V a 3

DEFINITION OF STABILITY AGAINST OVERTURNING OF RETAINING WALLS

ANDRÉ PADUART

Ingénieur A.I.Br., Agrégé de l'Enseignement Supérieur, Brussels

1. THE IDEA OF SECURITY AGAINST OVERTURNING.

Those projects which concern the building of retaining walls, indicate the necessity of studying stability under the threefold aspects of overturning, sliding and elastic stresses - While the two last aspects do not give rise to any special difficulty, and have become accepted criteria of good practice in construction, resistance to overturning raises more problems, and we have often noticed that the definition of security against overturning lacked precision, and could be interpreted in different ways by different designers.

The study of resistance to overturning is, however, as necessary as that of resistance to sliding, or as calculation of stresses.

The opinion has sometimes been expressed that the security against overturning is sufficient, provided that the stresses in the structure and the bearing pressure exerted on the ground through the foundations, do not exceed the permitted limits. This opinion is false, since it amounts to confusing a condition of static equilibrium with the strength of the materials used. These two conditions are essentially different, and it is possible for a retaining wall to turn over even if it is built on the hardest rock.

In what follows, we intend to analyse the manner in which security against overturning is usually analysed, and to find a more precise definition of it.

2. USUAL DEFINITION OF THE FACTOR OF SAFETY AGAINST OVERTURNING.

When one studies the resistance to overturning of a retaining wall, it is not enough to find out whether the wall will turn over or not. It is essential to see how far the wall is from overturning. That "how far" explains just what is meant by the idea of security. Usually, all the couples which act on a retaining wall are divided into a stabilizing moment and an overturning moment. The security varies directly as the quotient of those two moments, and it seems thus logical to adopt as a factor of safety against overturning, the dimensionless number represented by the quotient of the stabilizing moment and the overturning moment.

This definition seems quite simple, but we shall soon see that it is ambiguous end that it can lead, for a completely defined retaining wall, to different values of the factor of safety against overturning.

Indeed, this definition introduces two other definitions which are much more complicated: what is a stabilizing moment, and what is an overturning moment ? We have consequently defined an idea by introducing two new ideas, and we have not made the question any easier.

3. STABILIZING MOMENT AND OVERTURNING MOMENT.

One method of calculation consists in resolving all the forces into a vertical component and a horizontal component; the moment of all the vertical components about the point of overturning constitutes the stabilizing moment; the moment of all the horizontal components about the same point constitutes the overturning moment. But this definition is not logical. Indeed, the values which it gives depend essentially on the point of resolution. Thus, in the example given in fig. I, if we resolve the earth-pressure Q at point A, situated vertically above O, we find that the stabilizing moment due to the earth is zero, and if we resolve at point B situated at the same level as O, we find that the overturning moment is zero. This last case thus gives us an infinite factor of safety.

A second method of proceeding improves on the preceding method, as it stipulates that we must resolve each force at the point where it is applied to the structure. The drawback just mentioned is consequently eliminated. Yet this definition still appears artificial, and indeed, the result found can be altered if a group of forces is replaced by their resultant or by another group of components.

