for h = 0 becomes $\sin\alpha = \sin\rho$; $\alpha = \rho$
for h/y = 1 becomes $\sin\alpha = \sin\alpha = 2\,\mathrm{ctg}\rho\sin\rho$
for $\rho = 30^\circ$ becomes $\alpha = 60^\circ$
for $\alpha = 90^\circ$ becomes $h = \gamma\,\mathrm{ctg}\rho$ = vertically standing wall by vertical pressure of current

14) If in equation (8) we put
$k = 0; P_s = 0; P_t = \pm p'; s = 1; d = 1; p_w = 0$
we obtain in equation (8)

$$a = \frac{\pm p'}{\gamma} \quad ; b = 1$$

and

$$\cos\alpha = \frac{\pm\frac{p'}{\gamma}\mathrm{tg}\rho \pm \sqrt{1+\mathrm{tg}^2\rho - \frac{p'^2}{\gamma^2}}}{1+\mathrm{tg}^2\rho} \qquad (24)$$

The equations (24 and 25) differ from equation (21) and (22) only by the fact that in the denominator, instead of $\sin\rho$ the $\cos\rho$ stands. The values p'_1 to p'_4 as also the values p and p4 can be interpreted geometrically. In Fig. 4 p' is evaluated. In Fig. 5 there is in the triangle A C C' :

$$\frac{p'_1}{\gamma} = \frac{\sin(\rho-\alpha)}{\sin(90^\circ-\rho)} = \frac{\sin(\rho-\alpha)}{\cos\rho} \qquad (25)$$

i.e. If there is pressure of current in the direction of the inclination of the slope present which is greater than p'_1 then there is danger of landslide.

15) The equations (24) and (25) can, provided that for $p' = -\gamma_w \sin\alpha$ is put, be so converted that the following relation is obtained.

$$\frac{\gamma \pm \gamma_w}{\gamma} = \frac{\mathrm{tg}\rho}{\mathrm{tg}\alpha} \qquad (26)$$

Remark: The equation (26) has been obtained for $P_s = 0$ and $P_t = p'$. whereas the equation (18) was drawn up for $p_s = p$ and $p_1 = 0$ and has given

$$\frac{\mathrm{tg}\rho}{\mathrm{tg}\alpha} = \frac{\gamma}{\gamma \pm \gamma_w'}$$

i.e. equation (18) represents the reciprocal value of the equation (26).

In case of the validity of Darcy's law the frictional force is p'

$$p' = \gamma . nJ$$

and we obtain:

$$\frac{\mathrm{tg}\alpha}{\mathrm{tg}\rho} = \frac{J\,\text{critical}}{\gamma \pm n}$$

γ = weight of liquid
n = capacity of cavity
v = velocity of the trickling
J = fall

SUMMARY

The above formulae deduced by the author, are applied practically to determine the tendency to sliding of slopes with clayey material from red marl, Jura etc. The formulae are furthermore applied to determine the stability of hill-sides crossed by water-courses, of roads and reservoirs. Finally the equations have been applied to the calculations of the degree of safety of dams from shocks caused by earthquakes, battering with projectiles.

In addition, instead of the pressure of trickling water, the intensity of the wave of concussion has been given.

REFERENCE

1) Hvorslev M.J. About the characteristics of the solidity of disturbed, binding ground. Ingeniorvidensk. Skr. Ser.A, Nr. 45, Kopenhagen 1937.
Cf. A. Pitowar. About maximum slope of dry cones of rubble and dumps of rubble. p. 22 - 26, Zürich 1903)

-o-o-o-o-o-o-

I e 16

STATE OF STRESSES IN A HEAVY SOIL MASS

C. TORRE, VIENNA

1. INTRODUCTION.

In the following, we will discuss the state of stresses in a semi-infinite mass which consists of heavy, cohesionless grains of approximately equal size, and is bounded by a horizontal or inclined plane. In each point of the soil mass prevails limiting equilibrium. The load may be distributed uniformly over the ground surface. Instead of Rankine's 1) boundary condition

$$\tau = \pm u.\sigma \qquad (1)$$

$\mu = \mathrm{tg}\rho$, ρ = angle of friction), here, in general, a curved, everywhere constant and differentiable involute curve is assumed as boundary condition. It will be tried, to deal with the theory of breakage of a soil mass as a special case of the technical theory of breakage of solids. Considering the heavy soil mass, it is remarkable that, in all points of a plane parallel to the ground surface, identical states of stresses occur.

