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THE NATURAL OSCILLATION OF GROUND AND INDUSTRIAL FOUNDATIONS

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INTRODUCTION.

Some thousands of vibrographic and other records have been collected during the last four years on the behaviour of ground underneath and around heavy industrial foundations. The paper presents some of the results of this investigation, while some comparable research by others is re-examined, and a theory is advanced which it is hoped will produce a sound basis for the subject of machine foundation design in the oscillational principle.

As it is only possible to use somewhat uncontrolled methods with such installations, lack of accuracy is to some extent counteracted by the numerous readings, while the additional time needed for taking these has, by thus spreading the duration of their observation, provided previously unknown information. Owing to the non-linear behaviour which the Authors have exposed, and the very large number of readings at their disposal, they have thought it best to indicate general trends rather than to attempt a narrower and more statistical treatment of the subject at this early stage of the development of "SOIL DYNAMICS", the study of soil vibrations.

THE DYNAMIC ACTION OF THE GROUND.

I. GROUND SELF FREQUENCIES.

From 1928 to 1939 the Degebo organisation in Germany studied the phenomena of ground vibration, and were able to show that the ground on any site has a natural or self frequency of vibration peculiar to the site and to the type of soil; they employed a controlled oscillator for this study, and their results have been further substantiated in part by Andrews and Crockett in 1944, who independently discovered the self frequencies by a study of the resonance between heavy industrial plant and the ground upon which they were sited.

These self frequencies are clearly a function of (a) the physical properties such as the Modulus of Elasticity and the Density of the material, because they are found to be directly related to the measured velocity of the Rayleigh Wave in the same material, and (b) also to the thickness of the upper strata immediately adjacent to the foundation concerned. It has also been established that the ultimate bearing pressure of the ground is directly related to the self frequency of the ground; in 1946 Bergstrom and Linderholm in Sweden showed that this property provides a practical method of measuring bearing values of soil for ordinary building purposes. This represents an important advance in the technique of Soil Mechanics, but the bearing pressure allowable in Soil Statics can be a very different value from that in Soil Dynamics, because the static and dynamic elastic constants are not necessarily the same, and also because with the latter there is always the zone of resonance present. These differences do not appear to have been generally noted before. In view of their great importance a number of values observed by several workers have been shown in Table 1, although these have been obtained independently of one another they check mutually and thus form the basis of a reasonable working hypothesis. These measurements have been obtained either by variable speed oscillators not all having the same area of contact with the ground, or the same weight, or else by striking the ground with a heavy weight and distributing the blow over different areas. These areas have varied from one or two square feet up to nearly a hundred square feet. Resonant frequencies have in addition been measured from machine foundations having sizes up to a maximum of fifty feet square, or 2500 sq. ft. The range of frequencies is from 4 cycles per second for swamp, up to 40 cycles per second for good granite; the self frequencies of clay vary from about 10 to 25 cycles, the latter for very stiff clay, while the sands, gravels, and other broken materials cover much the same range.

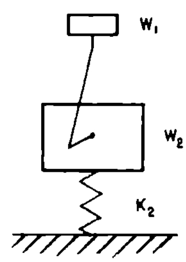


FIG.1

II. SPRING / WEIGHT EQUIVALENT OF THE GROUND AND FOUNDATION.

The displacement of the spring / weight system in Fig. 1 at any time in the steady state of oscillation is

$$\frac{n_o^2}{n_o^2 - N^2} \cdot a \times \sin 2\pi.N.t.$$

where n_o is the self frequency of the system,
N is the frequency of the impressed sinusoidal impulse,
t is the time,
a is the static elongation produced in the spring by the top weight W1.

Fig. 2 is the curve of the induced oscillations for steady conditions and showing the effect of damping.

It is here assumed that the stiffness K_2 of the spring obeys Hooke's Law, but if the load / deflection relation is curved or otherwise non-linear, the resonance peak loses its symmetry and bends over to one side or the other, as indicated in Fig. 3. The top of this peak has a frequency of 23.12 cycles per second, whereas if the amplitude induced had been smaller, the resonance frequency would have been higher than 23.12; probably about 28 cycles per second. The Fig. also indicates the curved and linear axes of the two kinds of peaks.