The factor of safety of the wall represented in fig. 2 will have three different

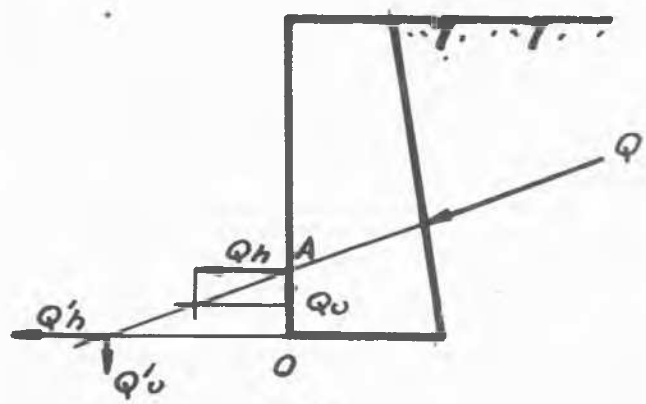

FIG.1

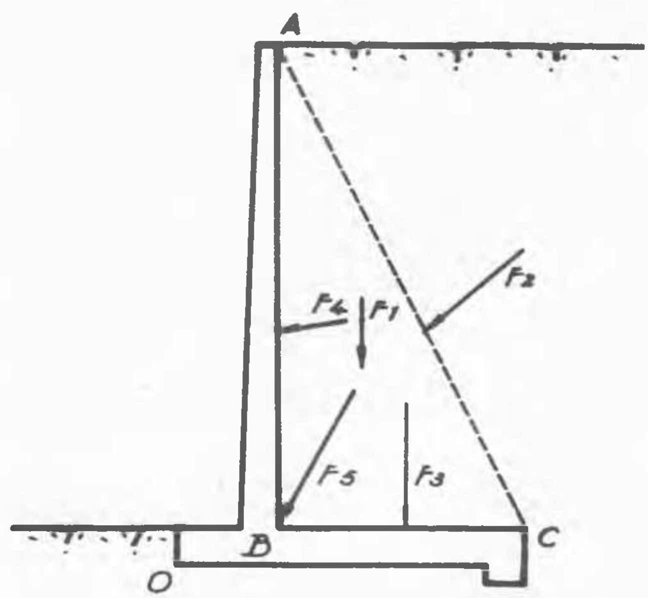

FIG.2

values according as the force due to the earth is expressed by the components F_1 and F_2 (hypothesis in which the earth-mass ABC is part of the wall) or by the components F_3 and F_4 or the resultant F_5. Thus we are left with yet another inexactitude and this method must be abandoned.

According to a third method of preceeding, we do not resolve the forces in vertical and horizontal directions, but calculate directly the moment of each one of them with respect to the overturning point. In this case, the stabilizing moment is equal to the moment of the stabilizing forces; the overturning moment is equal to the moment of the overturning forces. This conception only moves the difficulty further back, for we are led to inquire what a stabilizing force and an overturning force are. This question seems paradoxical, and one could answer that they are respectively a force which tends to produce it. The example given below shows that this answer is insufficient.

Let us consider a quay-wall, the stability of which we are to investigate, and let us begin by disregarding the upward pressure acting on the wall. We can then find the stabilizing moment M_S due to the weight of the wall, and the overturning moment M_o due to earth-pressure; from this we can deduce the value of the factor of safety μ as it has just been defined.

Let us now calculate the moment M_e due to upward pressure on the foundations. It is here that we must make a distinction between a stabilizing force and an overturning force. We may indeed affirm that the moment M_e tends to overturn the wall and is thus an overturning moment, but we do not make a mistake by saying that the effect of the upward pressure is to counteract the weight of the wall and so to reduce the stabilizing moment.

Thus there arise two possible conceptions, each as logical as the other, which lead to two different values of the factor of safety μ

$$\mu_1 = \frac{M_S}{M_o + M_e} \quad \text{and} \quad \mu_2 = \frac{M_S - M_e}{M_o}$$

Knowing that the value of a fraction approaches unity when one adds to both numerator and denominator the same positive quantity, we can see that if the wall is stable, μ_1 is smaller than μ_2 .

We will examine one more example. Fig. 3 represents a retaining wall, the face of which, in contact with the earth, presents two different inclinations, so that the force Q_{AB} on face AB passes to the right of the point of rotation O, while the force Q_{BC} on face BC passes above that point.

The results of the calculations will be different according to whether Q_{AB} is considered as a stabilizing force or as a component of the complete overturning force Q.

If we assume moreover that the wall may be used as a trackway for a crane producing vertical and horizontal reactions, the question once more will be to decide whether this effect must be considered as a stabilizing or an overturning influence.

