cos\alpha = \frac{\frac{-k\,b\,tg\rho}{s\gamma d} \pm \sqrt{1 + b^2 tg^2\rho - \frac{k^2}{s^2\gamma^2 d^2}}}{1 + b^2 tg^2\rho} \qquad (9)$$

2) When no porewater-pressure is present, the safety degree s is 1; then is for $P = 0$; $p_w = 0$, $s = 1$ and $a = \frac{k}{\gamma d}$; $b = 1$ from this

$$\cos\alpha = \frac{\frac{-k}{d\gamma} tg\rho \pm \sqrt{tg^2\rho - \frac{k^2}{d^2\gamma^2} + 1}}{1 + tg^2\rho} \qquad (10)$$

3) Let us be, as below (2) $P = 0$, $p_w = 0$, $s = 1$ add to this that the binding force (cohesion) $k = 0$ then the result is:

$$tg\,\alpha = tg\,\rho$$

i.e. the angle of slope α is equal to the angle

x) P_w may originate from lift from capillary force, from shock, etc.

xa) The forces P_s and P_t originate from the outer force P. P can be any force at all. e.g. as a result of capillary pressure, as a result of shock, impact of a projectile, as a result of pore-water pressure etc.

ρ of the inner friction. This is the case e.g. with not - binding gravel and sand.

ρ_s = Angle of apparent inner friction. Herewith is, according to Equations (2) and (7)

$$\left[(d\gamma \pm p_w)\cos\alpha \pm P_s\right] tg\rho_s = s\left[(d\gamma \mp p_w)\sin\alpha \pm P_t\right] \qquad (16)$$

TABLE.

Kind of Stone	Granulation	not grown over α	grown over α
Argillacevus slate	Smooth - breaking and slaty	26 - 30°	20°
Mica - slate	slaty	28 - 32°	
Lime - stone	Partly smooth - breaking	28 - 36°	30°
"	" bulky, large	32 - 40°	33°
Gneis	slaty	32 - 34°	32°
Sernifit (Conglomerate rich in quartz)	Quick - breaking, bulky	32 - 36°	--
Granite	Bulky, angular, rounded fragments	28 - 36°	

Evaluation of Experience for the angle of slope Values for dry Cones of rubble and rubble dumps.

From the above table it appears that by covering with grass, the angle of slope of a rubble embankment becomes flatter. This phenomenon is understandable for turfing is attended with the weathering and the breaking - up of stone - fragments. Through the breaking up of the blocks of stone the ground becomes loamy - clayey.
The angle ρ of the inner friction becomes smaller.

II. α-Value for artificial Banks.
The size of the natural angle of embankment α for the artificial embankment of not - binding matereal is shown by Table.

4) Let there be a lift present, i.e. let $p_w = -\gamma_w$ furthermore let P = 0; k = I then:

$$tg\alpha = \frac{d\gamma - \gamma_w}{s\gamma d} tg\rho \qquad (12)$$

5) For k = 0, γ_w = 1 (upward motion), $\gamma - \gamma_w = \gamma - 1 = \gamma_e^+$ and $d = 1$

$$\frac{tg\alpha}{tg\rho} = \frac{\gamma_e^+}{s\gamma} \qquad (13)$$

Expressed in words that means:
The tangent of the angle of slope is to the tangent of the angle of the inner friction as the weight per unit of volume of the material under water is to the weight per unit of volume of the material above water.

6) For the degree of safety s = 1

$$\frac{tg\alpha}{tg\rho} = \frac{\gamma_e^+}{\gamma} = \frac{\gamma - 1}{\gamma} \qquad (14)$$

If Equation (14) is written:

$$tg\alpha = \frac{\gamma_e^+}{\gamma} tg\rho$$

then it can be seen that the angle of embankment of a group diminishes quickly when water, which causes a lift of the material makes its appearance.