Degebo do not seem to have noticed this interpretation of non-linearity of the phenom-

TABLE I

FREQUENCY AND BEARING VALUES OF SOILS

SOIL	Frequency, c.p.s.	Admissible Pressure Tons/sq.ft.	
Marsh 10 feet thick on sand	4	0	All values from Degebo, vol,4 1936.
Marsh 5 feet thick on sand	13.1	-	
Marshy stratum about 5 feet thick on mud	about 15(x)	-	
Loose sand agglomeration	17.4	-	
Very wet softened clay	18.4	-	
Wet clay	19.2	-	
Finest sand	19.3	.9	
Clay with fine sand	20.7	-	
Moist Clay	20.8	-	
Coarse Gravel, compacted by Vibrator	21.2	-	
Very wet Sand above spring	21.6	-	
Moist tertiary clay	21.8	1.8	
Moist medium sand	21.8	1.8	
Dry medium sand	22	1.8	
Clayey sand over marl rubble	22.6	2.27	
Loose sand	22.8	-	
Medium-fine sand, stamped	23	-	
Dry loess of diluvial origin	23.5	-	
Gravel with stones	23.5	2.3	
Moist clay	23.5	-	
Medium sand, not compacted	23.7	-	
Marl rubble	23.8	2.7	
Medium sand, elutriated and stamped	24.3	-	
Dry clay	24.6(x)	-	
Dry clay with broken limestone	25.3	-	
Firm surface clay	25.5(x)	-	
Marl	25.7	3.6	
Berlin sand	27.5	-	
Very firm clay at depth of 3ft 6in.	28.5(x)	-	
Very firm sandy gravel 16 ft above spring water	29	-	
Fine tertiary sand, 3 ft 6 in deep	29.5(x)	-	
Tightly packed coarse gravel	30	4.1	
Sharp compact tertiary sand 3ft 6in deep	30.2(x)	-	
Weathered mottled sandstone	32	-	
Very uniform medium sand	33.4(x)		
Loose fill	19.1	1.0	(Lorenz. 1934)
Dense artificial cinder fill	21.3	1.5	
Fairly Dense medium sand	24.1	3.0	
Fine sand with 30% medium sand	24.2	1.4	
Very dense mixed grain sand	26.7	4.5	
Dense pea gravel	28.1	4.5	
Limestone	30	-	
Hard sandstone	34	-	
Peat	7.5	-	(Andrews & Crockett 1945 to 1947)
Waterlogged estuarine silt	10	0.7	
Very light soft clay	12	1	
light waterlogged sand	15	1.5	
Medium clay	15	2	
Layers of hardish peat and sand mixed	17	2	
Stiff clay	19	3	
Silt and sand mixed	23.3	3	
Sand and rubble loosely compacted	23.5	3	
Limestone	30	-	
Granite	40	-	

All Degebo frequencies and bearing pressures were measured using oscillators, but all frequencies measured by Andrews and Crockett were measured by setting the ground into oscillation by striking it with a hammer and measuring the frequency with a vibrograph: their admissible bearing pressures are " engineers " estimates for the sites.