4. DEFECT OF THE USUAL DEFINITION OF THE FACTOR OF SAFETY.

We have just seen that there exists no rigorous definition of the stabilizing or overturning moments. One might perhaps object that it would be possible to classify, once and for all, the effects of overturning and stabilizing forces, so as to exclude all ambiguity. The answer to this is that such a classification

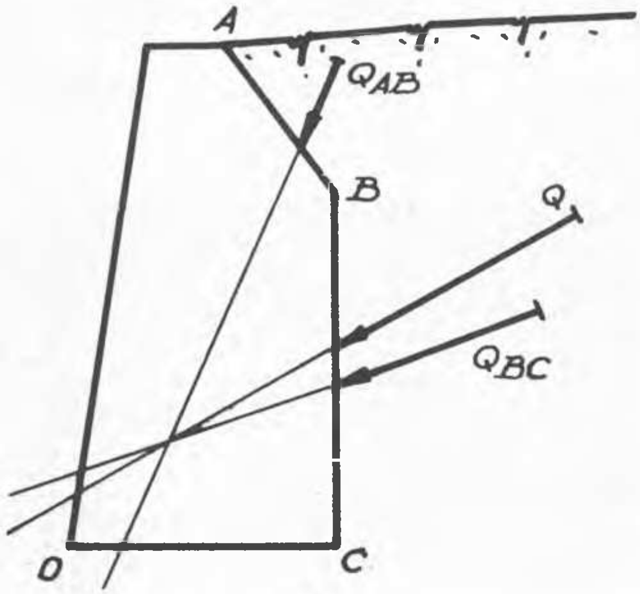

FIG.3

would always have a conventional character and would lack generality, for we cannot foresee the nature of all the forces which a retaining wall may have to resist.

Moreover, such a method of procedure could even lead, in some cases to a value of the factor μ which would in no way represent the idea of overturning stability as it must be conceived.

Thus if, in the last example we have just considered, the negative overturning moment of the crane were equal in absolute value to the overturning moment due to the pressure of the earth, the total overturning moment would be zero, and we should have an infinite value for the factor μ. Yet one could not deduce from this that the wall could never turn over when the crane rests on it.

It would seem that, despite all the hypothesis and conventions imaginable, it will never be able to give to the factor of safety μ a logical and unambiguous definition.

This arises from the fact that this definition contains a fundamental error of principle and stands in contradiction with the most elementary data of the exact sciences. Indeed, the factor μ is defined by the relationship of two quantities, some of the constituents of which can be transformed from one to the other, thanks to a mere change of sign. In other words though we may write the relations:

$$M_S - (M_o + M') = (M_S - M') - M_o$$

it would be false to deduce from it:

$$\frac{M_S}{M_o + M'} = \frac{M_S - M'}{M_o}$$

a deduction which is however implicit in the definition of the factor μ .

It should be noted that other factors of safety, for instance those that apply to the resistance of a beam to tension or bending, to the buckling of a compressed beam or to the sliding of a retaining wall, do not present this defect, as it is not possible to transpose any term by a mere change of sign, from numerator to denominator or inversely.

Indeed the factors of safety applicable

to the resistance of a beam are expressed by the relationship of a function depending only on the mechanical properties of that beam,with another function depending only on the external forces which are applied to it. The factor of safety for the sliding of a wall takes account of all the components of all the forces in two perpendicular directions, and this differentiation is something other than a mere change of sign.

From what goes before, it follows that, in spite of the analogy which the definition of the factor μ presents with that of other factors of safety, that factor is unable to represent exactly the "how far" we have been talking of above.