Without water is tg α = tg ρ

With water tg α is $tg\rho \frac{\gamma - 1}{\gamma}$

As $(\gamma - 1) < \gamma$ is, $\alpha < \rho$ that is: with appearance of water the slope must become flatter, so that equilibrium may be established.

7) Let k be the binding force (cohesion) and N the force operating vertically on the sliding surface, then according to Hvorslev 1)

$$k + N\, tg\rho = N(tg\rho' + tg\rho) = N\, tg\rho_s \qquad (15)$$

From Equation (16) may be calculated:

$$\cos\alpha = \frac{-a\,b\,tg\rho_s \pm \sqrt{-a^2 + b^2 tg^2\rho_s + 1}}{1 + b^2 tg^2\rho_s} \qquad (17)$$

with which is meant

$$b = \frac{d\gamma \pm p_w}{s\gamma d}\ ;\ a = \frac{P_s\, tg\rho_s \pm s P_t}{s\gamma d} \qquad \text{(cf. Equation 8)}$$

8) In case no other load is present P_s becomes 0 ; P_t = 0
In the above Equation (17)

$$\frac{tg\alpha}{tg\rho_s} = \frac{d\gamma \pm p_w}{s\gamma d} \text{ and for } d = 1\ ;\ \frac{tg\alpha}{tg\rho_s} = \frac{\gamma \pm p_w}{\gamma}$$

then bocomes(18)

9) For the degree of safety S = 1 and for the lift $P_w = -\gamma_w = 1$

Equation (18) becomes

$$\frac{tg\alpha}{tg\rho_s} = \frac{\gamma - 1}{\gamma} \qquad (19)$$

For the size of the angle α see page 54. In the formula (19) ρ_s means the angle of apparent inner friction, while the angle in Equation 14 is the true angle of the inner friction. In the angle ρ_s the binding force of the ground is taken into consideration. Fig. 5. Determination of danger of sliding with pressure of current towards the outside. The angle of slope of embankment is smaller than the angle ρ of the inner friction p vertical to the slope.

10) If in the equation (8) P_w = 0; k = 0; P_t = 0 is put on for P_s = -p = pressure of current vertical to the line of slope, wherewith the pressure of current is directed outwardly (see Fig. 5) then in Equation (8)

$$a = \frac{-p\, tg\rho}{s\gamma d}\ ;\ b = \frac{1}{s}$$

for s = 1 becomes b = 1. that is it becomes

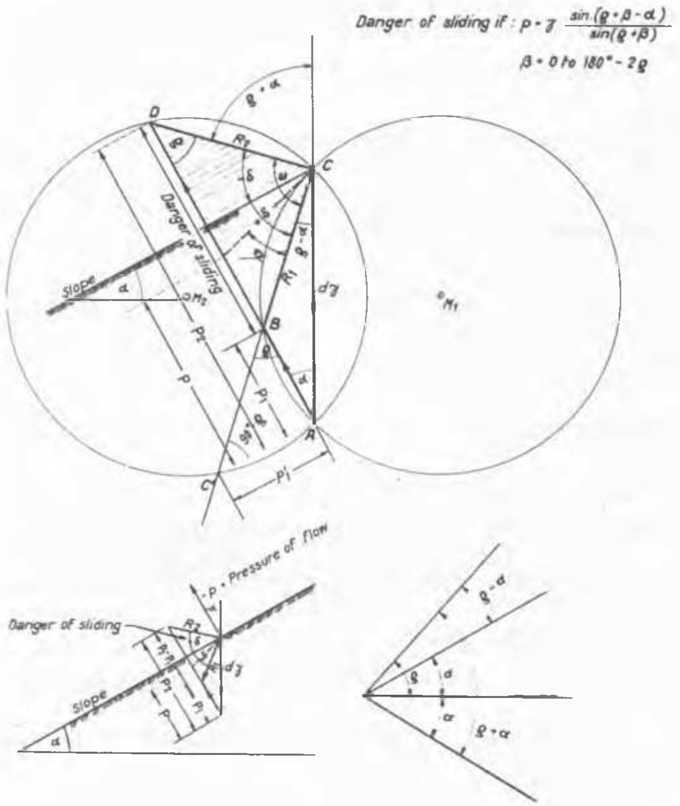
$$\cos\alpha = \left[\frac{p\, tg^2\rho}{\gamma d} \pm \sqrt{1 + tg^2\rho - \frac{p^2 tg^2\rho}{\gamma^2 d^2}}\right] \times \frac{1}{1 + tg^2\rho} \qquad (8a)$$

