199

INFLUENCE VALUES FOR ESTIMATING STRESSES IN ELASTIC FOUNDATIONS

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SUMMARY.

This paper deals with the problem of estimating the vertical normal stresses in quasi elastic foundations due to building loads. Contributions to this subject from the theory of elasticity are briefly reviewed. Influence values derived from the Boussinesq and Westergaard solutions are presented for loading conditions that are frequently encountered in building settlement investigations; namely, for the case of loads applied at a point, distributed uniformly along a line of finite length, and distributed uniformly over rectangular and circular areas.

Influence values for some of the above loading conditions as computed from the Boussinesq solution have been previously published. To the writer's knowledge, influence values corresponding to the Westergaard solution have not appeared in the technical literature. A major purpose of this paper is to summarize for ready reference and in a form, which the writer has found convenient, influence value data obtained from both the Westergaard and Boussinesq solutions that one may find desirable to have available when making a stress analysis for the computation of building settlements.

BRIEF REVIEW OF CONTRIBUTIONS FROM THE THEORY OF ELASTICITY

The determination of vertical normal stresses in clay-like strata underlying a structural foundation constitutes a major part of a settlement investigation. Rigorous solutions by Boussinesq 1), Burmister 2) and Westergaard 3) are available from the theory of elasticity that are strictly applicable to the case of semi-infinite masses that deform in accordance with Hooke's Law as a result of normal loads applied to their surfaces. The solution by Boussinesq is applicable to the case of a mass that has the properties of a homogeneous, elastic and isotropic solid. Burmister's solutions are applicable to layered systems each layer of which is assumed to be homogeneous, isotropic and elastic. Westergaard's solution is applicable to a homogeneous mass that is assumed to be reinforced internally in such a manner that horizontal displacements are entirely prevented.

Boussinesq's solution, 1885, has been applied extensively to investigations involving the analysis of stresses and displacements in clay-like soil masses. The equations for stresses and displacements derived from Boussinesq's solution that apply to various types of loading have been summarized by Gray 4). Jurgenson 5) has contributed influence tables and diagrams for stresses due to uniformly loaded long strips and uniformly loaded circular areas and for long strips with triangular and terrace loadings. Newmark has developed the solution and influence values for the vertical normal stresses due to a load uniformly distributed over a rectangular area 6); he has developed influence charts to simplify the computation of vertical stress on horizontal planes, the sum of the principal stresses (bulk stress), the horizontal stress on vertical planes and the components of shearing stress on horizontal and vertical planes due to distributed vertical loads 7). He has also contributed influence charts to simplify the computation of vertical displacements at the surface or within the interior of an elastic, homogeneous, and isotropic mass bounded by a plane horizontal surface and loaded by distributed vertical loads at the surface 8). Mathematical expressions for the displacement of the surface of a semi-infinite, elastic, homogeneous, and isotropic mass due to loading a rectangular area with a uniform pressure, a uniform shearing force per unit area x), and a moment have been developed by Vogt 9). These contributions have reduced the problem of determining the stresses and elastic displacements in an idealized, elastic, homogeneous and isotropic mass of semi-infinite extent due to normal loads applied to its surface to a comparatively simple task.

The more recent solutions by Westergaard, 1939, and Burmister, 1943, which are applicable respectively to masses in which horizontal displacements are prevented and to layered systems, have not as yet been used extensively even though the properties of the idealized masses to which they apply may, in some instances, represent more closely the properties of the real soil mass. Burmister's solution has found application in the design of airport runways 10). The writer has applied the Westergaard solution in a study of building settlements 11).

To simplify the application of Westergaard's solution to the problem of building settlements, the writer developed 11) the expressions and influence values for vertical normal stresses due to loading conditions that are encountered frequently in settlement investigations. Influence values for these loading conditions corresponding to both the Westergaard and Boussinesq solutions are presented herein.

INFLUENCE VALUES

The loads that are produced by a building can, in general, be treated as point loads, loads distributed uniformly along a line of finite length or distributed uniformly over a rectangular or circular area. To simplify a stress analysis, it is desirable to express the equation for the stress resulting from a given type of loading in terms of dimensionless ratios, the quantities constituting the ratios being dimensions that can be easily measured from a load plan. In this form, the right hand member of the equation can be solved once and for all for various values of the

x) Developed from Cerrutti's solution. See for example "A Treatise on the Mathematical Theory of Elasticity, "by A.E.H. Love, Cambridge University Press, 1927.

dimensionless rations; the results obtained are called influence values.

Influence values for the vertical stresses acting on horizontal planes and produced by loads applied at a point, applied uniformly along a line of finite length, and applied uniformly over rectangular and circular areas for both the Boussinesq and Westergaard solutions follow. For each loading condition a figure defining the coordinate system is given and the equations applicable to the case stated. Expressions for the stresses derived from the Boussinesq and Westergaard solutions are distinguished by the subscripts B and W respectively.

a. Load Applied at a Point

The vertical stress at B due to a load P applied at A on the surface of a semi-infinite mass is expressed in terms of the coordinate system shown in Fig. 1a and in accordance with the Boussinesq and Westergaard solutions respectively as follows:

$$\sigma_{zB} = \frac{3P}{2\pi} \cdot \frac{z^3}{(r^2+z^2)^{5/2}} \qquad 1_B$$

$$\sigma_{zW} = \frac{P \cdot K}{2\pi} \cdot \frac{z}{(r^2+K^2z^2)^{3/2}} \qquad 1_W$$

wherein $K = \sqrt{\frac{1-2\mu}{2(1-\mu)}}$ μ = Poisson's ratio x)

In dimensionless form these equations become

$$\sigma_{zB} \cdot \frac{z^2}{P} = \frac{3}{2\pi} \cdot \left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2} \qquad 2_B$$

$$\sigma_{zW} \cdot \frac{z^2}{P} = \frac{K}{2\pi} \cdot \left[\frac{1}{\left(\frac{r}{z}\right)^2+K^2}\right]^{3/2} \qquad 2_W$$

The values of the right hand members of these latter equations (designated P_{oB} 12) and P_{oW} respectively) for various values of the ratio r/z are given in Table 1.The values stated for P_{oW} are valid for $K = \sqrt{2}/2$; that is,for the case of $\mu = 0$.