5. NORMAL RESERVE OF STABILITY.

The only operation that we can perform with the stabilizing and overturning moments, without infringing the laws of mathematics, is an algebraic addition. By this procedure, we obtain the net moment representing in absolute value the couple necessary to cause the wall inquestion to rotate, and which consequently defines the normal stability reserve:

$$M_n = M_S - (M_o)$$

Since it is not dimensionless, this normal stability reserve is not as suitable as the abstract factor μ in defining the security of a structure. One might consider dividing it by a reference moment depending on the principal characteristics of the wall, as for example the product of its weight of the pressure of the earth and its height, but it is easy to see that there is no definition that could be considered as general and applicable in every case.

Nevertheless, the normal stability reserve lends itself quite well to comparing two walls performing a similar task and it could furnish a useful governing clause when drawing up a specification for the building of a structure with a definite task of earth retaining.

6. CRITICAL RESERVE OF STABILITY.

Because of the inadequacy of the idea of normal stability reserve, we must introduce a new element allowing us to estimate the security against overturning of any retaining wall. We shall find this element in the idea of security against failure, by analogy with some theories concerning reinforced concrete.

We propose to apply the factors of safety to the forces themselves and to give to all the elements occurring in the stability calculation more unfavourable values than could conceivably occur.

The designer should himself determine the conditions to adopt in order to place himself in an extreme position.

Those conditions will have to be chosen with all the more care in so far as information about their exact value is lacking.

The choice may also be influenced by the seriousness of the consequences of the overturning of the wall.

Having defined the calculation conditions, we can determine the net effective moment, or the algebraic sum of the stabilizing and overturning moments. If the result M_c, that we shall call "critical reserve of stability" is positive, we may deduce that the security of the structure is normal. The consideration of the critical reserve of stability offers interesting advantages compared with that of the usual factor of safety.

Indeed we have seen that the calculation of the stability reserve satisfies mathematical laws, and is always unambiguous, whatever the nature of the forces present may be. Moreover a few extreme cases may finally be considered and the critical reserve calculated,so that we can study the effect of introducing certain forces upon the security of the structure.

These advantages will be brought out by the few examples which end this paper.

7. EXAMPLES.

1. Quay-wall. Our first example is taken from the "Cours de Stabilité"by Professor G. Magnel, and concerns the quay-wall represented in fig 4.

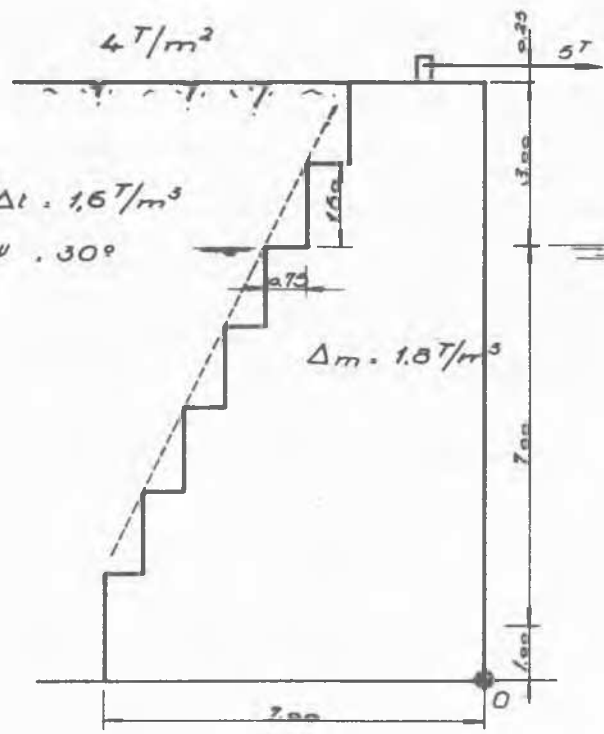

FIG.4

Considering as stabilizing agents; the weight of the wall, the vertical component of the earth-pressure, the pressure of the water in the basin; and as overturning agents: the pull on the bollards, the upward pressure on the foundations and the horizontal component of the earth-pressure and the water it contains, Prof. Magnel finds a factor of safety:

$$\mu = \frac{776}{504} = 1,53$$

But if we work through the calculation again, considering the resultant of the earth pressures as an overturning force, we find a different factor:

$$\mu_1 = \frac{368}{96} = 3,83$$

Furthermore we could cancel out the horizontal components of the water pressure on the opposing faces of the wall, since those forces are equal, and under these conditions we should find:

$$\mu_2 = \frac{282}{10} = 28,2$$

Finally if we take the upward pressure on the foundations as reducing the stabilizing moment, we obtain:

$$\mu_3 = \frac{86}{-186}$$

which has no longer any meaning.