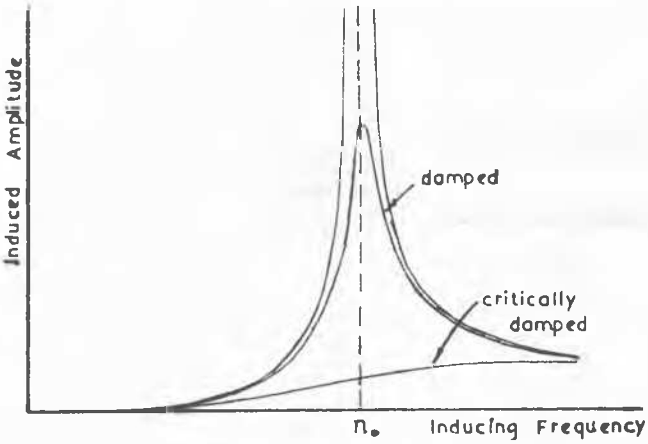


FIG.2

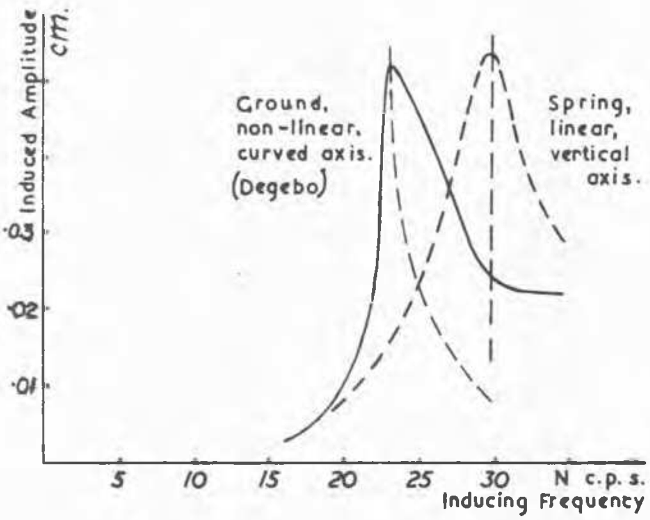


FIG.3

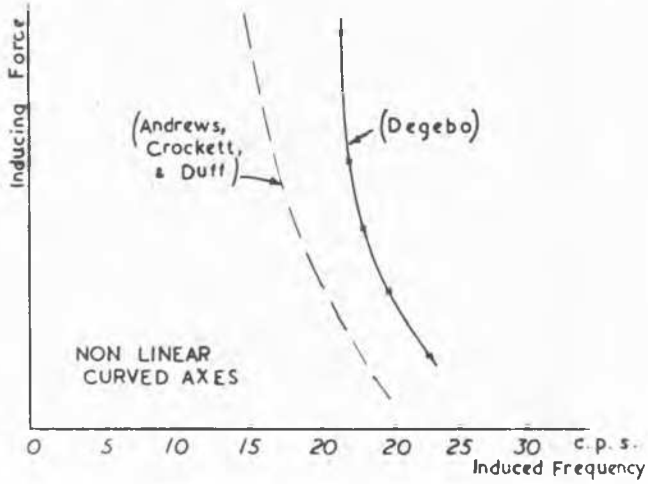


FIG.4

ena, but their published figures demonstrate its probable truth, as seen in Fig. 4, and these are further substantiated by the records of Andrews, Crockett, and Duff. When stating the self frequency of any site or soil, it is thus also necessary to state the amplitude of induced oscillation during the movement. In Table I, those higher frequencies based on