To find the value of a stress produced by a point load it is necessary only to measure the distance, r, from the point of load application to the point on the surface immediately above the point at which the stress is desired; divide this distance, r, by the vertical distance, z, between the plane on which the load is applied and the plane on which the stress is desired; select the influence value P_{oB} or P_{oW} corresponding to the value of the ratio r/z; and compute the stress from the respective equations:

$$\sigma_{zB} = \frac{P}{z^2} \cdot P_{oB} \qquad 3_B$$

$$\sigma_{zW} = \frac{P}{z^2} \cdot P_{oW} \qquad 3_W$$

b. Line Load of Finite Length

The vertical stress at B due to a load of p per unit of length applied along a line of length, y, is expressed in terms of the coordinate system shown in Fig. 1b and in accordance with the Boussinesq and Westergaard solutions respectively as follows:

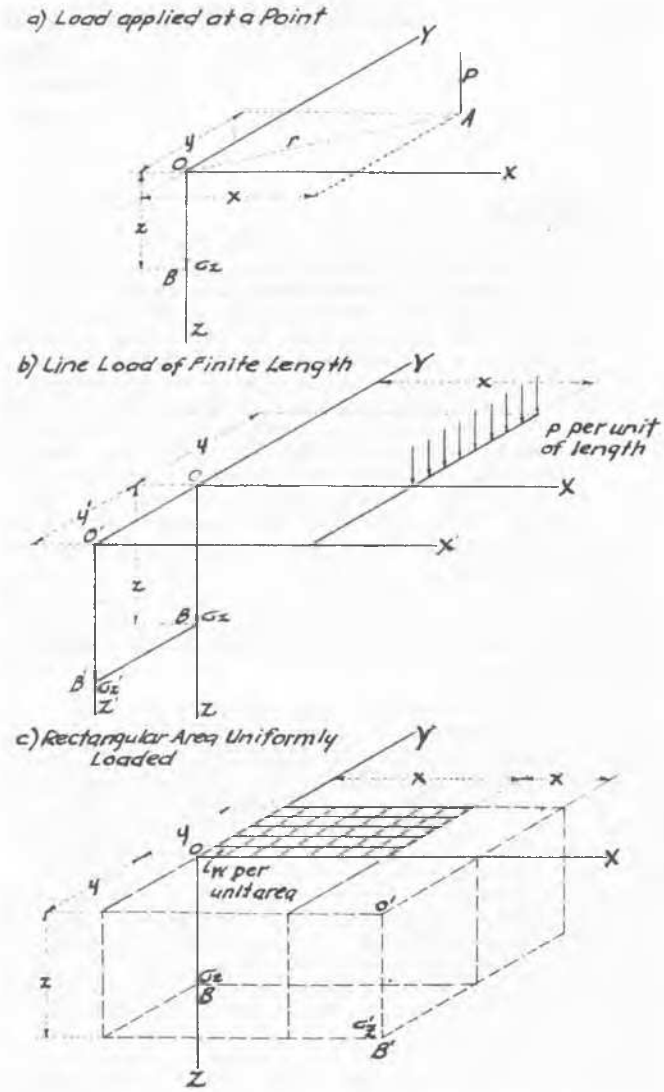

FIG. 1

$$\sigma_{zB} = \frac{p}{2\pi} \cdot \frac{yz^3}{(x^2+z^2)} \cdot \frac{1}{\sqrt{x^2+y^2+z^2}} \cdot \left[\frac{1}{(x^2+y^2+z^2)} + \frac{2}{(x^2+z^2)}\right] \qquad 4_B$$

$$\sigma_{zW} = p \cdot \frac{Kz}{2\pi} \left[\frac{y}{(x^2+K^2z^2)} \cdot \frac{1}{(x^2+y^2+K^2z^2)^{1/2}}\right] \qquad 4_W$$

In dimensionless form these equations become

$$\sigma_{zB} \cdot \frac{z}{p} = \frac{1}{2\pi} \cdot \frac{n}{(m^2+1)\sqrt{m^2+n^2+1}} \cdot \left[\frac{1}{(m^2+n^2+1)} + \frac{2}{(m^2+1)}\right] \qquad 5_B$$

$$\sigma_{zW} \cdot \frac{z}{p} = \frac{K}{2\pi} \cdot \left[\frac{n}{(m^2+K^2)} \cdot \frac{1}{(m^2+n^2+K^2)^{1/2}}\right] \qquad 5_W.$$

wherein $m = \frac{x}{z}$ and $n = \frac{y}{z}$.

The values of the right-hand members of these equations (designated p_{oB} and p_{oW} respectively) for various values of m and n are shown graphically in Figs. 2 and 3. The values shown in Fig. 3 correspond to $K = \sqrt{2}/2$; that is for $\mu = 0$.

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x) K is defined similarly in all the cases that follow.

TABLE 1.

INFLUENCE VALUES FOR VERTICAL NORMAL STRESSES DUE TO LOAD APPLIED AT A POINT

Boussinesq Solution: $\sigma_{zB} = \frac{P}{z^2} \cdot P_{oB}$

Westergaard Solution ($\mu = 0$): $\sigma_{zW} = \frac{P}{z^2} \cdot P_{oW}$

r/z	P_{oB}	P_{oW}
0.00	0.4775	0.3183
1	0.4773	0.3182
2	0.4770	0.3179
3	0.4764	0.3175
4	0.4756	0.3168
5	0.4745	0.3159
6	0.4732	0.3149
7	0.4717	0.3137
8	0.4699	0.3123
9	0.4679	0.3107
0.10	0.4657	0.3090
1	0.4633	0.3071
2	0.4607	0.3050
3	0.4579	0.3028
4	0.4548	0.3005
5	0.4516	0.2980
6	0.4482	0.2953
7	0.4446	0.2926
8	0.4409	0.2897
9	0.4370	0.2867
0.20	0.4329	0.2836
1	0.4286	0.2804
2	0.4242	0.2771
3	0.4197	0.2737
4	0.4151	0.2703
5	0.4103	0.2668
6	0.4054	0.2632
7	0.4004	0.2595
8	0.3954	0.2558
9	0.3902	0.2521
0.30	0.3849	0.2483
1	0.3796	0.2445
2	0.3742	0.2407
3	0.3687	0.2369
4	0.3632	0.2330
5	0.3577	0.2291
6	0.3521	0.2253
7	0.3465	0.2214
8	0.3408	0.2176
9	0.3351	0.2137
0.40	0.3294	0.2099
1	0.3238	0.2061
2	0.3181	0.2023
3	0.3124	0.1986
4	0.3068	0.1948
5	0.3011	0.1911
6	0.2955	0.1875
7	0.2899	0.1839
8	0.2843	0.1803
9	0.2788	0.1768
0.50	0.2733	0.1733
1	0.2679	0.1698
2	0.2625	0.1664
3	0.2571	0.1631
4	0.2518	0.1598
5	0.2466	0.1566
6	0.2414	0.1534
7	0.2363	0.1502
8	0.2313	0.1471
9	0.2263	0.1441
0.60	0.2214	0.1411
1	0.2165	0.1382
2	0.2117	0.1353
3	0.2070	0.1325
4	0.2024	0.1297
5	0.1978	0.1270
6	0.1934	0.1244
7	0.1889	0.1218
8	0.1864	0.1192
9	0.1804	0.1167