Considering the diversity of the results found by different approaches to the calculations, one can hardly judge the stability of the wall on the basis of the factor 1,53 and it would be well to consider what that factor means.

Concerning the normal stability reserve, we have a completely satisfactory value:
M_n = 776-504 = 368-96 = 282-10 = 86+186 =272 Tm.

In order to complete our information on the security of the wall, we can calculate the critical stability reserve by simultaneously considering the following factors:
the load on the coping: 4 T/m² x 1,5 = 6 T/m²
the pull on the bollards: 5 T/m x 2 = 10 T/m
the density of the masonry: 1,7 T/m³
the density of the dry earth: 1,9 T/m³
the angle of repose of the earth: 30° x 0,5 = 15°
lowering the water level in front of the wall: 1 m.
we find in this case: M_c = 899,7 Tm - 817 Tm = 82,7 Tm > 0
The stability of the wall against rotation is thus normal.

2. Overhanging retaining wall.

The moments with respect to point O, of the different forces acting upon the wall shown in the fig. 5 are as follows:
Weight of the wall : M_P = 41,9 Tm
Earth pressure = M_T = - 17,7 Tm
Pressure due to surcharged earth : M_S=-4,6Tm

From this we deduce a value of the factor of safety against overturning of:

$$\mu = \frac{41,9}{22,3} = 1,87$$

The normal stability reserve amounts to 19,6 Tm, which means that to make the wall overturn a horizontal force must be applied at the top of it equal to 2,9 T/m.

We will now calculate the critical reserve by considering the following factors:
Load on the coping: 1 T/m² x 2 = 2 T/m²
Density of masonry: 1,7 T/m³
Density of earth: 1,9 T/m³
Angle of repose: 30° x 0,5 = 15°

We can assume moreover that, owing to inadequate drainage, the water level rises behind the wall, to 2 m. above the base of the wall; we then obtain the following moments:
Due to weight of the wall: M_P = 39,4 Tm.
earth pressure: M_T = - 49,1 Tm.
water pressure: M_E = - 6,0 Tm.
surcharged earth: M_S = - 22,0 Tm.

The value of the critical stability reserve deduced from this is equal to - 37,7 Tm. This value being negative, we can deduce that the wall is insecure. It is interesting to notice that the comparison of the factors of safety μ (1,53 and 1,87) would have led one to believe that the stability of the overhanging wall is greater than that of the quay wall of the first example, which belief disagrees with the conclusion drawn from the comparison of the critical stability reserve.

3. Abutment of Cantilever Bridge.

We shall now deal with a more complex example, in order to show that the ideas which have just been defined may be applied to all structures, and that their calculation does not lead to any ambiguity. Let us consider the concrete abutment of a Cantilever bridge represented in fig. 6 - The reactions Rp and Rs due to the bridge correspond respectively to the dead load and live load.