By corresponding transformation of equation (8a) the relation is found

$$\frac{p}{d\gamma} = +\cos\alpha \pm \frac{\sin\alpha}{tg\rho} \qquad (8b)$$

i.e. there is a double solution present. Through the transformation of equation 8b is found

$$\frac{p}{d\gamma} = \frac{\sin(\rho \mp \alpha)}{\sin\rho} \qquad (20)$$



Determination of danger of sliding with pressure of current towards the outside.

FIG. 5

for d = 1 becomes $\frac{p}{\gamma} = \frac{\sin(\rho \pm \alpha)}{\sin \rho}$ (20a)

Equation (2) regarded as Sine - theorem can be interpreted geometrically as this happened in Fig. 5.

The resulting forces R_1 and R_2 in Fig. 5 are equally great but their directions are, by $2\delta = (180^o - 2\rho)$, different from each other i.e. $\pm\delta = 90^o - \rho$ i.e. the direction of the resultants by weight per unit volume and pressure of current include the angle $\pm\delta = 90^o - \rho$ to the direction of the slope.

Fig. 5 shows that there exists danger of sliding when the pressure of current p is greater than p_1 or smaller than p_2, that is, there is danger of a landslide when the pressure of current p from inside to outside has the magnitude of

$$p = \gamma \frac{\sin(\rho+\beta-\alpha)}{\sin(\rho+\beta)}$$

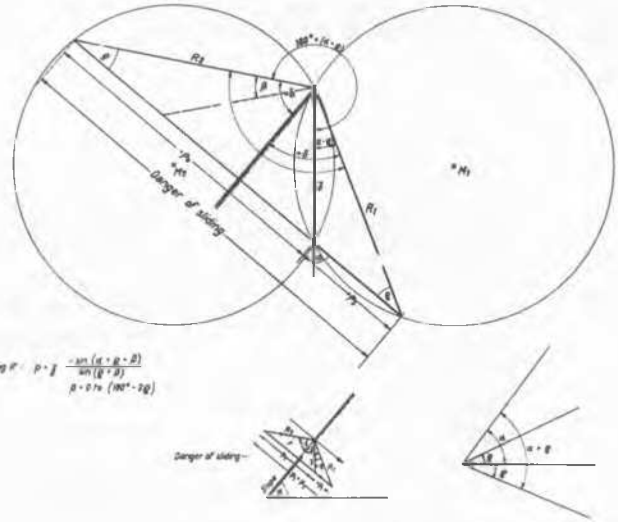
whereby β may be anything from 0 to $(180^o - 2\rho)$. The direction of unfavourable current is shown by the differentiation of the above equation, i.e. it is $dp/d\alpha = 0$.

11) If in Equation 8 is put

$$P_w = 0;\ k = 0;\ p_1 = 0;\ P_s = +p$$

p = pressure of current vertical to the line of slope. The pressure of the current is directed inwardly (see Fig. 6) Through corresponding transformation we obtain equation (8)

$$\frac{p}{\gamma d} = \frac{\sin(\alpha-\rho)}{\sin \rho}\ ;\ \frac{p}{\gamma d} = \frac{-\sin(\alpha-\rho)}{\sin\rho} \quad (21)$$



Determination of danger of sliding with inwards pressure of current.