r/z	P_{oB}	P_{oW}
0.70	0.1762	0.1143
1	0.1721	0.1119
2	0.1681	0.1095
3	0.1641	0.1072
4	0.1603	0.1050
5	0.1565	0.1028
6	0.1527	0.1006
7	0.1491	0.0985
8	0.1455	0.0964
9	0.1420	0.0944
0.80	0.1386	0.0925
1	0.1353	0.0905
2	0.1320	0.0887
3	0.1288	0.0868
4	0.1257	0.0850
5	0.1226	0.0833
6	0.1196	0.0815
7	0.1166	0.0799
8	0.1138	0.0782
9	0.1110	0.0766
0.90	0.1083	0.0751
1	0.1057	0.0735
2	0.1031	0.0720
3	0.1005	0.0706
4	0.0981	0.0692
5	0.0956	0.0678
6	0.0933	0.0664
7	0.0910	0.0651
8	0.0887	0.0638
9	0.0865	0.0625
1.00	0.0844	0.0613
1	0.0823	0.0601
2	0.0803	0.0589
3	0.0783	0.0577
4	0.0764	0.0566
5	0.0744	0.0555
6	0.0727	0.0544
7	0.0709	0.0534
8	0.0691	0.0523
9	0.0674	0.0513
1.10	0.0658	0.0503
1	0.0641	0.0494
2	0.0626	0.0484
3	0.0610	0.0475
4	0.0595	0.0466
5	0.0581	0.0458
6	0.0567	0.0449
7	0.0553	0.0441
8	0.0539	0.0432
9	0.0526	0.0424
1.20	0.0513	0.0417
1	0.0501	0.0409
2	0.0489	0.0401
3	0.0477	0.0394
4	0.0466	0.0387
5	0.0454	0.0380
6	0.0443	0.0373
7	0.0433	0.0366
8	0.0422	0.0360
9	0.0412	0.0354
1.30	0.0402	0.0347
1	0.0393	0.0341
2	0.0384	0.0335
3	0.0374	0.0329
4	0.0365	0.0324
5	0.0357	0.0318
6	0.0348	0.0313
7	0.0340	0.0307
8	0.0332	0.0302
9	0.0324	0.0297

r/z	P_{oB}	P_{oW}
1.40	0.0317	0.0292
1	0.0309	0.0287
2	0.0302	0.0282
3	0.0295	0.0277
4	0.0288	0.0273
5	0.0282	0.0268
6	0.0275	0.0264
7	0.0269	0.0259
8	0.0263	0.0255
9	0.0257	0.0251
1.50	0.0251	0.0247
1	0.0245	0.0243
2	0.0240	0.0239
3	0.0234	0.0235
4	0.0229	0.0231
5	0.0224	0.0228
6	0.0219	0.0224
7	0.0214	0.0220
6	0.0209	0.0217
9	0.0204	0.0214
1.60	0.0200	0.0210
1	0.0195	0.0207
2	0.0191	0.0204
3	0.0187	0.0201
4	0.0183	0.0198
5	0.0179	0.0195
6	0.0175	0.0192
7	0.0171	0.0189
8	0.0167	0.0186
9	0.0163	0.0183
1.70	0.0160	0.0180
1	0.0157	0.0178
2	0.0153	0.0175
3	0.0150	0.0172
4	0.0147	0.0170
5	0.0144	0.0167
6	0.0141	0.0165
7	0.0138	0.0163
8	0.0135	0.0160
9	0.0132	0.0158
1.80	0.0129	0.0156
1	0.0126	0.0153
2	0.0124	0.0151
3	0.0121	0.0149
4	0.0119	0.0147
5	0.0116	0.0145
6	0.0114	0.0143
7	0.0112	0.0141
8	0.0109	0.0139
9	0.0107	0.0137
1.90	0.0105	0.0135
1	0.0103	0.0133
2	0.0101	0.0131
3	0.0099	0.0130
4	0.0097	0.0128
5	0.0095	0.0126
6	0.0093	0.0124
7	0.0091	0.0123
8	0.0089	0.0121
9	0.0087	0.0120
2.00	0.0085	0.0118
1	0.0084	0.0116
2	0.0082	0.0115
3	0.0081	0.0113
4	0.0079	0.0112
5	0.0078	0.0110
6	0.0076	0.0109
7	0.0075	0.0108
8	0.0073	0.0106
9	0.0072	0.0105

r/z	P_{oB}	P_{oW}
2.10	0.0070	0.0103
1	0.0069	0.0102
2	0.0068	0.0101
3	0.0066	0.0100
4	0.0065	0.0098
5	0.0064	0.0097
6	0.0063	0.0096
7	0.0062	0.0095
6	0.0060	0.0094
9	0.0059	0.0092
2.20	0.0058	0.0091
1	0.0057	0.0090
2	0.0056	0.0089
3	0.0055	0.0088
4	0.0054	0.0087
5	0.0053	0.0086
6	0.0052	0.0085
7	0.0051	0.0084
8	0.0050	0.0083
9	0.0049	0.0082
2.30	0.0048	0.0081
1	0.0047	0.0080
2	0.0047	0.0079
3	0.0046	0.0078
4	0.0045	0.0077
5	0.0044	0.0076
6.	0.0043	0.0075
7	0.0043	0.0074
8	0.0042	0.0074
9	0.0041	0.0073
2.40	0.0040	0.0072
1	0.0040	0.0071
2	0.0039	0.0070
3	0.0038	0.0069
4	0.0038	0.0069
5	0.0037	0.0068
6	0.0036	0.0067
7	0.0036	0.0066
8	0.0035	0.0066
9	0.0034	0.0065
2.50	0.0034	0.0064
1	0.0033	0.0064
2	0.0033	0.0063
3	0.0032	0.0062
4	0.0032	0.0061
5	0.0031	0.0061
6	0.0031	0.0060
7	0.0030	0.0059
8	0.0030	0.0059
9	0.0029	0.0058
2.60	0.0029	0.0058
1	0.0028	0.0057
2	0.0028	0.0056
3	0.0027	0.0056
4	0.0027	0.0055
5	0.0026	0.0054
6	0.0026	0.0054
7	0.0025	0.0053
8	0.0025	0.0053
9	0.0025	0.0052
2.70	0.0024	0.0052
1	0.0024	0.0051
2	0.0023	0.0051
3	0.0023	0.0050
4	0.0023	0.0050
5	0.0022	0.0049
6	0.0022	0.0049
7	0.0022	0.0048
6	0.0021	0.0048
9	0.0021	0.0047