In Fig. 1 the involute curve $\tau = \tau(\sigma)$ is dashed. It is assumed as parabola. The pressure stresses are put on the positive σ-axis. The diameter of the stress circle (=circle of curvature) at the vertex of the involute curve furnishes the pressure stress, caused by the first layer of sand grains (see Fig. 2)

$$\sigma_2 = \sigma_3 = 0 \quad \text{and} \quad \sigma_1 = \sigma_0 = \gamma.h, \qquad (2)$$

where γ is the unit weight of the earth, and h the height of a sand grain. The horizontal stress $\sigma_2 = \sigma_3 > 0$ appears but only in the second sand layer (see Fig. 2). In Fig. 2 the x-axis is assumed horizontal, and y in the direction of gravity.

In Fig. 1 the real boundary curve $\tau = \tau(\sigma)$ 2) is dashed-dotted. Combining the vertexes of all the stress circles, we obtain the boundary curve $\tau_2 = \tau_2(\sigma_m)$, wherein

$$\tau_1 = (\sigma_1 - \sigma_3)/2 \quad , \quad \sigma_m = (\sigma_1 + \sigma_3)/2 \qquad (3)$$

For the general case (curved involute) the load may be assumed by

$$\bar{\sigma} = q + \sigma_0 \qquad (4)$$

where q is the load and σ_0 the stress according to Eq. (2). At the surface of the not loaded ground, the shearing angle is equal to zero, if the surface is horizontal. If the surface

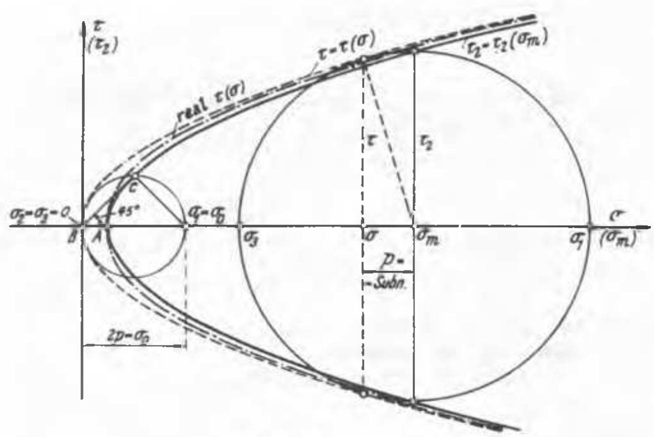

FIG. 1

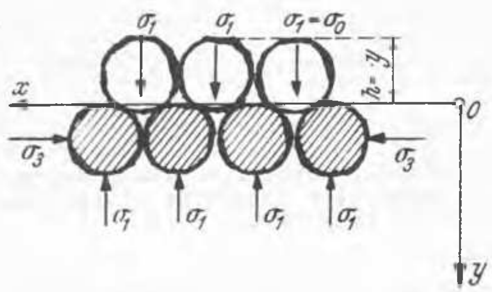

FIG. 2

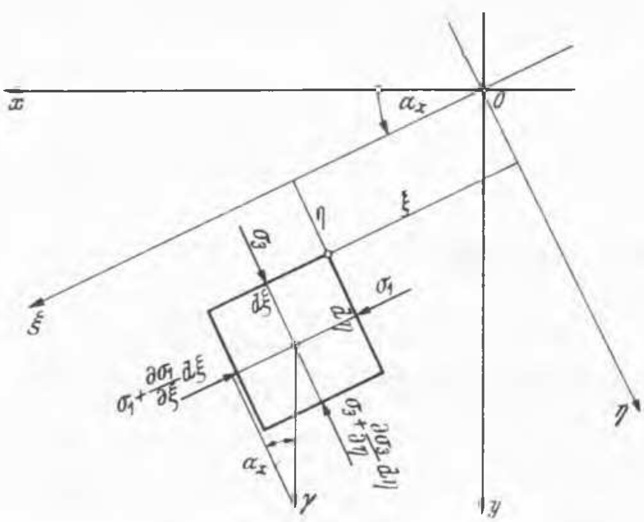

FIG. 3

is loaded, this shearing angle becomes greater than zero; its size will be found out according to the intensity of the load and the choice of the involute curve.