FIG. 6

For d = 1 becomes $\frac{p}{\gamma} = \frac{\pm\sin(\alpha \mp \rho)}{\sin\rho}$ (21a)

Equation (21) can, regarded as sine - theorem, be interpreted geometrically (see fig. 6). The resultants R_1 and R_2 are equally great and include the angle $2^- = 180^o - 2\rho$.

Fig. 6 shows that there is danger of a landslide with a slope whose angle of slope α is greater than ρ (ρ = Angle of inner friction) when the pressure of the current p inwardly has the magnitude

$$p = \gamma \frac{-\sin(\alpha+\rho+\beta)}{\sin(\rho+\beta)} \quad (21b)$$

where $\beta = 0$ to $(180^o - 2\rho)$.

12) Equation (20a) can also be brought to refer to equation (18) if, namely, the pore-water pressure in equation (18) is put down as $p_w = p/\cos\alpha$

so $N'' = \pm p_w \cos\alpha = \pm p$

becomes $\frac{tg\alpha}{tg\rho_s} = \frac{\gamma \pm \frac{p}{\cos\alpha}}{\gamma}$

By transforming this equation we obtain

$$p = \gamma \frac{\pm\sin(\alpha \pm \rho_s)}{\sin\rho_s} \text{ (cf equation 21a)} \quad (22)$$

ρ_s = Angle of apparent inner friction
$\rho_s = \rho$ for k = 0 that is for soils without binding force.

13) If in equation (21a) for the pressure $p = \gamma' h$ = pressure of water, is put down the form of the slope can be expressed by the angle of slope, having regard to the properties of the material of the dam. The properties of the materials of the dam are determined through Weight per unit of Volume $\gamma = \gamma_s(1-n)$ or $\gamma_s(1-n) + \gamma'$ Angle ρ or ρ_s .

Fig. 6 Determination of danger of landslide with inward pressure of current. The angle of slope α is greater than the angle ρ of inner friction.

If for the specific weight of water $\gamma' = 1$ is set then equation (21a) gives:

$$p = \gamma' \cdot h = \gamma \frac{\sin(\alpha-\rho)}{\sin\rho} = h$$

Reduced we obtain

$$\sin\alpha = \frac{\frac{h}{\gamma}\cot g\,\rho \pm \sqrt{\cot g^2\rho - \frac{h^2-\gamma^2}{\gamma^2}}}{1+\cot g^2\rho} \quad (23)$$

for h = 0 becomes $\sin\alpha = \sin\rho$; $\alpha = \rho$
for h/y = 1 becomes $\sin\alpha = \sin\alpha = 2\,ctg\rho\sin\rho$
for $\rho = 30^o$ becomes $\alpha = 60^o$
for $\alpha = 90^o$ becomes $h = \gamma\, ctg\,\rho$ - vertically standing wall by vertical pressure of current

14) If in equation (8) we put
$k = 0; P_s = 0; P_t = \pm p'; s = 1; d = 1; p_w = 0$
we obtain in equation (8)

$$a = \frac{\pm p'}{\gamma} \quad ; b = 1$$

and

$$\cos\alpha = \frac{\pm\frac{p'}{\gamma}tg\rho \pm \sqrt{1+tg^2\rho - \frac{p'^2}{\gamma^2}}}{1+tg^2\rho} \qquad (24)$$

The equations (24 and 25) differ from equation (21) and (22) only by the fact that in the denominator, instead of $\sin\rho$ the $\cos\rho$ stands. The values p'_1 to $p'4$ as also the values p and p4 can be interpreted geometrically. In Fig. 4 p' is evaluated. In Fig. 5 there is in the triangle A C C' :

$$\frac{p'_1}{\gamma} = \frac{\sin(\rho-\alpha)}{\sin(90^\circ-\rho)} = \frac{\sin(\rho-\alpha)}{\cos\rho} \qquad (25)$$

i.e. If there is pressure of current in the direction of the inclination of the slope present which is greater than p'_1 then there is danger of landslide.