r/z	P_{oB}	P_{oW}
2.80	0.0021	0.0047
1	0.0020	0.0046
2	0.0020	0.0046
3	0.0020	0.0045
4	0.0019	0.0045
5	0.0019	0.0045
6	0.0019	0.0044
7	0.0019	0.0044
8	0.0018	0.0043
9	0.0018	0.0043
2.90	0.0018	0.0042
1	0.0017	0.0042
2	0.0017	0.0042
3	0.0017	0.0041
4	0.0017	0.0041
5	0.0016	0.0040
6	0.0016	0.0040
7	0.0016	0.0040
8	0.0016	0.0039
9	0.0015	0.0039
3.00	0.0015	0.0038
2	0.0015	0.0038
4	0.0014	0.0037
6	0.0014	0.0036
8	0.0013	0.0036
3.10	0.0013	0.0035
2	0.0013	0.0034
4	0.0012	0.0034
6	0.0012	0.0033
8	0.0012	0.0033
3.20	0.0011	0.0032
2	0.0011	0.0031
4	0.0011	0.0031
6	0.0010	0.0030
8	0.0010	0.0030
3.30	0.0010	0.0029
2	00.009	0.0029
4	0.0009	0.0028
6	0.0009	0.0028
8	0.0009	0.0027
3.40	0.0009	0.0027
2	0.0008	0.0026
4	0.0008	0.0026
6	0.0008	0.0026
8	0.0008	0.0025
3.50	0.0007	0.0025
.6	0.0007	0.0023
.7	0.0006	0.0021
.8	0.0005	0.0020
.9	0.0005	0.0018
4.00	0.0004	0.0017
.1	0.0004	0.0016
.2	0.0003	0.0015
.3	0.0003	0.0014
.4	0.0003	0.0013
.5	0.0002	0.0012
.6	0.0002	0.0011
.7	0.0002	0.0011
.8	0.0002	0.0010
.9	0.0002	0.0009
5.00	0.0001	0.0009
.2	0.0001	0.0008
.4	0.0001	0.0007
.5	0.0001	0.0007
.6	0.0001	0.0006
.8	0.0001	0.0006

r/z	P_{oB}	P_{oW}
6.00	0.0001	0.0005
7.00	0.0000	0.0003
8.00	0.0000	0.0002
9.00	0.0000	0.0002
10.00	0.0000	0.0001
∞	0.0000	0.0000

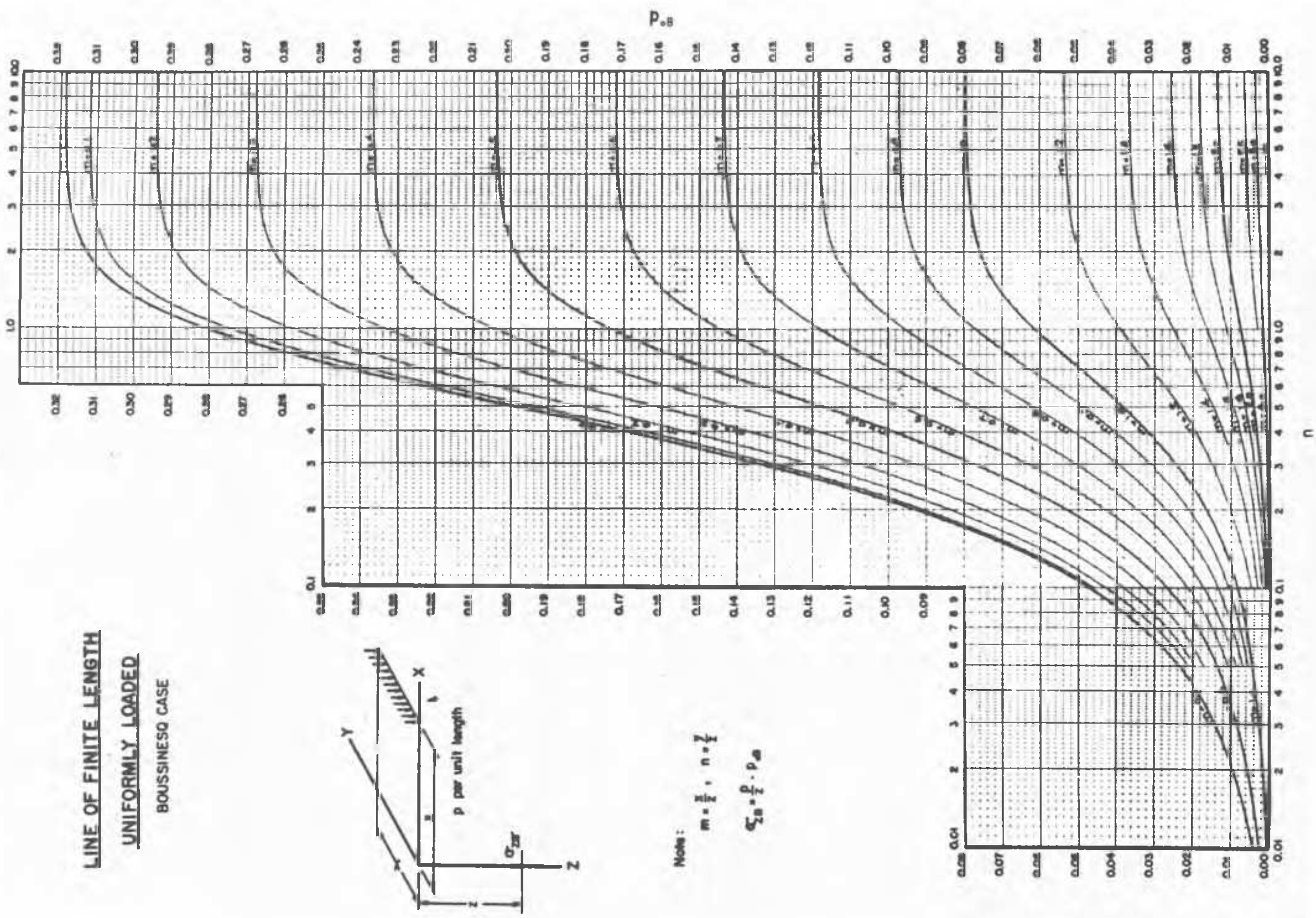

FIG. 2

To find the value of the stress at any point, B, it is necessary only to measure the distances x and y as defined in Fig. 1b; divide these distances by the depth z to obtain the ratios m and n respectively; and then select the influence value p_{oB} or p_{oW} from the respective chart. The stress is determined from the respective equations:

$$\sigma_{zB} = \frac{p}{z} \cdot p_{oB} \qquad 6_B$$

$$\sigma_{zW} = \frac{p}{z} \cdot p_{oW} \qquad 6_W$$

It is to be noted that the coordinate system shown in Fig. 1b is so chosen that the Y axis is parallel to the line that is loaded and that the X axis passes through one end of this line. By the principle of superposition the stress σ_z' at a point B' that lies in the plane z'O'x', which is parallel to the plane zOx, can be determined readily by subtracting from the stress due to a line load of length (y'+y) the stress due to the line load of length y'.

c. Rectangular Area-Uniformly Loaded

Expressions for the vertical stress at depth, z, below the corner of a rectangular area (x, y) uniformly loaded with a load w per unit of area are stated in terms of the coordinate system shown in Fig. 1c, as follows: x)

$$\sigma_{zB} = \frac{w}{4\pi} \cdot \left[\frac{2xyz(x^2+y^2+z^2)^{1/2}}{z^2(x^2+y^2+z^2)+x^2y^2} \cdot \frac{x^2+y^2+2z^2}{x^2+y^2+z^2} + \tan^{-1} \frac{2xyz(x^2+y^2+z^2)^{1/2}}{z^2(x^2+y^2+z^2)-x^2y^2} \right] \qquad 7_B$$

$$\sigma_{zW} = \frac{w}{2\pi} \tan^{-1} \frac{xy}{Kz} \frac{1}{(x^2+y^2+K^2z^2)^{1/2}} \qquad 7_W$$

Expressed in terms of dimensionless quantities these equations become

$$\frac{\sigma_{zB}}{w} = \frac{1}{4\pi} \cdot \left[\frac{2mn(m^2+n^2+1)^{1/2}}{m^2+n^2+1+m^2n^2} \cdot \frac{m^2+n^2+2}{m^2+n^2+1} + \tan^{-1} \frac{2mn(m^2+n^2+1)^{1/2}}{m^2+n^2+1-m^2n^2} \right] \qquad 8_B$$

$$\frac{\sigma_{zW}}{w} = \frac{1}{2\pi} \tan^{-1} \frac{mn}{K(m^2+n^2+K^2)^{1/2}} \qquad 8_W$$

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x) The equations and influence values for the Boussinesq problem were developed for this case by N.M. Newmark, loc.cit. (6). See also (13), "Tafeln zur Setzungs berechnung," Die Strasse, Oct 1934; and (14), "Stress Distribution and Deflection under Loaded Areas," by A. Gutierrez, a thesis submitted to the Massachusetts Institute of Technology, Cambridge, Mass., 1931.

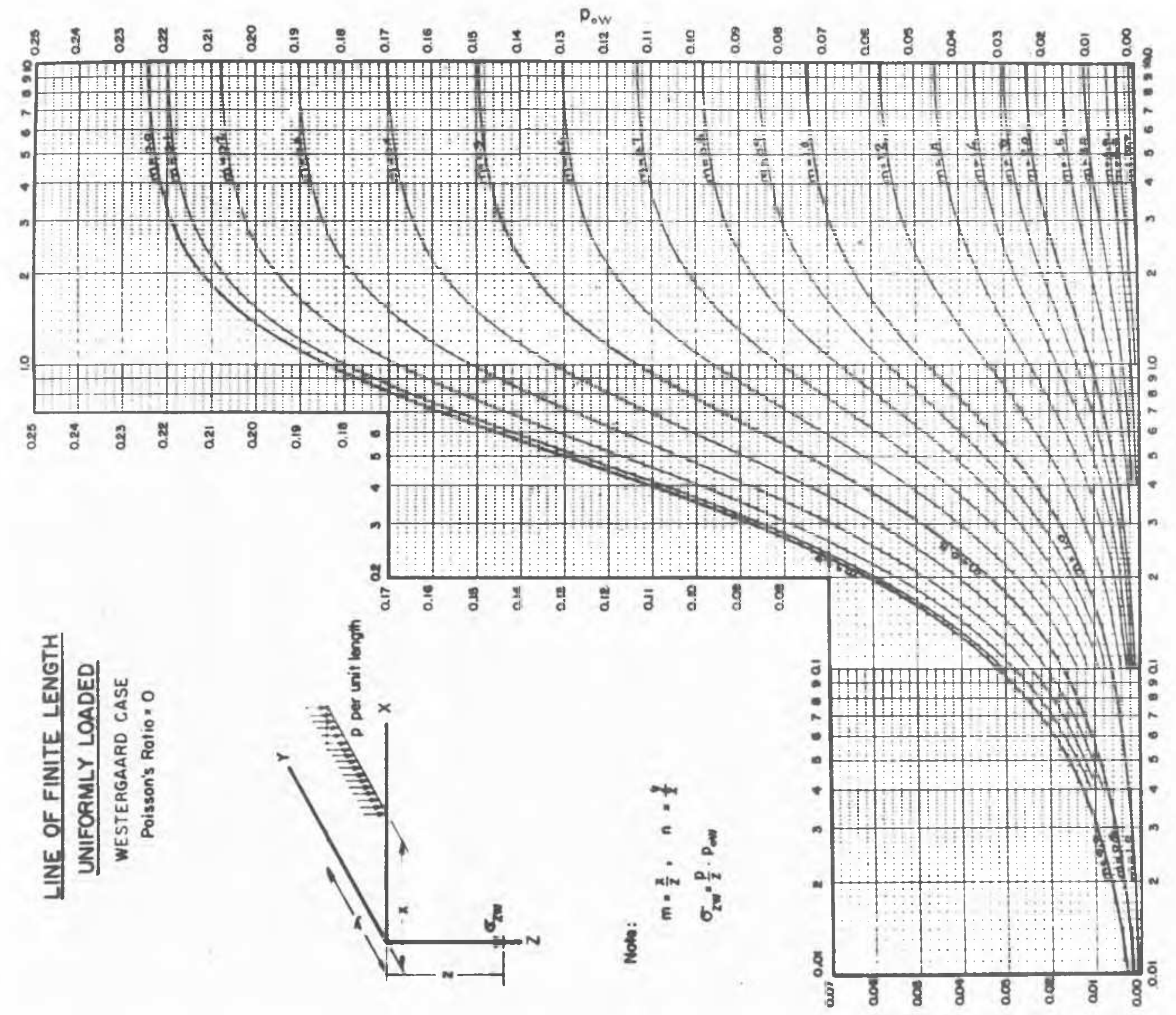

FIG.3

The values of the right-hand members of equations 8_B and 8_W, designated w_{oB} and w_{oW} respectively, are shown graphically in Figs. 4 and 5. The values shown in Fig. 5 correspond to a value of $\mu = 0$. To find the value of the stress at any point B below the corner of a rectangular area (x, y) uniformly loaded with a load of w per unit area proceed as follows:

1) Measure the distances x and y as defined in Fig. 1c.
2) Obtain the ratios m and n by dividing x and y respectively by z, the distance from the plane of loading to the plane on which the stress is desired.
3) Select from the chart the proper value of w_{oB} or w_{oW}.
4) Compute the stress, σ_z from the respective equations.

$$\sigma_{zB} = w \cdot w_{oB} \qquad 9_B$$

$$\sigma_{zW} = w \cdot w_{oB} \qquad 9_W$$

It is seen that the coordinate system shown in Fig. 1c is so chosen that the origin coincides with the corner of the loaded, rectangular area. The stress at some point, which is not below a corner, as for example at point B' in Fig. 1c, can be found from the principle of superposition as follows:

1. Determine the influence of value w_{o1} for the stress due to load uniformly distributed over the rectangular area (x+x'),(y+y'), which includes the area that is loaded and which has a corner above the point at which the stress is desired.
2. Determine the influence values w_{o2} and w_{o3} for the stress due to load uniformly distributed over the areas (x+x'), y' and x', (y+y') respectively.
3. Determine the influence value w_{o4} for the stress due to load uniformly distributed over the area x', y'. (this area was included twice in step 2.)
4. Compute the stress at B' from the equation:

$$\sigma_{z'} = w(w_{o1} - w_{o2} - w_{o3} + w_{o4}) \qquad 10$$

TABLE 2.

INFLUENCE VALUES FOR VERTICAL NORMAL STRESSES BENEATH THE CENTER OF A UNIFORMLY LOADED, CIRCULAR AREA.

Boussinesq Solution: $\sigma_{zB} = w \cdot w_{oB}$

Westergaard Solution: $(\mu = 0)$: $\sigma_{zW} = w \cdot w_{oW}$

r/z	w_{oB}	w_{oW}
0.00	0.0000	0.0000
1	0.0002	0.0001
2	0.0006	0.0004
3	0.0014	0.0009
4	0.0024	0.0016
5	0.0037	0.0025
6	0.0054	0.0036
7	0.0073	0.0049
8	0.0095	0.0063
9	0.0120	0.0080
0.10	0.0148	0.0099
1	0.0179	0.0119
2	0.0212	0.0141
3	0.0248	0.0165
4	0.0287	0.0190
5	0.0328	0.0218
6	0.0372	0.0247
7	0.0418	0.0277
8	0.0467	0.0309
9	0.0518	0.0343
0.20	0.0571	0.0378
1	0.0627	0.0414
2	0.0684	0.0452
3	0.0744	0.0490
4	0.0806	0.0531
5	0.0869	0.0572
6	0.0935	0.0614
7	0.1002	0.0658
8	0.1070	0.0702
9	0.1141	0.0748
0.30	0.1213	0.0794
1	0.1286	0.0842
2	0.1361	0.0890
3	0.1436	0.0938
4	0.1513	0.0988
5	0.1592	0.1038
6	0.1671	0.1089
7	0.1751	0.1140
8	0.1832	0.1191
9	0.1913	0.1244
0.40	0.1996	0.1296
1	0.2079	0.1349
2	0.2163	0.1402
3	0.2247	0.1456
4	0.2332	0.1510
5	0.2417	0.1564
6	0.2502	0.1618
7	0.2587	0.1672
8	0.2673	0.1726
9	0.2759	0.1781
0.50	0.2845	0.1835
1	0.2930	0.1890
2	0.3016	0.1944
3	0.3102	0.1998
4	0.3188	0.2053
5	0.3273	0.2107
6	0.3358	0.2161
7	0.3443	0.2215
8	0.3527	0.2268
9	0.3611	0.2322
0.60	0.3695	0.2375
1	0.3778	0.2428
2	0.3861	0.2481
3	0.3943	0.2534
4	0.4025	0.2586
5	0.4106	0.2638
6	0.4186	0.2690
7	0.4266	0.2741
8	0.4345	0.2792
9	0.4424	0.2843

r/z	w_{oB}	w_{oW}
0.70	0.4502	0.2893
1	0.4579	0.2943
2	0.4655	0.2993
3	0.4731	0.3043
4	0.4806	0.3091
5	0.4880	0.3140
6	0.4953	0.3188
7	0.5026	0.3236
8	0.5098	0.3284
9	0.5169	0.3331
0.80	0.5239	0.3377
1	0.5308	0.3424
2	0.5376	0.3470
3	0.5444	0.3515
4	0.5511	0.3560
5	0.5577	0.3605
6	0.5642	0.3649
7	0.5706	0.3693
8	0.5769	0.3736
9	0.5832	0.3779
0.90	0.5893	0.3822
1	0.5954	0.3864
2	0.6014	0.3906
3	0.6073	0.3948
4	0.6132	0.3989
5	0.6189	0.4029
6	0.6246	0.4069
7	0.6302	0.4109
8	0.6357	0.4149
9	0.6411	0.4188
1.00	0.6465	0.4227
1	0.6517	0.4265
2	0.6569	0.4303
3	0.6620	0.4340
4	0.6670	0.4377
5	0.6720	0.4414
6	0.6769	0.4451
7	0.6817	0.4487
8	0.6864	0.4522
9	0.6910	0.4558
1.10	0.6956	0.4593
1	0.7001	0.4627
2	0.7046	0.4662
3	0.7089	0.4695
4	0.7132	0.4729
5	0.7175	0.4762
6	0.7216	0.4795
7	0.7257	0.4828
8	0.7298	0.4860
9	0.7337	0.4892
1.20	0.7376	0.4923
1	0.7415	0.4955
2	0.7453	0.4985
3	0.7490	0.5016
4	0.7526	0.5046
5	0.7562	0.5076
6	0.7598	0.5106
7	0.7632	0.5135
8	0.7667	0.5165
9	0.7700	0.5193
1.30	0.7733	0.5222
1	0.7766	0.5250
2	0.7798	0.5278
3	0.7830	0.5306
4	0.7861	0.5333
5	0.7891	0.5360
6	0.7921	0.5387
7	0.7951	0.5414
8	0.7980	0.5440
9	0.8008	0.5466