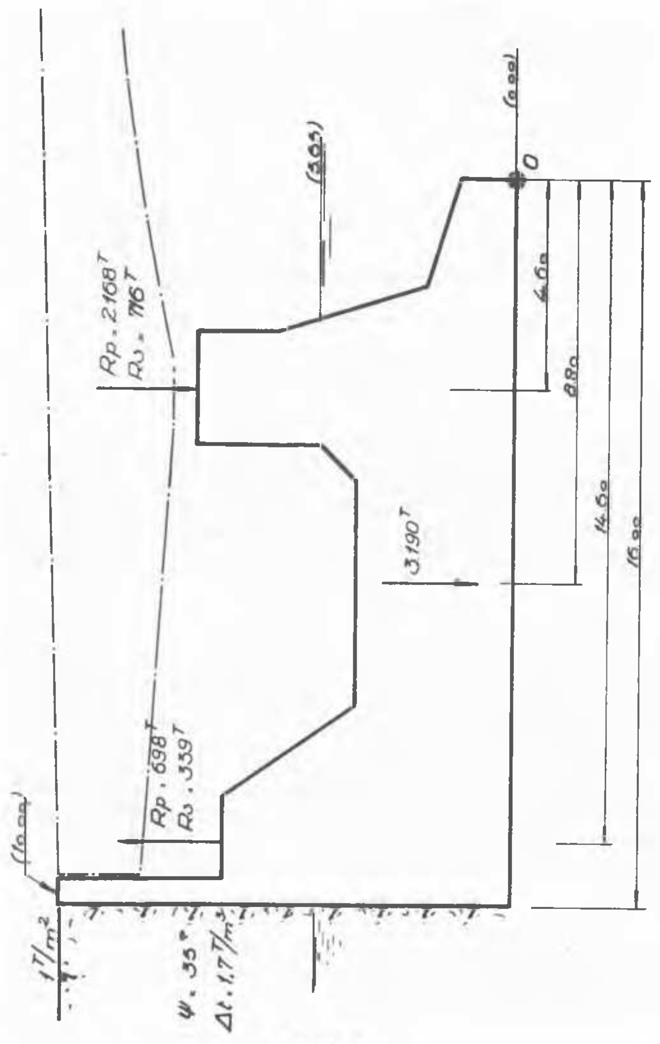

FIG.6

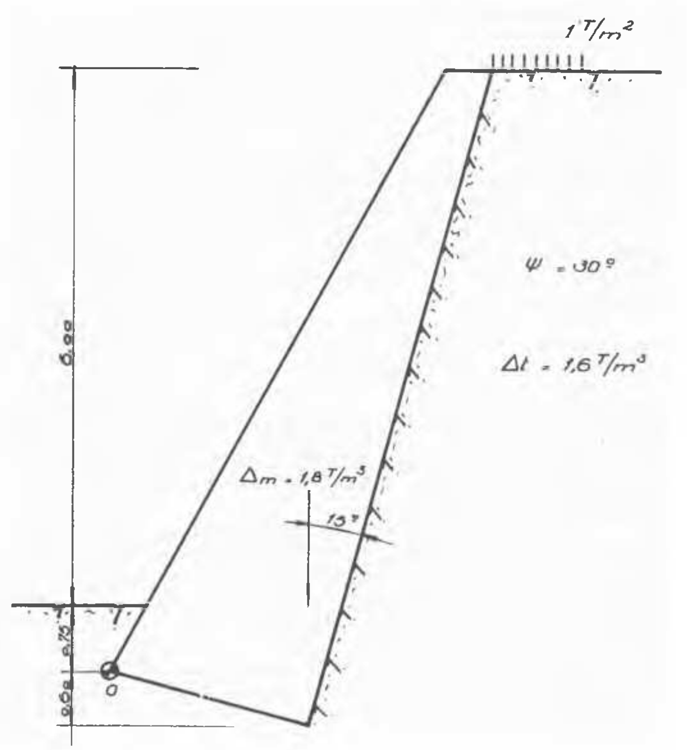

FIG.5

The abutment has a length of 17 meters and a weight of 3190 T.

The normal stability reserve is obtained by adding the following moments:

Due to dead load of the bridge:	-	220 Tm
live load on the bridge:	-	1655 Tm
weight of the abutment :		28390 Tm
upward pressure on foundations :	-	12290 Tm
earth pressure :	-	1370 Tm

We obtain from this: M_n = + 12.855 Tm.