15) The equations (24) and (25) can, provided that for $p' = -\gamma_w \sin\alpha$ is put, be so converted that the following relation is obtained.

$$\frac{\gamma \pm \gamma_w}{\gamma} = \frac{tg\rho}{tg\alpha} \qquad (26)$$

Remark: The equation (26) has been obtained for $P_s = 0$ and $P_t = p'$. whereas the equation (18) was drawn up for $p_s = p$ and $p_1 = 0$ and has given

$$\frac{tg\rho}{tg\alpha} = \frac{\gamma}{\gamma \pm \gamma_w'}$$

i.e. equation (18) represents the reciprocal value of the equation (26).

In case of the validity of Darcy's law the frictional force is p'

$$p' = \gamma . nJ$$

and we obtain:

$$\frac{tg\alpha}{tg\rho} = \frac{J\,\text{critical}}{\gamma \pm n}$$

γ = weight of liquid
n = capacity of cavity
v = velocity of the trickling
J = fall

SUMMARY

The above formulae deduced by the author, are applied practically to determine the tendency to sliding of slopes with clayey material from red marl, Jura etc. The formulae are furthermore applied to determine the stability of hill-sides crossed by water-courses, of roads and reservoirs. Finally the equations have been applied to the calculations of the degree of safety of dams from shocks caused by earthquakes, battering with projectiles.

In addition, instead of the pressure of trickling water, the intensity of the wave of concussion has been given.

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I e 16

STATE OF STRESSES IN A HEAVY SOIL MASS

C. TORRE, VIENNA

1. INTRODUCTION.

In the following, we will discuss the state of stresses in a semi-infinite mass which consists of heavy, cohesionless grains of approximately equal size, and is bounded by a horizontal or inclined plane. In each point of the soil mass prevails limiting equilibrium. The load may be distributed uniformly over the ground surface. Instead of Rankine's 1) boundary condition

$$\tau = \pm u . \sigma \qquad (1)$$

$\mu = tg\rho$, ρ = angle of friction), here, in general, a curved, everywhere constant and differentiable involute curve is assumed as boundary condition. It will be tried, to deal with the theory of breakage of a soil mass as a special case of the technical theory of breakage of solids. Considering the heavy soil mass, it is remarkable that, in all points of a plane parallel to the ground surface, identical states of stresses occur.

In Fig. 1 the involute curve $\tau = \tau(\sigma)$ is dashed. It is assumed as parabola. The pressure stresses are put on the positive σ-axis. The diameter of the stress circle (=circle of curvature) at the vertex of the involute curve furnishes the pressure stress, caused by the first layer of sand grains (see Fig. 2)

$$\sigma_2 = \sigma_3 = 0 \quad \text{and} \quad \sigma_1 = \sigma_0 = \gamma . h, \qquad (2)$$

where γ is the unit weight of the earth, and h the height of a sand grain. The horizontal stress $\sigma_2 = \sigma_3 > 0$ appears but only in the second sand layer (see Fig. 2). In Fig. 2 the x-axis is assumed horizontal, and y in the direction of gravity.

In Fig. 1 the real boundary curve $\tau = \tau(\sigma)$ 2) is dashed-dotted. Combining the vertexes of all the stress circles, we obtain the boundary curve $\tau_2 = \tau_2(\sigma_m)$, wherein

$$\tau_1 = (\sigma_1 - \sigma_3)/2 \quad , \quad \sigma_m = (\sigma_1 + \sigma_3)/2 \qquad (3)$$

For the general case (curved involute) the load may be assumed by

$$\bar{C} = q + \sigma_0 \qquad (4)$$

where q is the load and σ_0 the stress according to Eq. (2). At the surface of the not loaded ground, the shearing angle is equal to zero, if the surface is horizontal. If the surface