r/z	w_{oB}	w_{oW}
1.40	0.8036	0.5492
1	0.8064	0.5517
2	0.8091	0.5543
3	0.8118	0.5568
4	0.8144	0.5592
5	0.8170	0.5617
6	0.8196	0.5641
7	0.8221	0.5665
8	0.8245	0.5689
9	0.8269	0.5713
1.50	0.8293	0.5736
1	0.8317	0.5759
2	0.8340	0.5782
3	0.8362	0.5805
4	0.8385	0.5827
5	0.8407	0.5850
6	0.8428	0.5872
7	0.8450	0.5893
8	0.8470	0.5915
9	0.8491	0.5937
1.60	0.8511	0.5958
1	0.8431	0.5979
2	0.8551	0.6000
3	0.8570	0.6020
4	0.8589	0.6041
5	0.8608	0.6061
6	0.8626	0.6081
7	0.8644	0.6101
8	0.8662	0.6121
9	0.8679	0.6140
1.70	0.8697	0.6160
1	0.8714	0.6179
2	0.8730	0.6198
3	0.8747	0.6217
4	0.8763	0.6235
5	0.8779	0.6254
6	0.8794	0.6272
7	0.8810	0.6290
8	0.8825	0.6308
9	0.8840	0.6326
1.80	0.8855	0.6344
1	0.8869	0.6361
2	0.8883	0.6379
3	0.8897	0.6396
4	0.8911	0.6413
5	0.8925	0.6430
6	0.8938	0.6447
7	0.8951	0.6463
8	0.8964	0.6480
9	0.8977	0.6496
1.90	0.8990	0.6512
1	0.9002	0.6528
2	0.9014	0.6544
3	0.9026	0.6560
4	0.9038	0.6576
5	0.9050	0.6591
6	0.9061	0.6606
7	0.9073	0.6622
8	0.9084	0.6637
9	0.9095	0.6652
2.00	0.9106	0.6667
2	0.9127	0.6696
4	0.9147	0.6725
6	0.9167	0.6753
8	0.9187	0.6781
2.10	0.9205	0.6809
.15	0.9250	0.6876
.20	0.9291	0.6940
.25	0.9330	0.7002

r/z	w_{oB}	w_{oW}
2.30	0.9366	0.7061
.35	0.9400	0.7119
.40	0.9431	0.7174
.45	0.9460	0.7227
.50	0.9488	0.7278
.55	0.9513	0.7328
.60	0.9537	0.7376
.65	0.9560	0.7422
.70	0.9581	0.7467
.75	0.9601	0.7510
.80	0.9620	0.7552
.85	0.9637	0.7592
.90	0.9654	0.7631
.95	0.9669	0.7669
3.00	0.9684	0.7706
.10	0.9711	0.7776
.20	0.9735	0.7842
.30	0.9756	0.7905
.40	0.9775	0.7964
.50	0.9793	0.8020
.60	0.9808	0.8073
.70	0.9822	0.8123
.80	0.9835	0.8171
.90	0.9847	0.8216
4.00	0.9857	0.8259
.20	0.9876	0.8340
.40	0.9891	0.8413
.60	0.9904	0.8481
.80	0.9915	0.8543
5.00	0.9925	0.8600
.20	0.9933	0.8653
.40	0.9940	0.8702
.60	0.9946	0.8747
.80	0.9951	0.8790
6.00	0.9956	0.8830
.50	0.9965	0.8919
7.00	0.9972	0.8995
.50	0.9977	0.9061
8.00	0.9981	0.9120
9.00	0.9987	0.9217
10.00	0.9990	0.9295
12.00	0.9994	0.9412
14.00	0.9996	0.9496
16.00	0.9998	0.9559
18.00	0.9998	0.9608
20.00	0.9999	0.9647
25.00	0.9999	0.9717
30.00	1.0000	0.9764
40.00	1.0000	0.9823
50.00	1.0000	0.9859
100.00	1.0000	0.9929
∞	1.0000	1.0000