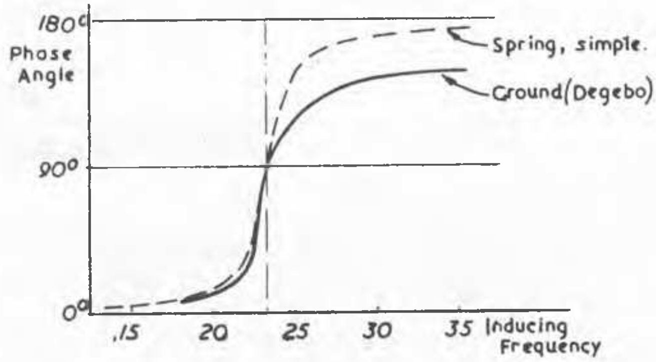


FIG.5

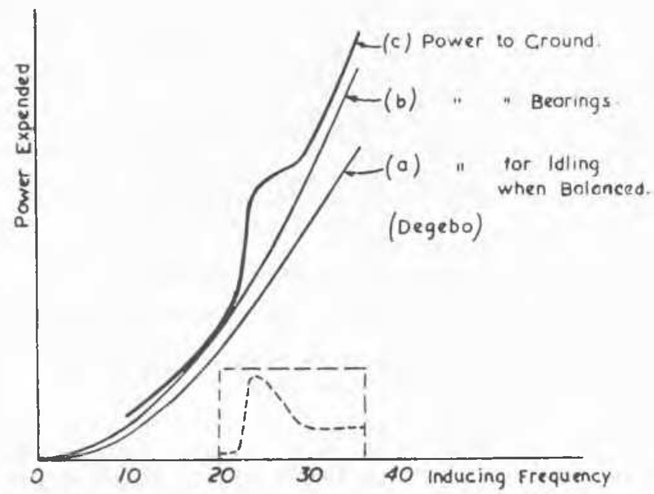


FIG.6

small amplitudes are marked with an asterisk. Self frequencies do not vary by more than about 25 per cent below their highest values for small amplitudes of movement. Degebo measured amplitudes up to about 0.06 cm., the present Authors up to about one quarter of an inch.

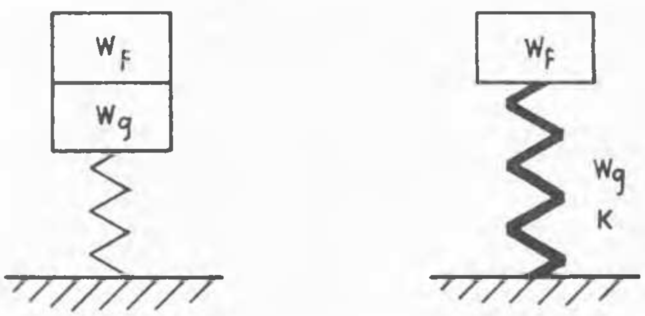
The relation between phase and induced frequency is a further demonstration of the similarity between a foundation with its surrounding ground and of a single spring / weight system. When N is lower than the self frequency of the ground n_o, then the displacement is in phase with or in the same direction as the inducing force, but at higher inducing frequencies than n_o the force and the movement are anti-phased or opposite. When the force is downward the ground moves upward, and vice versa. This illustrated in Fig. 5.

An interesting relation follows from this change of phase; extra power is needed to overcome the effect of resonance with the ground. Curve (a) in Fig. 6 represents the power needed in a certain well balanced oscillator to keep it idling at the various speeds; curve (b) gives the power required extra to overcome the additional friction in the bearings when the oscillator was put out of balance at various speeds. Curve (c) shows the large amount of extra energy necessary for inducing oscillations in the ground in the zone of resonance, and also the rather smaller amount needed at higher frequencies than this for maintaining the ground in oscillation, this latter movement being anti-phased or in the opposite direction to that of the oscillator. The oscillator has so to speak, to waste energy when it forces the ground to oscillate in this upper range, but it has to put forth only limited power below the resonance frequency. This energy loss is directly related to the induced amplitude, as may be seen in the subsidiary diagram. Andrews and Crockett have recognised such losses occurring in industrial plant.

III. RELATION BETWEEN ACTIVE WEIGHT, SELF FREQUENCY, AND DYNAMIC SPRING CONSTANT OF GROUND AND FOUNDATION.

This can be formulated sufficiently accurately with a non-linear system if the part of the movement is kept small. Although available information is not extensive, the Authors give a brief summary, and from it have arranged a working method which has provided reasonably good results within the limits of industrial requirements of today.

1) Active Weight. The active weight of ground W_g can be taken as that portion which is immediately underneath and acts with the concrete foundation or the oscillator base plate of weight W_f, but the full active weight is also springy, the system is not as shown in Fig. 7a, but rather that of the rigid weight W_f as in Fig. 7b, supported by a spring which has itself a weight of W_g.



a FIG.7 b

It has been shown by H. Hertz and S. Fuchs that under a circular area of contact between one body and another there is an ellipsoidal zone, adjacent to the area, in which all the principle lines of stress are in compression. Outside this zone some of the lines change to tension, whereby they form a tensional, or "hoop stressing", system round the outside of the inner ellipsoidal or compressed portion of the bulb, as indicated in Fig. 8a. While there is no experimental evidence to prove that the pressure portion of the bulb represents the equivalent active ground weight W_g, and while the assumption by one of the Authors that the self frequency of the ground is determined by the oscillational properties of this bulb is not at present checked, their combined ideas have been found to give reasonably accurate results in practice. This hypothesis is known as the Oscillation of the Bulb of Pressure.

