foundation upsets the old one's, resp. at what distance the new foundation has to be built without evoking dangerous tensions in the sphere of the old one. We know from practical experiences that piles too nearly driven cause density of soil and increase the carrying capacity. If there were piles driven again in the same sphere of density the resistence of the ground increases against this irruption. Through experiments on moulds the distance of the piles at which no influence of tension exists, can be fixed. Because of no "overlaping" (Uebergreifungs) tension a high degree of stability can be obtained. However the distance of piles can also be fixed, utilizing the density of soil which is caused by the "overlaping" tension. Because of the increased carrving capacity of ground material-expences can be reduced through retrenchment of the transverse section or through diminishing the number of piles.

Comparing the pictures of the two methods they nearly show the same results.

For the definition of the distribution of tension in grounds the two methods can equally be indicated as fit for use.

The use of these methods is of special value both at definition of foundation in groups and building erected on piles.

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VISCOUS FLOW TUBE MODEL

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SUMMARY

A model method is presented whereby variations of excess pore water pressures in a consolidating soil mass may be obtained in cases that include both expansion and compaction of soil mass. The various physical relationships between the prototype and model are discussed and a comparison of results obtained from a model and by formal mathematics are made. The basic idea of the model may also be used for the solution of frost penetration into the ground.

The viscous flow tube model is a hydraulic device which indicates approximately the variation of hydrostatic excess head of the pore water in a consolidating soil mass. The model is well suited for use, not only for illustrating such variations of head for the simple case of compression with one-dimensional flow, but also it may be used to obtain reasonable approximate solutions that are very difficult of solution by formal mathematical means. Various approximate mathematical solutions have been proposed 1) 2) but to the writer's knowledge a viscous flow tube model has never been used before for this purpose. Electrical models have been used for somewhat similar problems, such as heat flow and flow of compressible fluid flow, but the viscous flow tube model has a superiority over such electrical models because it can be adapted without difficulty to the cases where the same soil mass is subjected to expansion and then compression during the process of consolidation.

The upper sketch of figure 1 indicates a prototype foundation of width 2(H + L) centrally loaded for a width 2 H by an instantaneously applied uniform strip load. The compressible soil is assumed to be so highly stratified that all excess water forced out of the foundation by the load drains laterally. The prototype foundation is divided into equal segments, which are represented in the model by the vertical reservoir tubes. The amount of water forced out of the soil per unit load is simulated in the model by the amount of water drained out of the compression reservoir tubes per unit loss of head. The same conception holds for the soil expansion and the expansion reservoir tube. Thus the cross-sectional area of each tube represents the compressibility or expandibility of each segment in the prototype.

The water conductivity of the prototype soil per unit gradient is the permeability coefficient multiplied by the area normal to the direction of flow. In the model, the conductivity is simulated by the carrying capacity of each resistance tube per unit gradient. The hydrostatic excess head in the pore water of the prototype is represented in the model by its hydraulic grade line above the zero datum of the model.

The basic partial differential equation for consolidation at any given point in the prototype is 2):

$$\frac{k\,A\,\delta^2 u}{\gamma\,\delta x^2} = \frac{A\,a_v\,\delta u}{(1+e)\,\delta t} \qquad (1)$$

(net outflow rate of water) = (net rate of volume change of the soil mass)

where

k = coefficient of soil permeability in the horizontal direction,

A = unit cross-sectional area of prototype normal to the direction of flow,

γ = unit weight of water,

u = hydrostatic excess pore water pressure,

x = horizontal distance from the centre line of the prototype,

a_v = coefficient of soil compressibility

a_e = coefficient of soil expandibility

e = void ratio of soil,

t = time from application of load.

If the prototype soil is expanding because of an increase of hydrostatic excess pore pressure accompanied by a decrease in effective intergranular stresses, then equation (1) is valid if a_e is substituted for a_v. Average values of a_v and a_e may to obtained from laboratory consolidation test results.

For the model, the physical ideas of equation (1) may be expressed as follows:

$$\left[\left(\frac{\Delta h_1}{l'}\right)_{inflow} - \left(\frac{\Delta h_2}{l'}\right)_{outflow}\right] q = \alpha_c \frac{d\,h}{d\,t} \qquad (2)$$

(difference in flow rate to and from a reservoir tube) (outflow rate from a reservoir tube)

where

- l = unity,
- l' = length of each capillary resistance tube (all equal),
- Δh = difference in hydraulic head between two adjacent reservoir tubes,
- h = hydraulic head at reservoir tube,
- q = discharge rate of capillary resistance tube under unit gradient,
- α_c = cross-sectional area of compression reservoir tube,
- α_e = cross-sectional area of expansion reservoir tube,