2. LIMITING STRAINS.

In this chapter we shall express the boundary condition $\sigma_1 = \sigma_1(\sigma_3)$ in parameter representation x and y. In Fig. 3 a system of coordinates (x,y) is assumed, where the principal pressure direction (σ_1) and the x-axis form the constant angle α_x. Thus, the ground surface is not horizontal, but inclined to the x-axis at an angle δ (not shown in Fig. 3). By turning the coordinate axes round the angle α_x, we obtain the new system of coordinates (ξ, η), the axes of which coincide with the principal normal stresses. The equation of transformation are, as known (5a)

$$x = \xi \cos\alpha_x - \eta \sin\alpha_x \quad , y = \xi \sin\alpha_x + \eta \cos\alpha_x \tag{5b}$$

and $\xi = x\cos\alpha_x + y\sin\alpha_x \quad , \eta = y\cos\alpha_x - x\sin\alpha_x$

In Fig. 3 the lateral faces of the element of volume dξ.d .1 are parallel to ξ, η. Then, the condictions of equilibrium are

$$\frac{\partial\sigma_1}{\partial\xi} = \gamma\sin\alpha_x \ , \quad \frac{\partial\sigma_3}{\partial\eta} = \gamma\cos\alpha_x \tag{6}$$

The integral of Eq. (6) with α_x = constant, is

$$\left.\begin{aligned} \sigma_1 &= \gamma\cdot\xi\cdot\sin\alpha_x + \varphi(\eta) + C_1 \\ \sigma_3 &= \gamma\cdot\eta\cdot\cos\alpha_x + \psi(\xi) + C_2 \end{aligned}\right\} \tag{7}$$

The functions $\varphi(\eta)$ and $\psi(\xi)$ are computed from the boundary condition and especially from the comparison of variables. As an example, we take Rankine's 1) boundary condition, which reads

$$\tau_2 - \sigma_m \cdot \sin\rho = 0 \tag{8}$$

From Eqs. (8), (7) and (3) we find

$$\varphi(\eta)(1-\sin\rho) - \psi(\xi)(1+\sin\rho) = -\gamma\cdot\xi\cdot\sin\alpha_x(1-\sin\rho) + \gamma\cdot\eta\cdot\cos\alpha_x(1+\sin\rho) - C_1(1-\sin\rho) + C_2(1+\sin\rho) \tag{9}$$

From the comparison of variables, we obtain from Eq. (9)

$$\left.\begin{aligned} \varphi(\eta) &= (\gamma\cdot\eta\cdot\cos\alpha_x + C_2)\,\frac{1+\sin\rho}{1-\sin\rho} \\ \psi(\xi) &= (\gamma\cdot\xi\cdot\sin\alpha_x + C_1)\,\frac{1-\sin\rho}{1+\sin\rho} \end{aligned}\right\} \tag{10}$$

From Eqs. (7) and (10) we get the principal normal stresses to

$$\left.\begin{aligned} \sigma_1 &= (\gamma\cdot\xi\cdot\sin\alpha_x + C_1) + (\gamma\cdot\eta\cdot\cos\alpha_x + C_2)\,\frac{1+\sin\rho}{1-\sin\rho} \\ \sigma_3 &= (\gamma\cdot\xi\cdot\sin\alpha_x + C_1)\,\frac{1-\sin\rho}{1+\sin\rho} + (\gamma\cdot\eta\cdot\cos\alpha_x + C_2) \end{aligned}\right\} \tag{11}$$

C_1 and C_2 are the constant pressure stresses, acting in the direction σ_1 or σ_3. For the active ($\alpha_x = \pi/2$) and passive ($\alpha_x = 0$) earth pressure, and for the not loaded ground surface ($C_1 = C_2 = 0$), the Eqs. (11) are conform with the Rankine's equations 3)