Fig. 8c shows the modification to the ellipsoidal pressure portion of the bulb which must be applied when the foundation is partly below ground level; active ground weight is considerably increased where friction acts round the sides. However, the total weight is not altogether dependant on the outer dimensions of the concrete, because the foundation replaces some of the weight of the earth, while the con-

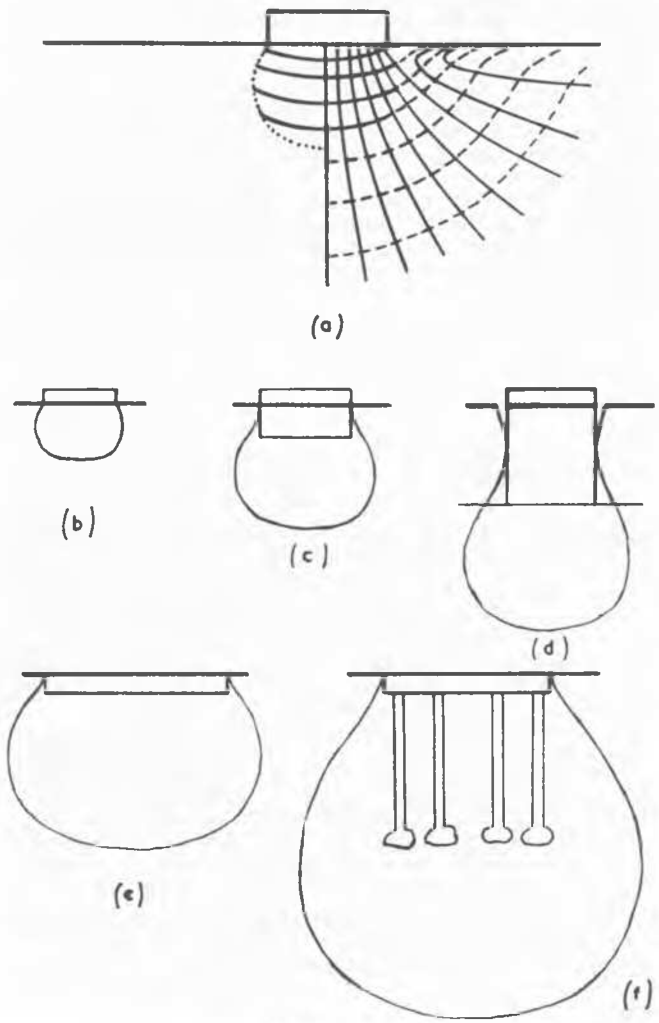


FIG.8

crete base might even be of lightweight cellular form. Fig. 8d represents the usual large block taken down to a "good bottom", and where at times the upper parts of the sides of the concrete may have little or no adhesion with the ground. It follows that if a great weight is needed in the foundation, a more economical means than the usual massive block is to form a large-area stiff raft, whereby W_g has a high value, as in Fig. 8e. Any form of piles may also increase the size and weight of the active ground, as in Fig. 8f. It is also clear that larger foundations will be influenced more by underlying strata than the smaller ones.

This all postulates that any change in the size of the bulb W_g is accompanied by an equivalent change in K, its spring constant. This is born out by Degebo, whose attempts to measure the weight W_g, with an oscillator have shown that average errors of 25, 50, and even greater percentages occur during the course of an experiment, though the self frequency remains about the same. It is thought that the proportions of the total bulb of pressure are linked with this comparative constancy of self frequency of all these foundations for any particular type of ground, despite the considerable variation in overall size from a foot or two square on the surface to a large and deep heavy industrial foundation.