The value α_c is used in the compression portion of the model, while α_e is used in the expansion portion. The following relationship between the prototype and model holds:

$$\alpha_c/\alpha_e = a_v/a_e$$

Equations (1) and (2) may be rewritten as

$$\frac{\partial^2 u}{\partial x^2} = \left[\frac{\gamma a_v}{k\,(1 + e)}\right] \frac{\partial u}{\partial t} \qquad (3)$$

$$\left[\left(\frac{\Delta h1}{l'}\right)_{inflow} - \left(\frac{\Delta h2}{l'}\right)_{outflow}\right] \frac{1}{l'} = \left[\frac{\alpha_c}{ql'}\right] \frac{d\,h}{d\,t} \qquad (4)$$

so that the right- and left-hand parts of the equations have the same physical dimensions. The prototype time factor 2) is

$$T_p = \left[\frac{k\,(1 + e)}{\gamma a_v}\right] \frac{t_p}{H^2} \qquad (5)$$

and the model time factor is

$$T_m = \left(\frac{ql'}{\alpha_c}\right) \frac{t_m}{(nl')^2} \qquad (6)$$

where n is the number of model compression reservoir tubes and nl' in the model corresponds to H in the prototype. Thus the time t_p required to obtain a certain per cent consolidation at a point in the prototype may be related to the time of the model t_m as follows:

$$t_p = t_m \frac{q\;\gamma\,a_v\,H^2}{\alpha_c\,n^2\;l'k\,(1 + e)} \qquad (7)$$

The compression reservoir tubes should extend to the grade line of the initial hydrostatic excess head of the model. If this initial head is not uniform then the reservoir tubes should be extended by expansion tubes to permit simulation of possible prototype expansion. A packing should be provided between the compression tube and the base of the expansion tube and fastened to the expansion tube, for the reason that if the rate of inflow becomes greater than outflow an expanding condition is indicated. To account for this in the model, the base of the expansion tube with its packing should be lowered to the liquid surface in the compression reservoir tube.

A gradual rate of increasing compressing load in the prototype may be simulated in the model by feeding water into the compression reservoir tubes at a rate proportioned by the relationship between prototype and model times t_p and t_m.

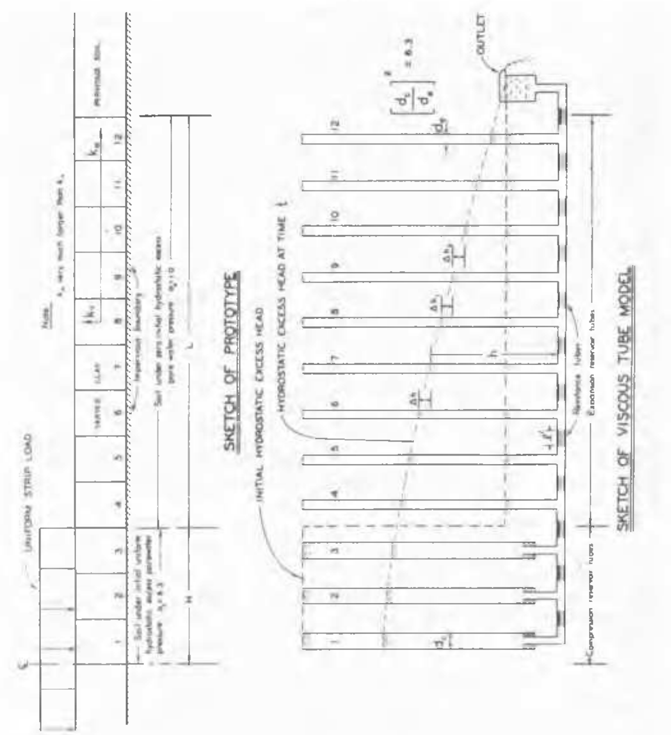
Consolidation in the prototype by two-dimensional flow can be accounted for in the model by a network of interconnected reservoir tubes. Each tube will represent a square in the prototype soil profile. In such a case the resistance tubes simulating resistance to vertical flow will be different from the horizontal flow resistance tubes to simulate differences in vertical and horizontal permeability.

For large temperature changes, corrections must be made to the time intervals between readings. The corrected time interval Δt_c is obtained from the uncorrected time interval Δt as follows:

$$\Delta t_c = \Delta t \left(\frac{\eta_c}{\eta}\right) \qquad (8)$$

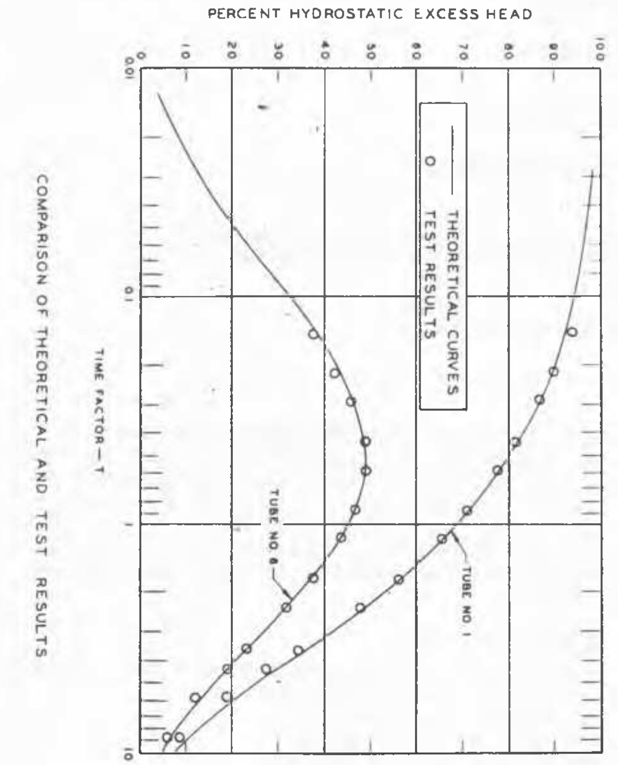
where η_c and η are the viscosity values at the standard temperature ϕ_c and average temperature ϕ of the time interval, respectively.