3. FIELD OF SHEARING LINES.

In Fig. 4 a system of coordinates (x,y) is assumed like that in Fig. 3. From the point M (x,y) start the shearing lines I and II. Their tangents from the same angles α_3 with the direction of the stress σ_1, and their normal lines from the same angle with the direction of the stress σ_1, respectively. The tangent at the shearing line I forms the angle ϑ with the x-axis; it is: $\vartheta = \alpha_x + \alpha_3$. If we denote the equation of the shearing line I by y=y(x), we have

$$\operatorname{tg}\vartheta = \frac{dy}{dx} = \frac{\operatorname{tg}\alpha_x + \operatorname{tg}\alpha_3}{1 - \operatorname{tg}\alpha_x\cdot\operatorname{tg}\alpha_3} \tag{12}$$

In Eq. (12) is $\operatorname{tg}\alpha_x$ = constant and

$$\operatorname{tg}\alpha_3 = \pm 1/\sqrt{\sigma_1'} \quad , \quad \sigma_1' = d\sigma_1/d\sigma_3 \tag{13}$$

In Eq. (13) σ_1' is the first derivation of the boundary condition $\sigma_1 = \sigma_1(\sigma_3)$.
From Eqs. (12) and (13) we obtain the differ-

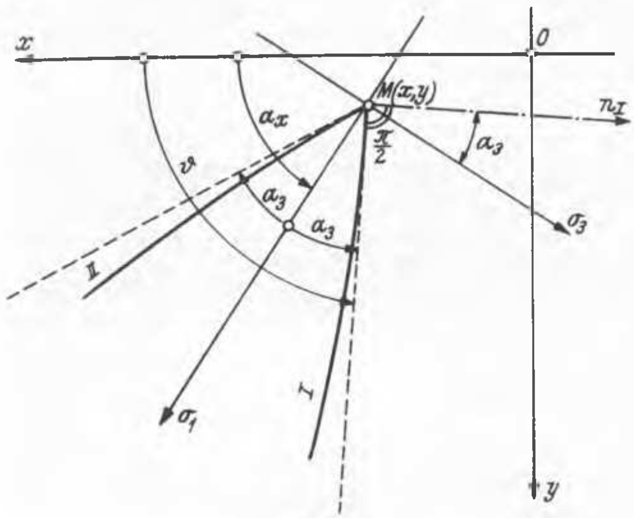

FIG. 4

ential equation of the field of shearing lines

$$\frac{dy}{dx} = \frac{1 \pm \sqrt{\sigma_1'} \cdot tg\alpha_x}{\pm\sqrt{\sigma_1'} - tg\alpha_x} \tag{14}$$

Here σ_1' must be expressed as function of x and y, which can easily be obtained, when σ_1 and σ_3 are expressed in parameter representation. From Eq. (7) results, when $d\eta = 0$ [$\eta = \sigma_1$ =const. are the straight lines parallel to the ξ axis, or according to Eq. (5b)

$$d\eta = dx \cdot \cos\alpha_x \, (dy/dx - tg\alpha_x) = 0$$

because $dy/dx = tg\alpha_x$]

$$\sigma_1' = \frac{d\sigma_1}{d\sigma_3} = \frac{d\sigma_1}{d\xi} \Big/ \frac{\partial\sigma_3}{\partial\xi} = \gamma \cdot \sin\alpha_x \Big/ \frac{\partial\psi(\xi)}{\partial\xi} \tag{15}$$

Eq. (15) is fit for the calculation of the case $\alpha_x = \pi/2$. If in our calculation occurs $\alpha_x = 0$, we have to calculate with $d\xi = 0$. Then, from Eq. (7) we have

$$\sigma_1' = \frac{d\sigma_1}{d\sigma_3} = \frac{\partial\sigma_1}{\partial\eta} \Big/ \frac{\partial\sigma_3}{\partial\eta} = \frac{\partial\varphi(\eta)}{\partial\eta} \Big/ \gamma \cdot \cos\alpha_x \tag{16}$$

If the introduce Eq. (15) or (16) and Eq.(5b) into Eq. (14), we then obtain the differential equation of the field of shearing lines.