2) Frequency Control. This rough constancy re-

quires a homogenious ground throughout the thickness of the containing stratum, and that this stratum should be not less than about three times the depth of the inner part of the bulb. This gives the "hoop stressing" enough room to be adequately developed. If the weight of the foundation W_f is increased, for example by adding the machine W_m to the top of it, and assuming that the bulb of pressure does not change its relative proportions, then the relation appears to be (for negligible damping and for a sufficiently small range of oscillation amplitude for linear conditions to be assumed):--

$$\frac{n_{gf}^2}{n_g^2} = \frac{W_g}{W_g + W_f}$$

or

$$\frac{n_{gfm}^2}{n_g^2} = \frac{W_g}{W_g + W_f + W_m}$$

where n_g is the self frequency of the ground of bulb weight W_g

n_{gf} is the self frequency of the ground and foundation of combined weight $W_g + W_f$,

n_{gfm} is the self frequency of the ground, foundation, and machine of combined weight $W_g + W_f + W_m$

This simple relation has been used successfully to obtain an idea of the change of frequency which it may be possible to obtain in an installation. A further complication can be introduced to allow for the effect of a heavy spring instead of a weightless one, but results do not seem to warrant it at present.

3) Dynamic Spring Constant of the Ground, etc. The approximate spring constant (K) can be found from the following equation:

$$n_{gfm}^2 = \frac{1}{(2\pi)^2} \cdot \frac{K}{W_g + W_f + W_m} \cdot g$$

This is not necessarily the same as the Static Spring Constant, particularly for high frequencies, but this has not been examined yet.

In practice, it is very difficult to make much difference to n_{gfm} within normal economic and practical limits; if the applied frequency N, i.e. that of the machine, is equal or nearly equal to n_{gfm} and it cannot be readily altered, it may be necessary to add considerable damping to prevent a serious resonant build up.

IV. DAMPING COEFFICIENT AND LOSS OF ENERGY OUTWARDS.

A single impulse given to the foundation on the ground sets the combination into oscillation; of that energy entering the ground, much the greater part radiates outwards away from the centre by means of stress waves, the small remainder being totally absorbed by the local damping effect and changed into heat. During any initial soil compacting process the damping coefficient is up to 5 per cent, and it is much smaller when compaction has ceased. This damping is approximately proportional to the velocity and has therefore a logarithmic coefficient. The former and larger portion of the energy is not at once all radiated outwards because, if such were the case, any resonant build up would be impossible. The records of Andrews, Crockett, and Duff show that the outward dispersal of energy is high if the amplitude is also large, and may be not more than about thirty per cent for each cycle, thus demonstrating why a steady input can build up, and that such a build up cannot be very large. At very small amplitudes there is a reduction of only two or three per cent per cycle. Fig.9 show a typical vibrogram of the movement of a machine foundation set into motion by a blow, and its subsequent oscillations, wherin it is seen that the energy removal is very great for the large early oscillations, but is only slight for the later ones.

Furthermore, the previousely described increasing frequency accompanying the reduction of amplitude is very clear, which appears to be the normal non-linear behaviour.

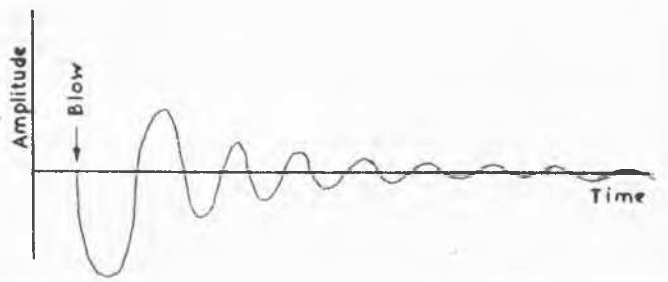


FIG.9

V. UNSTABLE HARMONIC FREQUENCIES.

Andrews, Crockett, and Duff have stated that ground oscillates in resonance most often at its "most stable self frequency", and that it can oscillate on a range of harmonics, and probably on sub-harmonics. These frequencies do not always all respond at the same time, presumably because change of moisture content, ambient temperature, and barometric pressure produce daily and hourly differing damping coefficients, thereby inhibiting the response of one or another of the range of possible frequencies from time to time. In complex oscillatory systems it is generally common for such variable inhibitary actions to be present; they do not reduce but actually prevent some modes of oscillation from ever starting. Typical daily frequency responses on one site over an observation period of ten weeks have been:--