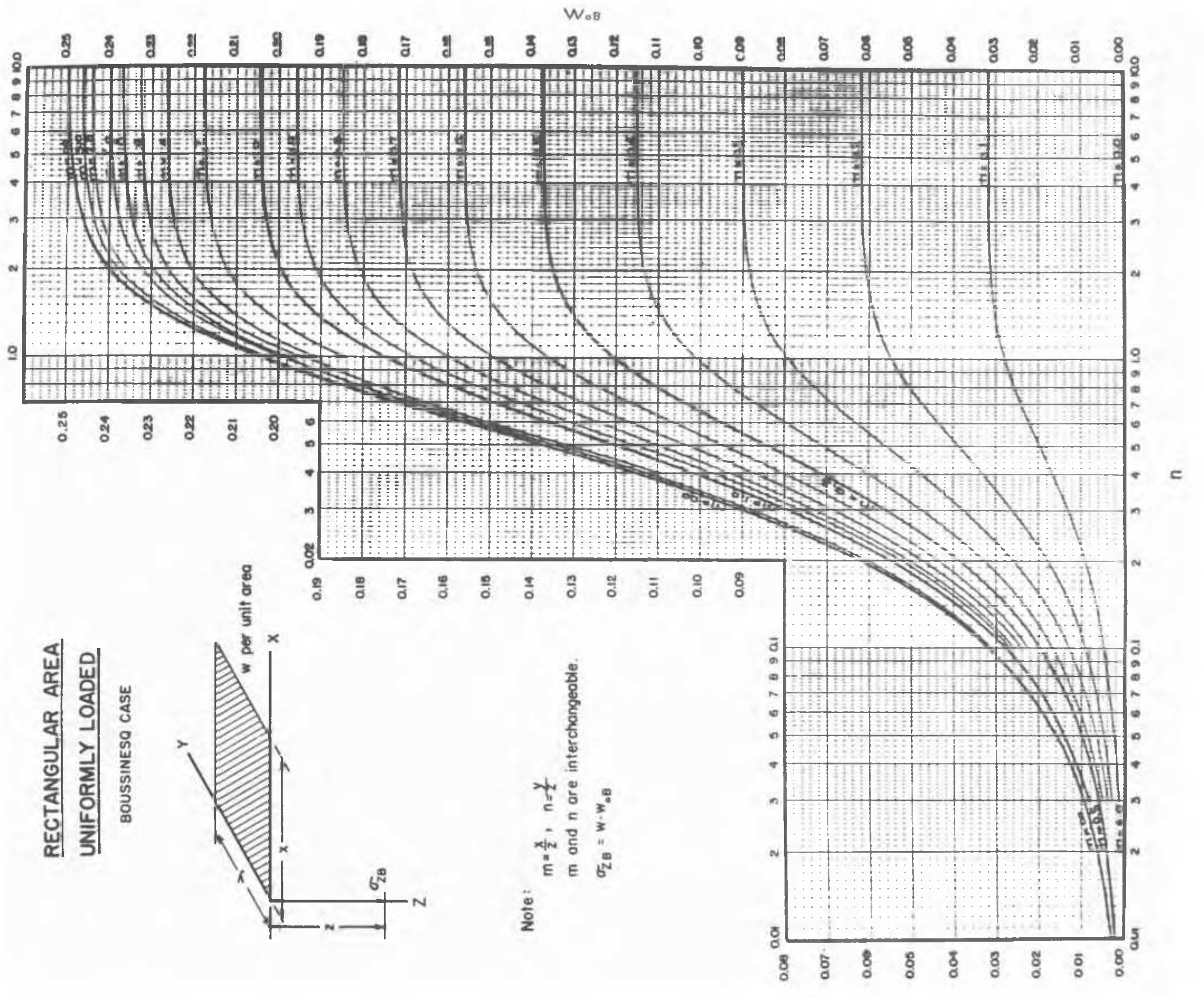

FIG. 4

d. Circular Area Uniformly Loaded

Expressions for the stress beneath the center of a uniformly loaded, circular area are as follows:

$$\frac{\sigma_{zB}}{w} = 1 - \left[\frac{1}{1 + (r/z)^2}\right]^{3/2} \qquad 11_B$$

$$\frac{\sigma_{zw}}{w} = 1 - \frac{K}{\left[K^2 + (r/z)^2\right]^{1/2}} \qquad 11_W$$

In Table 2 are given the values of the right-hand members of these equations, w_{oB} and w_{oW}, for various values of the ratio r/z. The values of w_{oW} correspond to a value of $\mu = 0$. The value of the stress below the center of a uniformly loaded, circular area is determined with the aid of the values given in Table 2 from equations similar to Eq. 9_B and 9_W.

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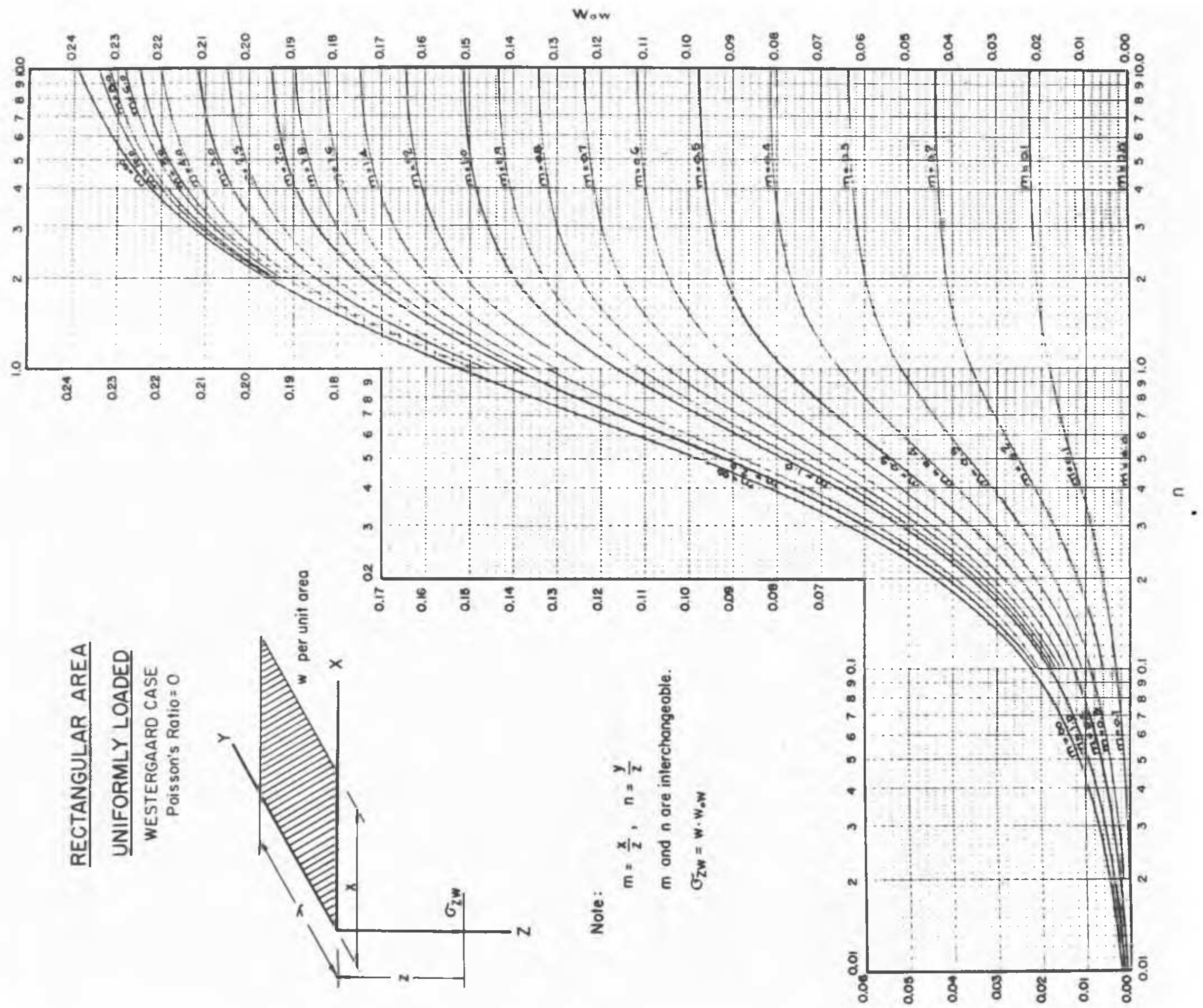

FIG.5

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