As an example, we take the state of stresses after Rankine. Then, according to Eq. (10)

$$\frac{\partial\psi(\xi)}{\partial\xi} = \gamma \cdot \sin\alpha_x \cdot \frac{1-\sin\rho}{1+\sin\rho} \tag{17}$$

and according to Eqs. (15) and (17)

$$\sigma_1' = \frac{1+\sin\rho}{1-\sin\rho} = \cot^2\alpha_s \quad , \quad \alpha_s = \frac{\pi}{4} - \frac{\rho}{2} \tag{18}$$

From Eq. (18) and (14) we obtain the differential equation

$$\frac{dy}{dx} = \frac{tg\alpha_x \pm tg\alpha_s}{1 \mp tg\alpha_x \, tg\alpha_s} = tg(\alpha_x \pm \alpha_s) = tg\vartheta_{I,II}$$

or after integration $y = x . tg\vartheta_{I,II} +$ const. (19)

Therefore, the field of shearing lines consists of two groups of straight lines, which form the constant angle $\vartheta_{I,II}$ with the x-axis.

Using Eq. (16), we should obtain the same result.

4. STATE OF STRESSES ACCORDING TO THE INVOLUTE PARABOLA.

As boundary condition we use the involute parabola from Fig. 1. Its equation is

$$\tau^2 = 2p . \sigma \tag{20}$$

According to the equations of transformation (they give the limiting stresses σ_1 and σ_3, when the involute curve $\tau = \tau(\sigma)$ is given; 2)

$$\sigma_{1,3} = \sigma + \tau\tau' \pm \sqrt{\tau^2 + (\tau\,\tau')^2} \quad , \tau' = d\tau \;\, d\sigma \tag{21}$$

we obtain from Eq. (20) the boundary condition in the form

$$(\sigma_1 - \sigma_3)^2 - 4p(\sigma_1 + \sigma_3) + (2p)^2 = 0 \tag{22}$$

From Eq. (22) follows for $\sigma_3 = 0 \ldots . \sigma_1 = + \sigma_0$: $2p = \sigma_0$ and hence

$$(\sigma_1 - \sigma_3)^2 - 2\sigma_0(\sigma_1 + \sigma_3) + \sigma_0^2 = 0 \tag{23}$$

or according to Eq. (3) we get the boundary condition to

$$\tau_1^2 - \sigma_0 \cdot \sigma_m + (\sigma_0/2)^2 = 0 \tag{24}$$

From Eqs. (7) and (24), by comparing the variables and solution of the quadratic equation, we get the functions

$$\varphi(\eta) = -(C - \sigma_0) + \gamma \cdot \eta \cdot \cos\alpha_x + \sqrt{(C-\sigma_0)^2 + 4\sigma_0 \cdot \gamma \cdot \eta \cdot \cos\alpha_x} \tag{25}$$

$$\psi(\xi) = +(C + \sigma_0) + \gamma \cdot \xi \cdot \sin\alpha_x - 2\sqrt{\sigma_0(C + \gamma \cdot \xi \cdot \sin\alpha_x)} \tag{26}$$

Following, the ground surface may be horizontal and not loaded. Then, according to Eq. (4) are : $q = 0$, $\bar{C} = C = \sigma_0$.

Further may be $\alpha_x = \pi/2$. Then, according to Eq. (25) is $\varphi(x) = 0$ and according to Eq. (26)

$$\psi(y) = 2\sigma_0 + \gamma \cdot y - 2\sqrt{\sigma_0(\sigma_0 + \gamma \cdot y)} \tag{27}$$

From Eqs. (14) and (15), when $\alpha_x = \pi/2$, we obtain the differential equation

$$\pm\sqrt{\frac{1}{\gamma} \cdot \frac{d\psi(y)}{dy}} \cdot dy = dx \tag{28}$$

From Eqs. (27) and (28) we get an integral, which, after substituting

$$z = \sqrt{1 + \gamma \cdot y/\sigma_0} \tag{29a}$$

gives

$$x = \pm \frac{2\sigma_0}{\gamma} \int \sqrt{z^2 - z} \cdot dz + K \tag{29b}$$

or, after integration, when $h = \sigma_0/\gamma$, according to Eq. (2) (about this integral see 5)

$$x = \pm \frac{h}{4}\left[2(2z-1) \cdot \sqrt{z^2-z} - \cosh^{-1}(2z-1)\right] + K \tag{29}$$