	23.3	and 70	
	23.3 and 46.7		
10.5	and 46.7		and 93.3

Of these, 23.3 cycles per second was the self frequency which responded on most occasions and this is therefore called the "most stable self frequency", and it is clear that this is the one measured by other workers as in Table I. The above values show a harmonic range which at first sight is not pure; 10.5 is less than half of 23.3, but this is accounted for by the larger amplitude at the lower frequency, and non-linearity. From these and other site observations the Authors infer that the ground can respond to a frequency series something like that in Fig. 10, which bears a proper harmonic relation. The intermediate and smaller peaks have been included because on some sites one or another of them have been in evidence; they correspond to a "half harmonic".

VI. SELF FREQUENCIES IN OTHER DEGREES OF FREEDOM.

The self frequencies of the ground so far described have been those purely in the vertical translational degree, with impulses applied symmetrically over finite surfaces.

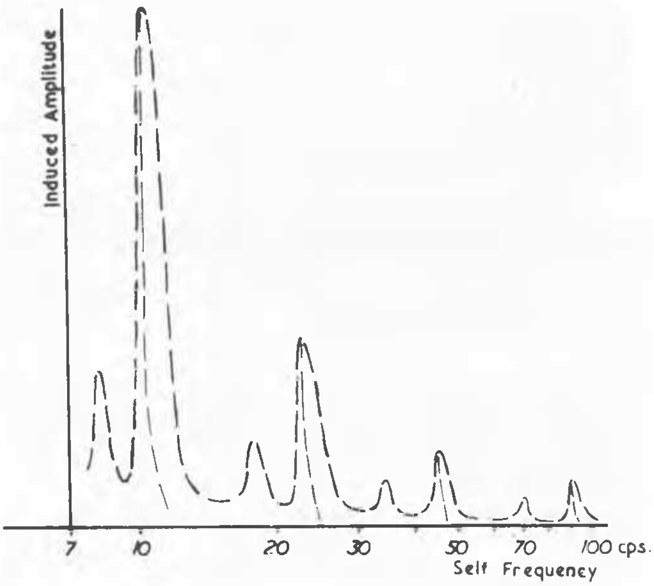


FIG.10

Similar studies have not yet been undertaken of the other five degrees of oscillational freedom - two in horizontal translation and three in rotation. It is unreasonable to assume that their principles would be widely different, and Andrews, Crockett, Duff, and Walker have obtained several records of beat notes between machines and ground in rotational modes of oscillation.

VII. FREQUENCIES OF SEVERAL UNDERLYING STRATA.

The self frequencies already discussed apply only where the thickness of the stratum containing the foundation is about three times the depth of the pressure part of the bulb, but it often happens that there are several strata in this depth and then the self frequency is more complicated. Each stratum can oscillate almost entirely by itself, particularly when the wavelength of the induced oscillation is of the same order as or greater than the thickness of the stratum itself. In such a case there would be a seperate frequency for each stratum in the foundation / ground system, probably in addition to the normal bulb frequency, though one of them would be likely to predominate.

VIII. CONCLUSIONS.

The Authors consider that it is now established that ground can oscillate as a mechanical entity having peculiar self frequencies dependent on its physical properties. They suggest that the mechanism can be termed the "oscillation of the bulb of pressure", and treated in practical design work as such, that it behaves in a non-linear manner, and that this mechanism provides a satisfactory framework for subsequent observation and research.

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