The model shown in figure 1 was constructed and tested in the Soils Laboratory of the former Providence District, Corps of Engineers, under the direction of Mr. K. Linell, based upon ideas developed by the author. The reservoir tubes representing the compressing segments had a cross-sectional area 6.3 times as large as that for the tubes representing the segments subject to expansion as the excess water flows out of the compression zone to the side drain-



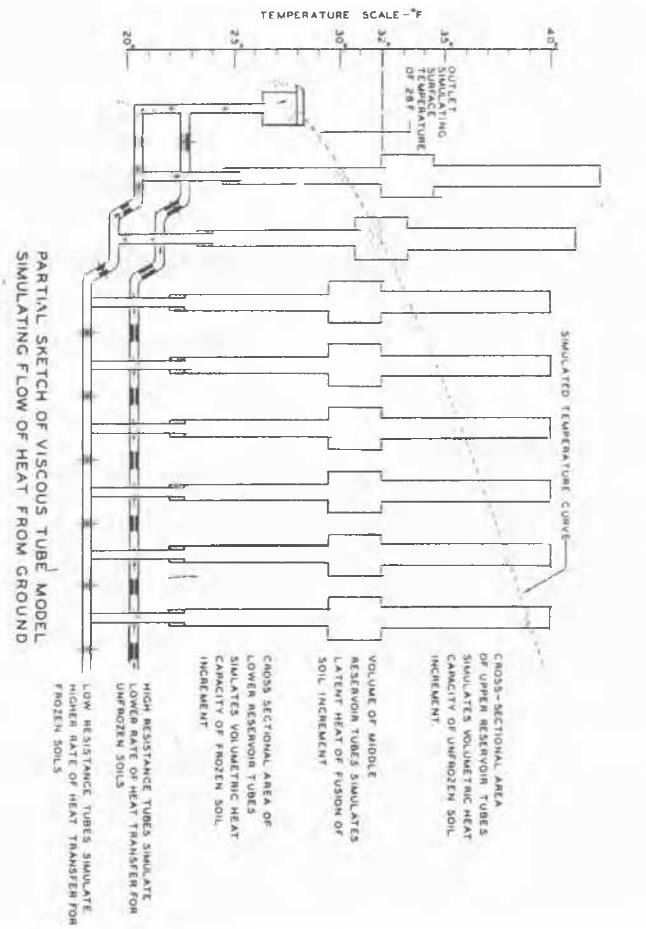
Sketch of viscous tube model

FIG.1



Comparison of theoretical and test results.

FIG.2



Partial sketch of viscous tube model simulating flow of heat from ground.

FIG.3

age faces. Thus the ratio α_c/α_e = 6.3 represents a_v/a_e = 6.3 in the prototype soil.

Data regarding unit discharge rate and length of resistance tubes for the model noted above were not determined. Therefore the test data shown on figure 2 were related to theoretical data by equating the corrected test time when tube 1 was at 50 per cent hydrostatic excess head to the theoretical time factor for the equivalent position in the prototype to obtain a time versus time factor equation of t (in min.) = 0.205 T.

For any given case, the greater the number of reservoir tubes used in the model, the more closely will the model results approach the theoretical prototype results. For the case of consolidation by two-dimensional flow, some assumptions must be made as to distribution of hydrostatic excess head in the soil. Also, assumptions have to be made as to values of a_v and a_e. However, it is believed that the viscous flow model offers a means of obtaining approximate results for cases which can not be obtained by any other means except, perhaps, by the laborious solutions of simultaneous difference equations.

The viscous flow tube model can also be used to study the penetration of frost into the ground due to variable surface temperatures. The cross-sectional area of the upper and lower reservoir tubes, as shown in figure 3, simulate the volumetric heat capacity of thawed and frozen soils, respectively. The large middle tube simulates the latent heat of fusion of the soil moisture. When the water in a reservoir reaches the elevation representing the freezing temperature, it is maintained there until all the water has flowed out of or into the central tube to simulate freezing or thawing. This may be done by constantly adjusting the position of the tube. The difference in heat transmission of frozen and thawed soils is accounted for by two sets of different resistance tubes; one set being used when the reservoir tubes are in a simulated thawed state, the other when in the simulated frozen state. A surface resistance to heat flow or emissivity may be simulated by an additional resistance tube of proper resistance.

In conclusion, it appears that the viscous flow model offers a means of readily obtaining solutions to problems regarding variation of excess pore water pressures that are very difficult to obtain by formal mathematics. The example given indicates that model data may be obtained that are well within the range of practical accuracy.

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