According to Eq. (29a) we may calculate from z the depth y

$$y = (z^2 - 1) . h \tag{30}$$

Therefore, the involute parabola furnishes the equation for the stresses and shearing lines, that depend on the depth y and the grain size h. A defect of this calculation is that, referring to the very small parameter $2p = \sigma_0$, the state of stresses is almost like that of the hydrostatical pressure. If, for instance, $h = 0.1$ cm, $\gamma = 0.0018$ kg/cm^3, then $\sigma_0 = \gamma . h = 0.0018$ kg/cm^2. According to Eqs. (27) and (7) the principal normal stresses are

$$\sigma_1 = \gamma \cdot y + \sigma_0 \; , \; \sigma_3 = \gamma \cdot y + 2\sigma_0 - 2\sqrt{\sigma_0(\sigma_0 + \gamma \cdot y)} \tag{31}$$

and hence

$$\tau_1 = (\sigma_1 - \sigma_3)/2 = \sqrt{\sigma_0(\sigma_0 + \gamma \cdot y) - \sigma_0} \tag{32}$$

For the above numerical values in a depth of

y = 10 m, the principal shearing stress Eq.(32) is τ_2 = 0·0178 kg/cm², opposite to a principal normal stress Eq. (31): σ_1 = 1.80 kg/cm². For the pebbles, when h = 10cm, y = 10 m, γ =1.8 t/m³, is τ_2 = 0.163 kg/cm² opposite to σ_1 = =1.82 kg/cm².
The involute parabola therefore furnishes very little shearing stresses. The dependence of the shearing stresses from the grain size is caused by the choice of the involute curve.

In Table 1 the numerical values for x and y are calculated according to Eqs. (29) and (30). Besides, in this Table individual terms of Eq. (29) are calculated in order to show their influence at increasing numbers z.

the term arc cosh (2z - 1) in Eq. (29) can be neglected against the first term, what is visible from Table 1. Besides, 1 may be neglected against z. Then, according to Eq. (30) is : $y = z^2 . h$ and from Eq. (29)

$$x = \pm y + C \qquad (33)$$

Thus, we see that in great depths the shearing lines Eq. (29) approach the straight lines, which form an angle of 45 with the x - axis (= ground surface)

From Eq. (29) is min z=+1 : for z<1 the abscissa x is imaginary. When $y = -\sigma_0/\gamma = -h$, from Eq. (29) results : z=0; y=-h is the coordinate of the ground surface.

TABLE 1.

Diameter of grain h = 0,1 cm.

z	Eq(30) y (cm)	$\sqrt{z^2-z}$	2z-1	2.(c).(d)	$\cosh^{-1}(2z-1)$	Eq.(29) x-K (cm)
(a)	(b)	(c)	(d)	(e)	(f)	(g)
1	0	0	1	0	0	0
2	0,3	1,4142	3	8,4852	1,7628	± 0,1681
3	0,8	2,4495	5	24,4950	2,2925	± 0,5551
4	1,5	3,4641	7	48,4974	2,6339	± 1,1466
5	2,4	4,4721	9	80,4978	2,8871	± 1,9403
6	3,5	5,4772	11	120,4984	3,0889	± 2,9352
7	4,8	6,4807	13	168,4982	3,2566	± 4,1310
8	6,3	7,4833	15	224,4990	3,4023	± 5,5274
9	8,0	8.4853	17	288,5002	3,5255	± 7,1244
10	9,9	9,4868	19	360,4948	3,6432	± 8,9214