We now assume that the critical situation corresponds to the following conditions:

Dead load reactions of the bridge 10% increased
Live load reactions of the bridge multiplied by 2
Weight of the abutment reduced by 10%
Superimposed load on coping multiplied by 2
Angle of repose: 15^o
Density of dry earth: 1,9 T/m^3
Impact factor on the bridge: 1/7
Rising of the water level in the earth: 50 cm

The moments acting on the wall become:

Due to dead load reactions of the bridge:	-	240 Tm.
live load reactions	-	3310 Tm.
weight of the abutment	-	25550 Tm.
upward pressure on foundations	-	13330 Tm.
earth and water pressure	-	4410 Tm.
impact	-	780 Tm.

It follows that the critical stability reserve is equal to M_c = 3.480 Tm. which is a satisfactory value.

Let us note in conclusion that if we had based our opinion on the value of the factor of safety μ, we should have found a value of

$$\mu_1 = \frac{15.880}{3.025} = 5{,}25$$

when taking as sole overturning factors, the live load reactions of the bridge and the earth pressure, and a value of

$$\mu_2 = \frac{28.390}{15.535} = 1{,}83$$

when adding to the group of overturning factors the dead load reactions of the bridge and the upward pressure on the foundations.

-o-o-o-o-o-o-o-o-

Va 4 REDUCTION OF LATERAL COHESIVE SOIL PRESSURE ON QUAYWALLS BY USE OF SAND DIKES

HARRIS EPSTEIN

Head Designing Engineer, Waterfront Structures, Bureau of Yards and Docks, U.S. Navy Department, Washington, D. C.

SUMMARY

The paper introduces, for economical reasons, a borrowed fill dike method for construction of quaywalls when the available fill material on the site is of a poor cohesive unconsolidated nature. It analyzes theoretically dikes of various shapes determining their variation in effectiveness with their size as well as the sizes required for full effectiveness. Finally the paper gives the results fo several tests on such dikes performed at Princeton University for the Bureau of Yards and Docks, U.S. Navy Department, and compares same with the analytical results.

ECONOMICAL CONSIDERATIONS

The sheet piles and the backfill, which constitute the main cost of quaywall construction, are economically interdependent. Heavier and longer sheet piles will permit the use of an inferior backfill and a better backfill will result in shorter and lighter sheet piles. When the material on the site of a proposed quaywall is of a poor unconsolidated nature, it may be economical at times, in order to save on the cost of sheet piles, to import from a distance at a greater cost a good sandy granular material for the fill. In some cases it may even pay to dredge off and dispose of the poor local material and substitute for it a good borrowed fill. The use of the borrowed fill method depends on the unit cost and the quantity of the borrowed material required. It is not necessary however, to have the entire fill made of imported material; enough good material may be placed to form a dike against the quaywall and the remainder of the fill may be pumped in hydraulically from the local inferior material, the dike being analagous to an earth dam, where an earth fill is depended upon to retain the full water pressure. However, the dike dimensions can be made much smaller than the dimensions of a similar earth dam, where in addition to retaining the full water pressure-entirely absent in the case of the quaywall dike, such considerations as imperviousness to water seepage, accessibility to land construction equipment etc. enter into the determination of the top berm.

The borrowed fill quantity required is determined by the size and dimensions of the dike necessary to render same fully effective against a hydraulic clay fill. The full effectiveness of the dike is said to be attained when the resultant pressure against the wall is the same as for a level fill of the dike material.

The reduction in the horizontal pressure from a hydraulically placed clay fill by means of an intervening sand dike can be conceived as resulting from the three following causes:

a) The drainage afforded by the continuous sand dike.
b) The increased friction of the sand against the sheet piles as compared to the friction of the fluid clay.
c) The structural strength or the shearing resistance of the sand dike.

The size of the sand dike required depends largely on the relative contribution of each of the above 3 causes to the reduction of fluid