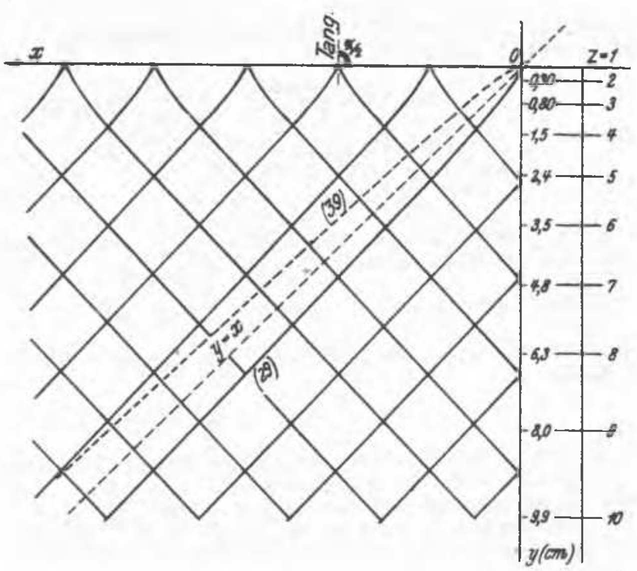

FIG. 5

In Fig. 5, the field of shearing lines is drawn according to the data of Table 1. The shearing lines, calculated according to the involute parabola have as we suppose, the advantage of intersection at an angle which, at increasing depth, approaches the angle of 90°.

When z = 1 and y = 0, respectively, the shearing line in Fig. 5 has a vertical tangent. By increasing $z \rightarrow \infty$, i.e. z becomes $z \gg 1$, then,

If we have to deal with passive earth pressure (α_x = 0), then, from Eq. (26) with q = 0 : $\psi(x)$=0 and according to Eq. (25), we obtain

$$\varphi(y) = \gamma \cdot y + 2\sqrt{\sigma_0 \cdot \gamma \cdot y} \qquad (34)$$

From Eqs. (16) and (34) we obtain

$$\sigma_1' = \frac{1}{\gamma} \cdot \frac{d\varphi(y)}{dy} = 1 + \frac{\sqrt{h}}{\sqrt{y}} \qquad (35)$$

From Eqs. (14) and (35), when α_x = 0, we obtain the differential equation

$$\pm\sqrt{1 + \frac{\sqrt{h}}{\sqrt{y}}} \cdot dy = dx \qquad (36)$$

After substituting

$$y = z^2 . h \qquad (37)$$

the integral Eq. (36) is

$$x = \pm 2h \int \sqrt{z^2+z} \, . \, dz \qquad (38)$$

with the solution

$$x = \pm \frac{h}{4}\left[2(2z+1)\cdot\sqrt{z^2+z} - \cosh^{-1}(2z+1)\right] + K_1 \qquad (39)$$

In Fig. 5 a shearing line Eq. (39) for K_1 = 0 is dashed. The numbers in the last column of Table 1 hold also here, when z = z + 1. At the position y = 0 of the shearing lines Eq. (39) the tangents are horizontal.

REFERENCES

1) W.M.I. Rankine. London Phil. Trans. 147, part I (1857) p.9
2) C. Torre. Oesterr. Ing.-Archiv 1, 36 u. 316 (1946/47).
3) Handbuch der Physik, vol. VI.; Essay from

A. Nádai: Plastizität und Erddruck, p. 428, Springer-Verlag 1928.

4) Eq. (13) is adapted to the denotions of this country. For $\sigma_1' = \sigma_1/\sigma_3$ Eq. (13) furnishes that angle α_3, which gives the greatest inclination between the normal lines of the section area and the resulting stress See, for instance, F. Wittenbauer: Aufgaben aus der technischen Mechanik; vol. II, Festigkeitslehre; problem Nr. 12, edition 3, Springer-Verlag 1918.-
See also H. Krey Erddruck, Erdwiderstand und Tragfähigkeit des Baugrundes. Edition 5; completed by J. Ehrenberg: Eq. (14), (15) p.32, 33. Ernst und Sohn 1936.

5) F. Tölke. Praktische Funktionslehre, p. 127, Eq. (229). Springer-Verslag 1943.

ACKNOWLEDGEMENTS:

The author is indebted to Prof.Dr. Karl Wolf for his valuable assistance in working out the analytic part of this paper and to Prof.Dr. O.K. Fröhlich for his assistance concerning the sphere of Soil Mechanics.
This work with alterations will also be published in the "Transactions of the Akedemie der Wissenschaften in Wien (Math.-Naturw. Klasse)."

-o-o-o-o-o-o-

SUB-SECTION If

EARTH PRESSURE

If 3

MINIMUM VALUE OF EARTH PRESSURE

Prof. Dr. JÓZSEF JÁKY

1. ASSUMPTIONS OF THE CLASSICAL EARTH PRESSURE THEORY.

According to Classical Earth Pressure Theory active earth pressure E acting on the back of the retaining wall is a component of the weight (G) of the wedge sliding along plane AC (Fig. 1).

Coulomb assumed a plane sliding surface starting from lower corner (A) of the retaining wall and inclined at angle to the horizontal. With varying $\sphericalangle\alpha$ the earth pressure passes through a maximum decisive for proportioning the wall. Force Q acting on the sliding plane consumes at instant of sliding the entire internal friction, therefore it includes angle φ with the normal line, the other reaction E includes unknown angle δ with the normal to the wall, and ψ with the vertical, respectively. Therefore, the earth pressure is essentially a function of two variables $E = f(\alpha, \psi)$ particularly

$$E = G(\alpha)\frac{\sin(\alpha-\varphi)}{\sin(\alpha-\varphi+\psi)} \qquad (1)$$

The classical earth pressure theory assumes after Coulomb the angle ψ as a constant that is as known and determines the value of α where

$$\frac{dE}{d\alpha} = 0 \quad \text{i.e.}$$

$$\frac{dG}{d\alpha}\,\frac{\sin(\alpha-\varphi)}{\sin(\alpha-\varphi+\psi)} + G\,\frac{\sin\psi}{\sin^2(\alpha-\varphi+\psi)} = 0 \qquad (2)$$

Since

$$\frac{G\sin\psi}{\sin(\alpha-\varphi+\psi)} = Q \quad \text{and} \quad \frac{dG}{d\alpha} = -\frac{h^2\gamma}{2}$$

introducing these values into Eq. (2)

$$Q = \frac{h^2\gamma}{2}\sin(\alpha-\varphi) \qquad (3)$$

Rebhann's wellknown theorem is obtained from which the different theorems of the classical theory originate (equality of areas, Poncelet's graphical solution). These determine

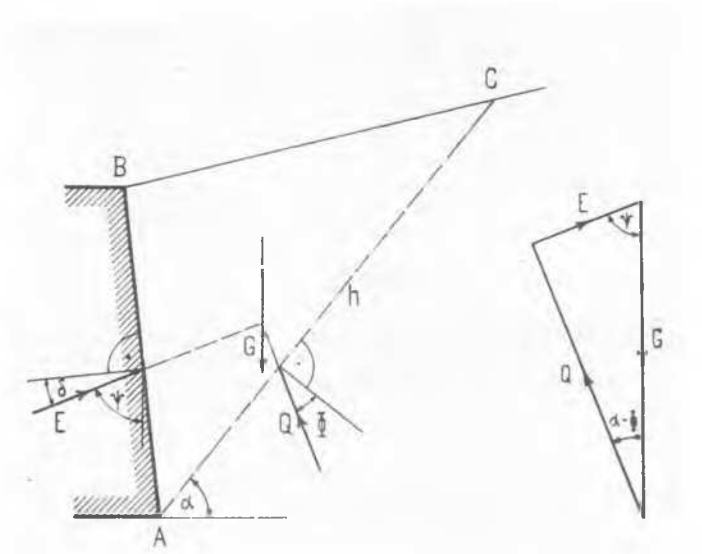

FIG. 1

the maximum value of earth pressure corresponding to a given wall friction angle. Since angle may be chosen between $\delta = 0 - \varphi$ the value of E remains indefinite as long as $\sphericalangle\delta$ is not influenced by any given relationship.

Another shortcoming of the classical earth pressure theory is that the points of attack are unknowns, the theory does not give the points of attack either of E of of Q, and will not know instead of forces.

2. NEW THEOREMS OF THE CLASSICAL THEORY.

Plotting values of active earth pressure corresponding to various values of α and δ in a spacial system of coördinates, the saddle shaped surface on Fig. 2. is obtained. Saddle point P gives the absolute minimum of earth pressure.

It is probable and it may be proved theoretically as well as experimentally that this is the decisive earth pressure for calculating